



Consumption patterns of public water supply using linear programming techniques

Ibrahim ABDULLAHI ¹, Ismail ABUBAKAR ², Iyadunni Ndububa OLUFUNMILAYO ³,
Lawal Batagarawa RABI'A ⁴, Adebola Ajibike MA'ARUF ⁵ and Tsoho USMAN ⁶

^{1,4,6} Department of civil Engineering Nigerian Defence Academy, Kaduna State, Nigeria

³ Department of Civil Engineering Federal University Oye-Ekiti, Ekiti State, Nigeria

^{2,5} Departments of Water and Environmental Engineering Ahmadu Bello University Zaria,
Kaduna State, Nigeria

Emails: ¹ aibrahim@nda.edu.ng, ² abuismail1@yahoo.com, ⁴ OINdubaba@yahoo.com,
⁴ kilishir@yahoo.com, ⁵ idowu_ajibike@yahoo.com, ⁶ udtsoho@yahoo.com

Corresponding author phone: +2348026906930. +23407062407917

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Abstract

This report presents the outcome of a survey of water use pattern in Kaduna metropolis, Kaduna State Nigeria. The water use pattern study extends to various water consumers such as residential, institutional, commercial and industrial.

The study was conducted by distribution of questionnaires and oral interview. The questionnaires contain questions related to water demand and uses. The research work presents the outcome of a linear Programming Techniques Model and sampling techniques of water use pattern carried out. The study was conducted by the application of linear Programming Approach and sampling techniques. The sampling technique contains questions related to water demand and uses. The information obtained were analysed and the result show that the residential water consumption varies from 46l/c/d to 115l/c/d with lower values for high and medium density areas, while higher value is for low density areas because of high standard of living. The daily values for commercial, industries and Public facilities sampled within the metropolis were; 29,915.854, 366,666.666 and 29, 308.804 litres respectively. A Water consumption

model was developed using Linear programming modelling technique to the allocation of 176,360,000 L of water produced daily by the State Water Corporation and supplied to the four sectors of the town. The result shows that the model gave an optimal solution of 275, 170, 410 L to the four sectors with an allocation of 150,441,410 L to residential sector, 101,325,000 L to commercial sector, 223,040,000 L to public sector and 1,100,000L to industrial sector. It is concluded that the water supply by the Water cooperation is less than the water demanded.

Keywords

Allocation; Management; Linear programming; Sector; Utilized

Introduction

Water consumption varies from one area to another depending on socioeconomic standard of the people, the level of education and development, nature of prevailing climate, the hygienic characteristics of the people, level of provision of sanitation facilities and general sanitary habits of the people [1]. However, it generally agreed that a minimum of 50 l/c/d is needed by an individual to satisfy his basic need irrespective of socio-cultural background [2].

Kaduna metropolis, like many other cities in Nigeria does not have accurate water demand figures that could be used for the design and improvement of the water supply system.

For effective service delivery, many studies have been carried out in the area of water demand management in developing countries. In a study conducted by [3] in Ilorin, Nigeria, the basic per capita consumption for planned residential areas was found to be 89 litres per day. This demand is however suppressed because of intermittent pumping [4]. [3] Proposed a probable minimum demand of between 92 to 96 liters per capita per day depending on the 9 liters or 10 liter's capacity water flush toilets.

This survey and evaluation study on water use pattern in Kaduna metropolis is to establish water consumption rates for the residential, institutional, commercial and industrial consumers. The results will be useful for the purpose of upgrading the water supply scheme, planning expansion of water treatment plant, design of pumping stations, and the design of storage and distribution system for Kaduna metropolis.



The purpose of this research work is too carried out the surveyed of the water consumption pattern for both Residential, Public, Commercial and Industrial sectors in Kaduna metropolis using random sampling techniques and Questionnaire method. From the method adopted in the research, the consumption pattern of residential water in Kaduna metropolis indicated that residential water use in low density areas ranged between 70.25l/c/d to 115.22l/c/d with an average value of 92.74l/c/d. The residential water use for medium density areas ranged from 55.50l/c/d to 66.94l/c/d, with an average of 61.22l/c/d. Also the residential water use in high density areas was between 46.07l/c/d and 96.37l/c/d with an average value of 71.22l/c/d. The differences that exist in the water use rates among the study areas are as result of variation in socio economic level, standard of living, age of community, population, availability of water and level of sanitation awareness. The daily consumption pattern for all commercial, industrial and institutional facilities sampled within the metropolis were 29,915.854, 366,666.6 and 29,308.804 litters respectively.

Material and method

Study area

The map of Kaduna metropolis is obtained indicating the various areas of study. Kaduna metropolis consists of four Local Government Areas (LGAs), namely: Kaduna North, Kaduna South, Igabi and Chukun which comprises the study area as showed in Figure 1 above and is situated at the centre of the northern Nigeria. It is cited on gently sloping ground on the west bank of River Kaduna, which serves as a good source of water [5]. It has a population of about 767,306 only for Kaduna North and South Local Government. While Igabi and Chukun Local Govt. Areas have a population of about 803,025. However there are some parts of these local governments which are not part of the metropolitan.

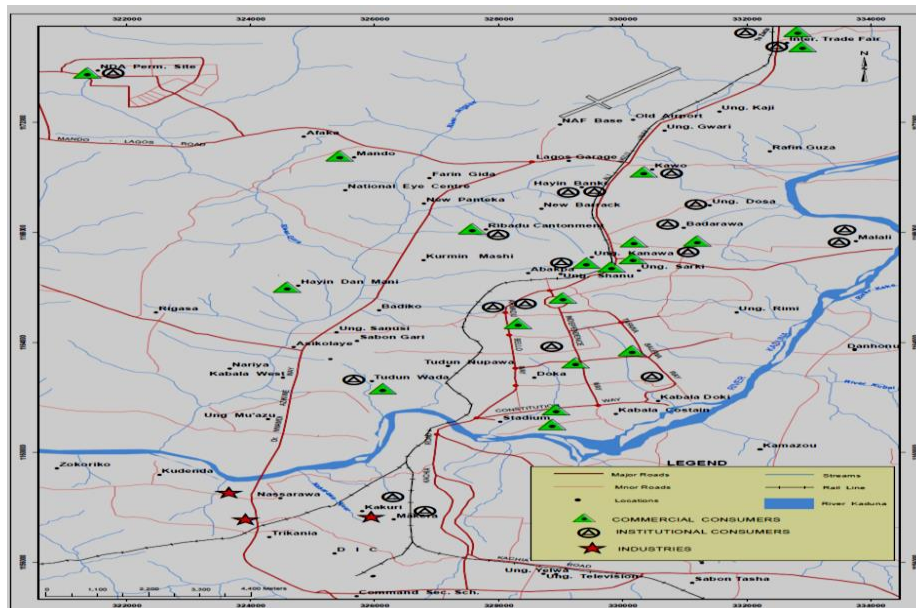


Figure 1. Map of Kaduna metropolis showing various areas of study

All the four Local Government Areas constitute a total sum of about 1,570,331 including some few places in Igabi and Chukun Local Government Areas, but has now grown to 465,323 for Kaduna North, 514,023 for Kaduna South 549,789 for Igabi and 475,147 for Chukun making a total number of 2,004,282 by 2016 when the 2006 figure was projected with 2.47% annual growth rate recommended in Software world for population growth (2003).

The water supply intake is situated at Malali village in Kaduna north where treatment plant was built. The first treatment plant (old plant) was established in 1972, while the second plant (new plant) was established in April 1987. The second plant was established to supplement the first plan; the old treatment works plant has a capacity of 90million litter per day while the new treatment works has a capacity of 150million litter per day.

The total capacity of the treatment works is 240 million litter per day. On average a total of about 180 million litter per day is expected to be pumped everyday out of which 85 percent is supplied per day, in addition to these two treatment plans another treatment plant is located in Kaduna South which has a capacity of 27million litter per day it was commissioned in 1927. All these three treatment plans (Kaduna North new water works, Kaduna North old water work and Kaduna South water works) has the production capacity of producing 105.64, 54.86 and 15.86million litter per day of treated eye water supply to the metropolis respectively, making total sum of 176.36million litter per day as the expected total production of treated water supply



to the metropolis. Kangimi dam was also built to supplement the main Kaduna treatment plant which is located at about 30Km away from Kaduna city; it has a capacity of 58.5 mm³. Kaduna like other urban areas in the developing countries has a number of sectors. Four of such sector namely Residential, Commercial, Industrial and the Public institutions are prominent in Kaduna.

The residential structure of Kaduna show 48 wards with Kakuri/Makera being the largest ward followed by Rigasa, Nasarawa, Tudun-Wada, Doka respectively while, Janruwa being the least, in terms of population according to Kaduna State Census 91. Also there are well established commercial concerns; among them are Hotel establishments, car washing centres, laundry and dry cleaning services, mini and supermarkets, banks, etc.

The public institutions sector has Fire Fighting establishments, army and police barracks, public hospitals and schools among its prominent establishments and finally the industrial sector has a wide range of industries including the processing and the manufacturing ones. Two sampling techniques were utilized, namely the stratified and simple random sampling. For our stratified sampling, each sector was divided into strata in which every stratum was allocated.

Data collection

Primary data were, therefore, collected from Kaduna State Water Board, Kaduna Bottling Company and Kaduna Petrochemical Refinery Company (NNPC) for the four sectors and also field work were conducted while secondary data were obtained from Published materials. The determination of the water use pattern in Kaduna Metropolis would have been easier and perhaps more accurate if households and other consumers have rate-measuring meters. In the absence of meters, Questionnaires and verbal interview method was adopted to facilitate the estimation of water use in the city. The survey method involved drawing up a set of questions for the households in each of the study areas (low, medium and high density areas) and also for commercial institutional and industrial users. Similarly data on water consumption, number of total connection for the sectors was also obtained from Kaduna State Water Board for the purpose of the research. Population data was also collected from National Population Commission and Published materials.

Sampling method

Were one surveys every person or a whole set of units in a population you are taking a census. However, this method is often impracticable; as it's often very costly in terms of time and money. For example, a survey that asks complicated questions may need to use trained interviewers to ensure questions are understood [13]. This may be too expensive if every person in the population is to be included. To overcome these problems, samples are taken from populations, and estimates made about the total population based on information derived from the sample. A sample must be large enough to give a good representation of the population, but small enough to be manageable.

Random sampling

In random sampling, all items have some chance of selection that can be calculated. Random sampling technique ensures that bias is not introduced regarding who is included in the survey [65]. Five common random sampling techniques are:

- simple random sampling,
- systematic sampling,
- stratified sampling,
- cluster sampling, and
- multi-stage sampling

Simple random Sample: With simple random sampling, each item in a population has an equal chance of inclusion in the sample [19]. Simple random sampling is the most widely-used probability sampling method, probably because it is easy to implement and easy to analyse.

Stratified Sample: A general problem with random sampling is that you could, by chance, miss out a particular group in the sample. However, if you form the population into groups, and sample from each group, you can make sure the sample is representative. In stratified sampling, the population is divided into groups called strata. A sample is then drawn from within these strata.

Linear programming technique

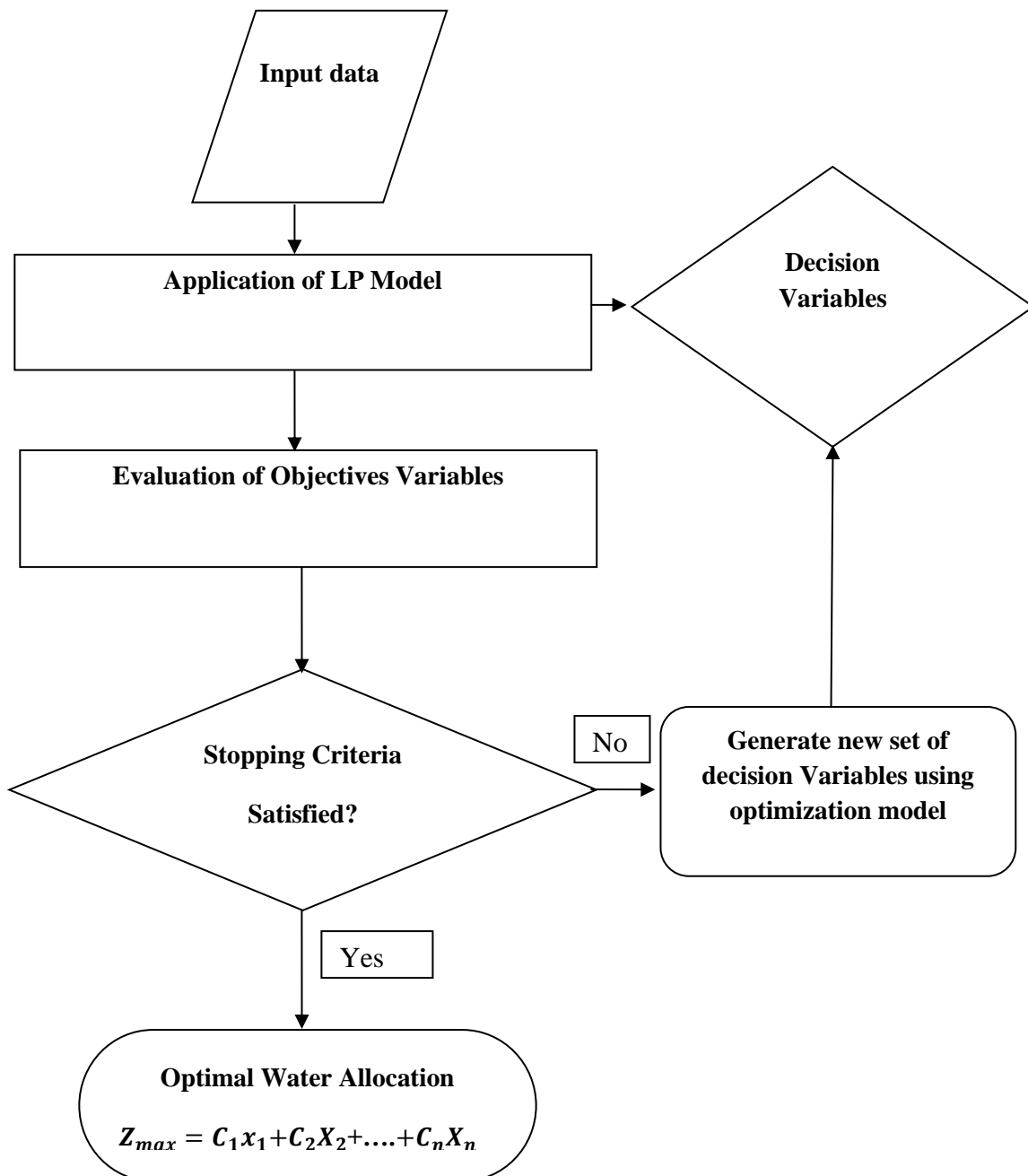


Figure 2. General framework of simulation – optimization modelling approach

For water allocation in Kaduna metropolis

First, the given problem must be presented in linear programming form. This requires defining the variables of the problem, establishing inter-relationships between them and formulating the objective function and constraints as showed in figure 2 above for the General

framework of simulation. A model, which approximates as closely as possible to the given problem, is then to be developed. If some constraints happen to be nonlinear, they are approximated to appropriate linear functions to fit the linear programming format. In case it is not possible, other techniques may be used to formulate and then solve the model [14].

To solve such L.P.P there are two methods:

1. The Big-m-method or method of penalties
2. The two-phase simplex method.

The formulation of linear programming problem as a mathematical model involves the following key steps [23].

Step 1: Identify the decision variables to be determined and express them in terms of algebraic symbols such as x_1, x_2, x_3 ;

Step 2: Identify all the limitations in the given problem and then express them as linear equations or inequalities in terms of above defined decision variables.

Step 3: Identify the objective which is to be optimized (maximized or minimized) and express it as a linear function of the above defined decision variables.

Subject to the constraints we express Eq. [1],

$$Z = C_1X_1 + C_2X_2 + C_3X_3 \dots + C_nX_n \quad (1)$$

Where: $X \geq 0, i = 1,2,3 \dots, n$

$$a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n = b_1 \quad (2)$$

$$a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n \geq b_1 \quad (3)$$

$$a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n \leq b_1 \quad (4)$$

Where: $i = 1,2,3 \dots \dots m$.

We can state the objective function as Eq. [5]:

$$MaxZ = 150,441.41x_1 + 101,325x_2 + 22,304x_3 + 1,100x_4 \quad (5)$$

Where: Z - the maximum water to be allocated (in liters), x_1, x_2, x_3, x_4 - the various sector ie Residential sector, Commercial Sector, Public Institutions Sector and Industrial Sector.

Result and discussion

This survey and evaluation study on water use pattern in Kaduna is to establish water consumption rates for the residential, institutional, commercial and industrial consumers. The results will be useful for the purpose of upgrading the water supply scheme, planning expansion of water treatment plant, design of pumping stations, and the design of storage and distribution system for Kaduna metropolis. Similarly the data collected from water board on water supply to the sectors will be use to optimize the water allocation.

Analysis of residential water demand

The total volume of water used by all households divided by the total population gives the per capita water use.

The population of households affects total water use. The population distribution is shown in Table 1 for the various residential areas.

Table 1. Analysis of surveyed data distribution of residents in households

Number of people in household	Number of households		
	Low density area	Medium density area	High density area
1-4	59	56	28
5-8	86	122	73
9-12	18	31	38
13-16	03	16	19
>16	00	24	15

122 households out of 281 sampled in medium density areas have the number of residents between 5 and 8. For the low density areas, most of the households have between 5 and 8 (i.e. about 50% of the total households sampled in this category).

None of the households in the low density residential areas have its number of occupants greater than 16.

Table 2 gives the detail analysis of the type of houses that exists in the study areas under each category. For the densely populated area, 47% of the people live in multi-tenant buildings popularly called 'face to face', 23% in story buildings, 20% in flats or apartments and 10% in bungalows.

Table 2. Analysis surveyed type of houses in the residential areas

Type of houses	Low density area	Medium density area	High density area	Total
Flat	80	69	32	181
Bungalow	50	85	18	153
Storey Building	36	30	43	109
Multi-tenant houses	00	65	80	145
Total	166	249	173	588

Bungalows type house were most common for medium populated areas, followed by flats, multi-tenant houses, and story buildings in that order. Most of residents do not depend on direct public supply (tap water alone), they also source for water from combination of wells and boreholes.

Table 3 shows that in the low and medium density areas, more households depend on a combination of taps and borehole, while in the high density areas, residents depend on a combination of taps and wells.

Table 3. Analysis surveyed of sources of water supply to the residential areas

Source of water	Number of households indicating various sources of water		
	Low density area	Medium density area	High density area
Only Tap	25	55	30
Only Well	05	35	14
Only Borehole	19	15	12
Tap and Well	49	34	45
Tap and Borehole	58	75	16
Well and Borehole	04	30	29
Borehole, Well and Tap	06	05	27

Analysis of commercial, institutional and industrial water demand

The daily values for all commercial, industrial and institutional facilities sampled within the metropolis were 29,915.854, 366,666.6 and 29,308.804 litres respectively.

Tables 4 to 6 show the summary of all the commercial, institutional and industrial consumers surveyed respectively.

Table 4. Analysis of surveyed commercial consumers

Type of Establishment	No	%
Retail Shops	20	14.18
Hotels	12	8.51

Canteens	25	17.73
Slaughter Slabs	10	7.09
Abattoir	14	9.93
Car Wash	30	21.28
Dry Cleaners	10	7.09
Hair Dressing Saloons	08	5.67
Others	12	8.57
Total	141	100

Table 5. Result of surveyed of institutional consumers

Type of Establishment	No	%
Banks	8	8.51
Clinics	10	10.64
Hospitals	10	10.64
Nursery and Primary Schools	25	26.60
Secondary Schools	20	21.28
Post-Secondary Schools	3	3.19
Government Ministry / Parastatals	10	10.64
Fire services	4	4.25
Barracks	1	1.06
Others	3	3.19
Total	94	100

Table 6. Analysis of surveyed of industrial consumers

Type of Establishment	No	%
Block Making	50	44.64
Bakery	20	17.86
Pure Water Factories	35	31.25
Others	07	06.25
Total	112	100.00

Car wash has the highest number followed by the food canteens as commercial users. Other types of establishment include business centres, petrol filling stations, video clubs, etc. Out of 94 institutional consumers sampled, nursery and primary schools have the largest number and closely followed by secondary schools. Banks, ministries and parastatal have the least number. A total number of 112 industrial consumers were surveyed consisting mostly of block making industries, pure water, and bakery. Others include bottling company, NNPC, IBB, Yoghurts Production, etc.

Analysis of population forecast

In the analysis of this research work only one method of the four method of population forecast I the methodology was applied in order to forecast the future population of Kaduna metropolis which is known as geometric increase method the most popularly used method of future population forecast and the method is describe below while Table 7 is the result of the forecast from 2006 to 2018:

Table 7. Census population of 2006 and projected population forecast to 2016 ofKaduna State

S/No	L.G.A	Senatorial District	2006 Population			2016	2018
			Male	Female	Both Sexes	Population	Population
1	Chikun	B	187,433	184,839	372,272	475,147	498,908.849
2	Igabi	B	217,414	213,339	430,753	549,789	577,283.5009
3	Kaduna North	B	187,075	177,500	364,575	465,323	488,593.5381
4	Kaduna South	B	204,969	197,762	402,731	514,023	539,729.1756

Source: Field work (2016)

The method of estimation can be in the following method:

1. Arithmetic increase method
2. Geometric increase method
3. Method of varying increment or incremental increase

Arithmetic increase method: This method is based upon the assumption that the population increases at a constant rate i.e. the rate of change of population with time i.e ($\frac{dp}{dt}$) is constant. Thus Eq. [6] or [7]:

$$\frac{dp}{dt}=k \tag{6}$$

$$\int_{p_1}^{p_2} dp = k \int_{t_1}^{t_2} dt \tag{7}$$

By integration of Eq.(6-7) then it becomes Eq. [8]:

$$P_2 - P_1=k ((t_2 - t_1)) \tag{8}$$

Where: suffixes 1 and 2 represent the last and the first decade or census, respectively. Thus t_2-t_1 - no. of decade.

The general form of arithmetic increase is given by Eq. [9]:

$$P_n=\{P_0+n. \mu\} \tag{9}$$

Where: p_n - prospective or forecasted population after n decades from the present (i.e. last known census), p_o - population at present (i.e. last known census), n - no, of decade between now and future, μ - average (arithmetic mean) of population increase in the known decade.

Geometric increase method: In this method, the per decade percentage increase or percentage growth rate (r) is assumed to be constant, and the increase is compounded over the existing population every decade. This method is therefore also known as uniform increase method. The difference between arithmetic and geometric method is that in geometric there is compounding in every decade while in arithmetic there is no compounding. Hence geometric increase is given by Eq. [10]:

$$P_n = P_0 + \frac{r}{100P_0} \quad (10)$$

Thus equation (9) can simplify as Eq. (11):

$$p_n = p_o(1+r/100) \quad (11)$$

Where: r - is in percentage, p_n - population after 2 decade for successive decade.

Similarly Eq. [12] can be simply to give Eq. [12]:

$$p_n = p_o(1+r/100)^n \quad (12)$$

Where: p_o - initial population, i_e - the population at the end of last known census, p_n - future population after a decades, r - assume geometric rate (%).

Method of varying increment or incremental increase: In this method, the per decade growth rate is not assume to be constant as in the arithmetic or geometric progression methods: but is progressively increasing or decreasing, depending upon whether the average of the incremental increases.

The population for a future decade is worked out by adding the mean arithmetic increase (say μ) to the last known population as in “arithmetic increase method” and to this is added the average of the incremental increases \bar{y} , once for the first decade, twice for the second decade, thrice for the third decade, and so on. Incremental increase is given by Eq. [13]:

$$P_n = \{P_0 + n \cdot \mu\} + n \frac{(n+1)\bar{y}}{2} \quad (13)$$

Where: p_n - population after n decade from present (i.e. last known decade), μ - average increase of population of known decade, average of incremental increase of the known decade.

From the method of population forecast described above one of the method will be used in order to forecast the population of the residential, commercial public and industrial consumption of the study area. In this case I applied (Geometric increase method).

The formula is given as Eq. [14]:

$$p_n = p_o(1+r/100)^n \tag{14}$$

Where: p_o - initial population, either population at the end of last known census, p_n - future population after a decades, r - assume geometric rate(%) and $r = 2.47(\%)$ according to the growth rate.

The table above (Table7) is the population forecast of the four local governments in Kaduna Metropolis using the population census of 2006 given by the National Population census. The population was projected to 2016 and later on projected to current year which is 2018 as indicated in Table7 (see Appendix).

Table 8 and Figure 3 shows the result of total water supply, total water demand, deficiency and % deficiency for the four sectors. In each of the sector the total water demand, total water supply, deficiency and % deficiency are represented in blue, red green and grey colours respectively.

Table 8. Total daily water demand and supply for the four sectors of Kaduna Metropolis area

Sector	Total water demand (L)	Total water supply (L)	Deficiency	% Deficiency
Residential	158,398,518l	69,125,133	89,273,385	77.43
Commercial	101,325,000	3,377,500	97,947,500	96.67
Public Institution	22,304,000	743,466	21,560,534	96.67
Industrial	1,100,000	366,667	733,333	66.67

Source: Field work (2016)

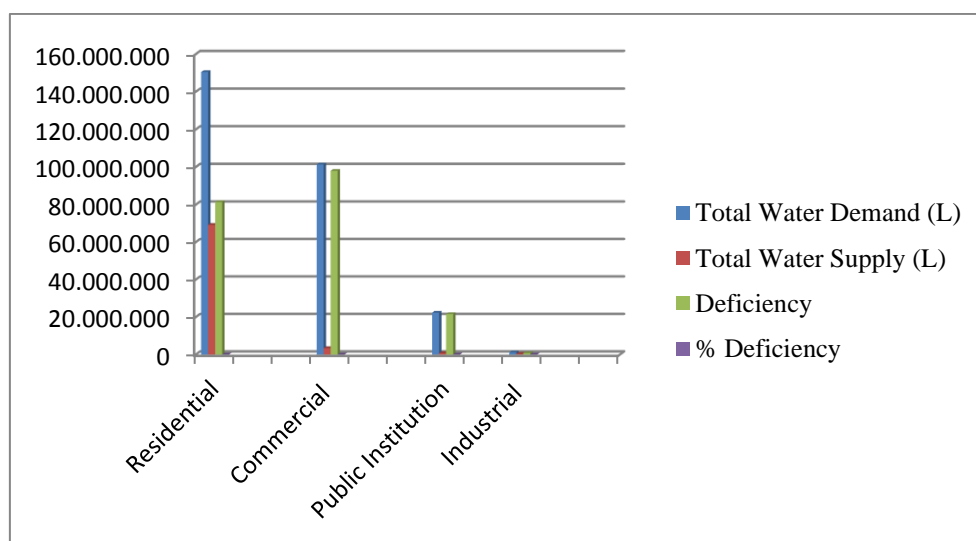


Figure 3. Total daily water demand, supply, deficiency and % deficiency for the four Sectors of Kaduna metropolis area

In each of the sectors there is a combination of four colours representing the comparison between total water demand, total water supply, deficiency and % deficiency. The result shows that the % deficiency is high for commercial, public, while that of industries is lower compared to residential sectors indicating that those areas relies more with their personal source of water supply than that of the public source. It is very clear from the figure that the demand is higher than the supply for both commercial, public and industrial sectors.

Table 9 and Figure 4 above is the comparison between model allocation and the water supply in the four sectors of the metropolis.

Table 9. Comparison between model allocation and Water supply to the sectors

Sectors	Water supply by the water corporation (L)	Model allocation to sectors (L)
Residential	69,125,133	158,398,518
Commercial	3,377,500	101,325,000
Public Institution	743,466	22,304,000
Industrial	366,667	1,100,000

Source: Field work (2016)

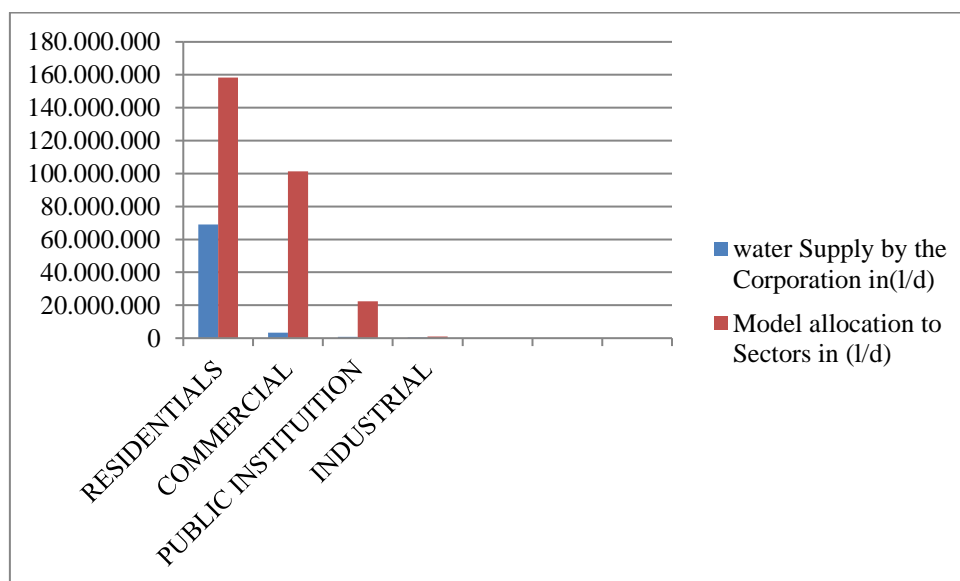


Figure 4. Comparison between model allocation and water supply to the sectors

The blue colour of the bar chart is the water supply by the water board while the maroon colour is the bar chart of the water supply by the model allocation In all of the four sectors water supply is lower than the model allocation this indicated that there is a short of water for all the sectors.

Table 10 above is the optimal allocation of Water Supply for the four Sectors of Kaduna Metropolis Area.

Table 10. Optimal allocation of water supply to the sectors

Total quantity supply(L)	Sectors; Total quantity allocated				Total
	Residential x_1 (L)	Commercial x_2 (L)	Public x_3 (L)	Industrial x_4 (L)	
176,360,000	158,398,518l	101,325,000	22,304,000	1,100,000	283,127,518l/d

Source: Field work (2016)

The Table 10 represented the total quantity supply by the Water Board, the optimal allocation obtained by the model for the residential, commercial, public institutions and industrial sectors and the total optimal water allocated to the four sectors respectively, the table indicated that the total optimal water allocated is higher than the water supply by the Water Board to the sectors.

Conclusion

From our analysis, there is a clear difference in the quantity of water being allocated to the sectors by the State Water Corporation is inadequate because the Optimal Water allocated by the model for the four sectors is 283,127,518 l/d while the Water board supply 176,360,000 l/d and only 73,612,760 l/d is allocated. This indicated that there is a deficiency of 106,767,518 l/d of water supply compare with the total water produced by the board. There is a need for the Water Corporation to enhance their supply so as to meet the optimal allocation of the supply. The basic lesson learnt from this research is that: The prevailing allocation method by the State Water Corporation is inefficient to the extent that water is undersupplied to the sectors. This calls to question the water supply criteria being used by the Water Corporation. Since Kaduna metropolitan town has the same social-economic character of other Nigerian towns and to a great extent with urban areas in developing countries, any plan dealing with the allocation of public water supply to the urban sectors within these regions should be based on the Criteria specified by this model. Thus the allocation of water with this model technique assures of optimal supply of water to strategic consumers on the satisfaction of the prevalent constraints. The implication of this research which can be applied internationally is that water is a scarce resource in most



urban areas especially in developing countries. The reoccurring water supply shortages relative to demand have been a source of worry to many public water supply managers in these urban areas. The situation as presently experienced is that water which is manufactured at a very great financial cost is often undersupplied to the sectors. This calls the attention of managing the little resources available by making law enforcement for water abuses.

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Appendix

The algebraic expression is written as follows:

$$5x_1 + 8x_2 + 8x_3 + 14x_4 \leq 35$$

$$19x_1 + 19x_2 + 19x_3 + 24x_4 \leq 81$$

$$588x_1 + 141x_2 + 94x_3 + 112x_4 \leq 935$$

$$23x_1 + 23x_2 + 23x_3 + 3x_4 \leq 72$$

$$2x_1 + 2x_2 + 2x_3 + 6x_4 \leq 12$$

$$110x_1 + 110x_2 + 110x_3 + 120x_4 \leq 460$$

$$5x_1 + 5x_2 + 5x_3 + 1x_4 \leq 16$$

To calculate the demand for the sectors we use the formula as follows:

Customers water demand – $Q_n = P_n \times q_d$.

Where: Q_n - recorded average daily consumption at year n; P_n - population at year n; q_d - computed per capita daily demand at year n.

Residential Sector = $75.06 \text{ l/d} = 158,398.518 \text{ l/d} \times 0.07506 \text{ m}^3/\text{d} = 158,398.518 \text{ m}^3/\text{d}$

Commercial Sector = $29,915.854 \text{ l/d} = 3,387 \times 29.916 \text{ m}^3/\text{d} = 101,325 \text{ m}^3/\text{d}$

Public Institutions Sector = $29,308.804 \text{ l/d} = 761 \times 29.309 \text{ m}^3/\text{d} = 22,304 \text{ m}^3/\text{d}$

Industrial Sector = $366,666.666 \text{ l/d} = 3 \times 366.667 \text{ m}^3/\text{d} = 1,100 \text{ m}^3/\text{d}$

- Kaduna metropolis optimal water supply = $283,127,518 \text{ l/d} = 283,127.518 \text{ m}^3/\text{d}$.

- The allocation of various sectors in the metropolis are:

Residential sectors = $158,398,518 \text{ l/d} = 158,398.518 \text{ m}^3/\text{d}$.

Commercial sectors = $101,325,000 \text{ l/d} = 101,325 \text{ m}^3/\text{d}$.

Public sectors = $22,304,000 \text{ l/d} = 22,304 \text{ m}^3/\text{d}$.

Industrial sectors = $1,100,000 \text{ l/d} = 1,100 \text{ m}^3/\text{d}$.

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