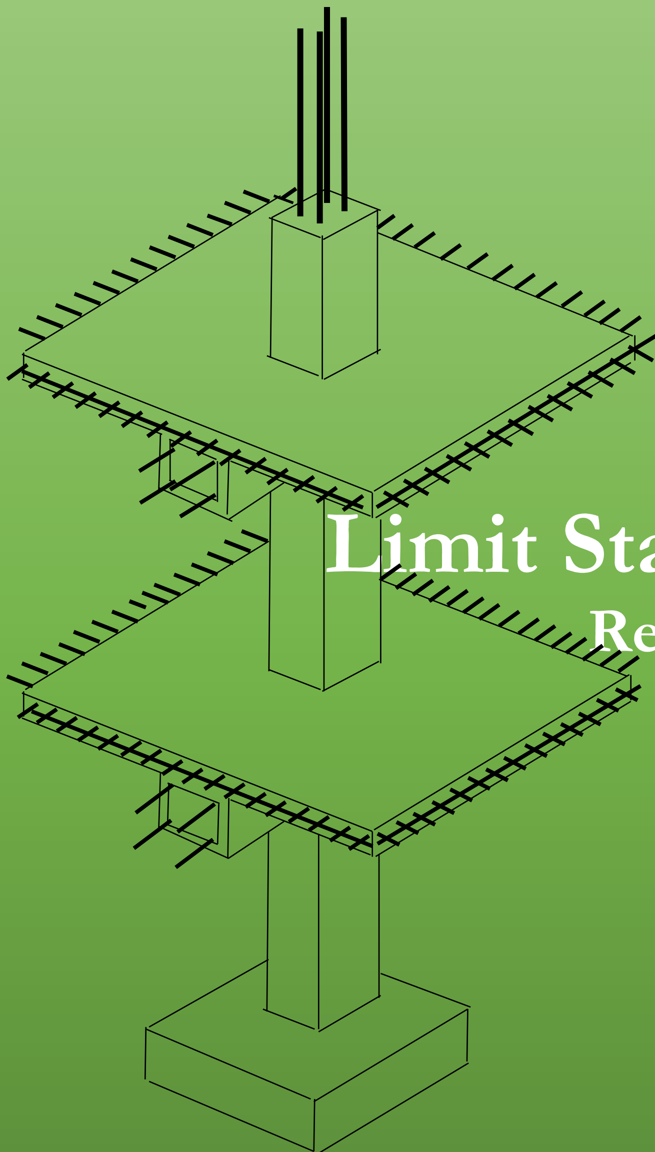

Limit State Design of Reinforced Concrete Structural Elements

Christopher Ajiboye FAPOHUNDA

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Dedicated

To

The Apostolic brotherhood of Peter and Paul together with their Lord and God
Jesous Christ, the crucified but resurrected and enthroned one Man immortal and
incorruptible King, the Peace-doing Image and Son of God of Peace,
also, my Lord and God, but through them.

Contents

Preface

This text on the limit state design of reinforced concrete elements was developed out of my class notes, prepared for teaching and training undergraduate students studying Civil engineering and related disciplines, the design of reinforced concrete structures. The book intends to provide introduction and solid foundation to the students in the principles of limit state design, especially in Nigeria where there is a dearth of indigenous books in the area.

Though the book is primarily aimed at students of civil and structural engineering degree and diploma courses, other students in allied professions such as architects, builders and surveyors may also find it suitable. Practicing professionals in all these areas, without the knowledge of the limit state design who require introduction to it, will doubtless find the book useful. However, prior or parallel courses in structural analysis is necessary for complete and integrated understanding of the subject of this book.

Dr. C. A. Fapohunda had BSc., MASc. and PhD degrees in Civil Engineering respectively at University of Ife (now Obafemi Awolowo University), Nigeria, University of Toronto, Canada and the University of Lagos, Nigeria, specializing in Structural Engineering. He is a registered engineer and a member of the Nigerian Institution of Structural Engineers. He has previously lectured at the Polytechnic Ibadan, Osun State College of Technology Esa-Oke, and Caleb University, Imota, all in Nigeria. He presently lectures in the Department of Civil Engineering, Federal University, Oye-Ekiti, Nigeria.

Acknowledgment

Since the book's major audience is undergraduate students, the materials used are selected from numerous works, re-arranged and presented in a format suitable for the intended audience. Being conscious of this, the author wishes to acknowledged the many authors used. The author also acknowledges those who at various times have helped with the preparation and checking of the manuscript. Special thanks to Dr. Adetayo, Dr. Oyelade and Dr. Ukponu for helping to improve the content and substance of this book. The followings, namely, Mr. Hussein Bello, Mr. Moyosoluwa Odewumin, Mr. Samuel Odunuga, and Ms. Blessing Adigo also help to check the calculations in the manuscript to minimize arithmetical errors. The author is most grateful to them.

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List of Some Symbols

The symbols listed below are those occurring frequently in the text and are in accordance with BS 8110 (1997). Other symbols are defined in the text where necessary.

A	= cross sectional area of member
A_c	= area of concrete
A'_c	= area of compression reinforcement
A_s	= area of tension reinforcement
A_{sc}	= area of longitudinal reinforcement for column
$A_{s.}$ prov	= area of tension reinforcement provided
$A_{s.}$ req.	= area of tension reinforcement required
A_{sv}	= cross sectional area of the two legs of a link
b	= width of the section
b_w	= breadth of web or rib of a member
d	= effective depth of tension reinforcement
d'	= effective depth of compression reinforcement
E_c	= static secant modulus of elasticity of concrete
E_s	= modulus of elasticity of steel
F	= ultimate load
F_k	= characteristic load
f_{bs}	= bond stress
f_{cu}	= characteristic cube strength of concrete
f_k	= characteristic strength
G_k	= characteristic dead load
g_k	= characteristic dead load per unit area or per unit length
h	= overall depth of section in the plane of bemding
h_{agg}	= maximum size of aggregates
I	= second moment of area
i	= radius of gyration
k	= stiffness of a member
l_e	= effective height of a column
l_{ex}	= effective height for bending about the major axis
l_{ey}	= effective height for bending about the minor axis
lx	= length of the shorter side of rectangular slab
ly	= length of the longer side of rectangular slab
M_{add}	= additional moment

Q_k	= characteristic imposed load
q	= distributed imposed load
q_k	= characteristic imposed load per unit area
s_v	= spacing of links
V	= shear force due to ultimate load
v	= shear stress
v_c	= ultimate shear stress of concrete
Vu	= ultimate shear resistance of a reinforced concrete section
z	= lever arm

Chapter *1* - Introduction

1.1 Structure

The word **structure** has various meanings and interpretations, depending on the environment. It is not uncommon to hear something like:

- Political structure
- Social structure
- Family structure
- Grammatical structure
- Organizational structure
- And so on.

While some of these may be considered intangible, however, the word structure was from a tangible construction and building background derived from the Latin word "*structura*", meaning "to construct, to fit together". The purpose of which is to create an undistorted protected space. All civil engineering structures – buildings of various forms, bridges, tunnels, etc. - are undistorted protected space to perform specific function (s). Structure can thus be conceptualize as being a system or a body consisting of many parts or many members but connected (or coupled together) in such a way as to form perfect unity which resists any agent of distortion (or disintegration), without fragmentation or collapse. Human body is an example of such a system. When a man moves about, no part of the body, say an eye, or a hand, is left behind. All move together as one. When he falls down, all the parts are also involved. When situation demands, and he wrestles with another man, all the parts of his body are involved and contribute their part to prevent being defeated. In the same vein, when a civil engineering structure, say a building, had part of its wall collapsed, or the roof is blown off, it can no longer provide the function of secure habitation, in such a distorted form. Structure then, can be understood to mean a unitary organized system of suitable members or materials, for the purpose of resisting adverse effects. Inherent in this definition is the expectations of strength, robustness and durability of any structure.

Structure may be dynamic or static in relation to the way it is analyzed. In dynamic analysis, structures are analyzed and its responses determined to a time-varying load. The term response encompasses the displacements, velocities and accelerations of the various components of the system, as well as the strains and stresses induced in the system. Whereas for static structure, the load is assumed constant.

Thus, dynamic structures include:

- human body
- airplane
- ship
- and so on.

Each of the dynamic structures resists different adversity based on their environment of operation. Static structure on the other hand, includes:

- buildings of all types – religious, commercial, educational, institutional, office, etc.

- bridges
- towers
- transmission lines
- civil engineering infrastructure, and so on.

Static structures consist of horizontal and vertical members which are rigidly connected for the purpose of transferring loads from one point, in space to another, to the foundation without failure or excessive deformation. Such structure transferred myriads of loads like:

- Weight of people
- Weight of Building itself
- Furniture
- Equipment/Machineries
- Environment elemental loads like snow, winds, etc.
- Unforeseen loads like terrorist, bomb blast, etc.
- And so on.

This book is devoted to static structures. Some of the properties that a static structure is expected to demonstrate are:

- Strength. A structure should be strong and well able to resist load expected loads with adjustment for partial safety factor for material and buckling (local or overall).
- Stability. Structure is in stable equilibrium when small perturbations do not cause large movements.
- Rigidity. That is, the structure is stiff and does not deform under applied loads.
- Robustness. This is the ability of a structure to withstand events like fire, explosions, impact or the consequences of human error, without being damaged to an extent that it becomes unfit for its intended use.
- Durable. A good structure must have long service lifespan with deterioration within acceptable limit.
- Sustainability: This is an environmental issue that deals with: (i) the embodied energy of the structure, (ii) the CO_2 emissions of the construction materials and (iii) the depletion of resources. The structure should be developed and maintained with minimal environmental impact.

All the above are achieved through the process of **structural engineering.** The aims that Structural Engineering set to achieve are that:

- The structure must be safe, because the society demands security in the structure it inhabits
- The structure must fulfil its intended purpose during its life span
- The structure must be economical in cost and maintenance

In modern times, structural engineering has widened its scope to include:

- The study of the manner in which the members of a structural system are selected, arranged and coupled together to achieve a structure that is strong, stable, rigid and robust.
- Investigation into the fitness of a material for structural function
- Making a material fit for structural function, and
- Sustainability issues in Structural engineering.

Structural Engineering is a creative process of finding a safe, economic and environmentally-friendly solution to structural problems. It is an iterative or cyclic process until acceptable design is obtained. In design, it is not uncommon to have more than one solution to the same problem. Structural engineering, in practice is divided roughly into six, namely: (i) conceptualization of structural systems (ii) loads determinations, (iii) structural analysis, (iv) structural design (v) structural drafting (vi) supervision of the implementation of the structural design Structural design is usually preceded by structural analysis, and analysis on the other hand makes use of the loads that have been appropriately determined.

Conceptualization of Structural Systems involves the selection of appropriate structural elements and structural sub-systems bearing in mind the intended use of the structure and the intended location. Selection of appropriate or available materials also forms part of consideration in the conceptualization of structural system for a structure. Knowledge of the soil properties in relation to its bearing capacity and settlement characteristics will also help in arriving at the most suitable structural systems.

Load determination is the stage in which the type and magnitude of loadings to which a structure may be subjected to, during its entire design life, is determined. It is usual to classify loadings on the structure into three, namely: dead load, imposed load and wind load. The documents used in the determination of loadings on structure are:

1) BS 648 – Schedule of weights for building materials
2) BS 6399 – Design loadings for buildings:
    a. Part 1: Code of Practice for dead and imposed loads
    b. Part 2: Code of Practice for wind loads
    c. Part 3: Code of Practice for imposed roof loads

Loadings will be discussed further in subsequent chapters.

Structural Analysis involves determining the force actions produced by a given loading system on a model, which is an idealization of the actual structure. In analysis, three things are usually in view, viz:

1) The distribution of forces throughout the whole members that make up the structure with a known load is determined
2) The stress distribution within individual members resulting from the applied load/actions is ascertained to ensure that limits are not exceeded
3) The deflection (the extent of movement of the structure) under a particular set of loads/actions is satisfactory.

Structural design, on the other hand is used to describe the whole creative process of finding a safe, economic and environmentally-friendly solutions to structural problems. It involves the element-sizing calculations which are carried out to ensure that they will have sufficient strength and rigidity to resist the internal forces arising from the various combinations of loadings/actions.

Structural Drafting. Having checked the adequacy of the analysis and design under erecting and service condition, detailed drawings with specifications are then issued to the constructor. The drawings and specifications constitute instructions to the constructor, who will now erect the structure under the supervision of the designer, the Structural engineer. Through all these activities, the structural engineer is guided by Code of Practice (discussed in Section 1.6) which are compendia of good practice drawn up by experienced engineers.

Supervision of Implementation of Structural design. This is the actualization stage in which the structure is constructed as designed

1.2 The Goal of Structural Design

The goal of structural design is to arrive at a structure which meets the requirements of the client at the reasonable cost. The requirement may include any or all of the followings:

1) That the structure shall not collapse
2) That the structure shall not require excessive repair in the event of accidental overload or incidence of weather action.
3) That the appearance of the structure or the motion of the structure should not be such that it causes no discomfort to the users.
4) That the building shall be sufficiently fire-resistant to allow occupants to escape in the events of fire outbreak.

1.3 Design Process

The **Client** initiates the work. The Client may be Government (at all levels), Institutions, Companies, Individuals. Design processes, which is after the client brief, are carried out in two stages, namely, (i) Pre-design activities and (ii) Design activities.

1.3.1 Pre-Design Activities

This is the first stage in design process. It involves activities which are not only wholly structural engineering issues but formed bases of the design decisions. Some of these steps include: These are:

- Liaising with all the other members of the design team like Architect, Builders, Civil Engineer, Soil (geotechnical) Engineer, Quantity Surveyor, Mechanical Engineers, Electrical Engineers, etc.
- Preliminary investigations of the site, which involves gathering as much information on things like soil conditions, results of soil investigations, site location, conditions of adjacent structures, etc.

- Analysis of information and the data obtained for the purpose of making a sound structural decision.

1.3.2 Design Activities

Design activities are broadly divided into two stages: preliminary design and detailed design. These stages are however interrelated and may be carried out simultaneously.

Preliminary Design

In preliminary design, the design considers different structural schemes that satisfy the client brief. Also, the structural elements sizes are determined with approximate methods; and approximate cost of the different schemes are determined. Details of required are activities include:

- Development of general layout and geometry of the Structure
  i. selection of a general layout from many possible alternatives and the basic geometry of the structure with particular attention paid to member hierarchies (which member supports other members) and spanning direction.
  ii. selection of best location or adapting the structure to a site yet to be determined.
  iii. other considerations like financial, sociological, cultural, environmental, legal, etc. aspects.
- Investigation of loads

  This involved the determination of loads for which a given building should be designed. Most of these loadings are specified in codes and specifications. All manner of loading forms like: dead loads, live, loads, moving loads, bomb blasts, etc. must be properly identified and assessed. Soil investigation is also necessary to obtain the bearing capacity of the soil and settlement characteristics.
- Stress analysis

  Here the stress analysis is carried out to determine the internal forces in various members of the structure, and also the displacement at some controlling points. The principles governing this phase of design are usually discussed in the theory of structures.
- Selection of materials

  This stage involves the selection of materials to be used for the building/structure. Cost, maintainability, and durability are factors apart from strength that will be considered.
- Selection of sizes and shapes of members.

  This is the selection of suitable sizes and shapes of members and their connections. This stage depends on the results of the structural analysis together with the design provisions in codes and specifications. Usually, a trial and error approach may be used in the search for the proportioning of the elements that will be economical and structurally adequate. **A sound**

knowledge of strength of materials, the process of fabrication and assemblage are also essential.

Detailed Design

This stage usually involves the use of commuter programs to analyze the scheme considered best in preliminary design. The members' sizes are optimized and structural drawings are issued.

The final design is obtained from detailed design. The final design is used to determine the actual cost of the structure.

A typical sequence of design process can be illustrated with design flow chart in Figure 1.1.

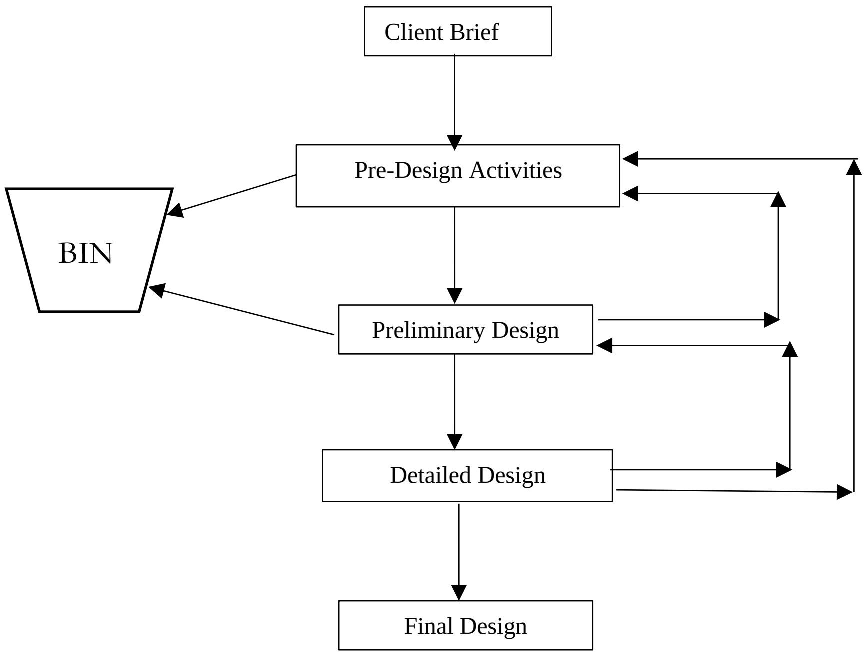


Figure 1.1: Typical Design Process

Throughout the whole design process, the designer should be conscious of the cost involved, and lately, its environmental impact. The cost includes:

1) Initial cost: site preparation, materials, construction, consultants' fees, supervisory fees, miscellaneous.
2) Maintenance Cost
3) Insurance – chiefly against fire
4) Demolition expenses

1.4 Considerations in Design

1) The structure being designed is to perform satisfactorily during its intended life. That the structure should sustain all the loads and deformations of normal construction and use, as well as have adequate resistance and durability to the effects of misuse and fire
2) The use of suitable materials, good quality control and good supervision are very important for the production of safe, serviceable and durable structures.
3) That the quality of concrete, steel and other materials as well as the workmanship will be adequate for safety, serviceability and durability.

That design process, including the design for durability, construction and use in service; is to be considered as a whole. The design lifespan recommended for building of all types is shown in Table 1.1.

Table 1.1: Intended working lives as recommended in BS EN 1990

Description of Structure	Intended Working Life (In Years)
Temporary Structures	10
Replaceable structural parts	10 - 25
Agriculture and similar Structure	15 - 30
Building structures and other common structures	50
Monumental building structures, bridges and other Civil Engineering Structures	100

Although, the structural engineer will come with good design, experience have shown that, in many instances, he may not be consulted or have control over the manner of its construction. In such instances, it is the responsibility of the client to ensure that the construction of the project is carried out in such a way that will result in good service life performance expectations. The construction should be carried out by personnel having the **appropriate skills and experience**, who will:

a. Ensure adequate supervision and enforce compliance to design specifications
b. Not compromise on quality control procedures.
c. Use the right materials and products as specified.
d. Will consider his/her professional obligation as sacred

It is also assumed that the client will use the structure in accordance with the design brief.

1.5 Structural Design Principles/Philosophies

There are many design principles that are employed in the design of reinforced concrete structures. Some of these are:

1) Modular ratio or Elastic-Stress Design Method
2) Load factor Design Method
3) Limit State Design Method

1.5.1 Modular Ratio

In this method, the moments and forces acting on a structure are calculated from the actual applied loads, but the permissible stress in concrete and reinforcement are restricted to only a fraction of their true strength. For example, a material, say steel may be limited to permissible stress value of 165N/mm^2 while it can actually withstand a stress of 275N/mm^2 before yielding. This ensures that in the event of failure, it is in the desirable form (that is, reinforcement yielding and thus gives advance warning that failure is imminent, rather than concrete crushing, which may happen unexpectedly and explosively. Some of the assumptions on which this method is based are:

1) That the stress is proportional to the strain for both steel and concrete, that is, a constant "modular ratio"
2) That the plane section remains plane after bending. That is, strain varies linearly across the section in bending
3) That there is a perfect bond between the concrete and the steel
4) That the concrete has negligible tensile strength.

The major shortcoming of this method is that, as failure approaches, the assumed linear relationship between the stress and the strain is no longer valid.

1.5.2 Load Factor Method

This method was developed as an answer to the shortcoming of modular-ratio when failure is approached. Here the analysis of the section is carried out by relating the actual strength of a section to the actual load causing the failure. The actual load causing the failure is determined by "factoring" the design load by appropriate factor of safety. A simple definition of factor of safety (FS) is:

$$FS = \frac{Load\ to\ cause\ failure}{Actual\ load\ on\ the\ structure}$$

1.5.3 Limit State Design

This was the method that originated from the recommendations of European Concrete Association in 1972 and has thus been adopted in all Europe. In this method, the design of each individual member or section of a member must satisfy two criteria:

- The Ultimate Limit State (ULS)
- The Serviceability Limit State (SLS)

To ensure acceptable compliance with these limit states, various partial safety of factors is employed in the limit state. **This Book is concerned with the Limit State Method of Design**. The rules governing this method of design are contained in the Code of Practices CP 110, BS 8110 and Eurocode.

1.6 What are Design Codes in Reinforced Concrete?

Design codes can be defined as a set of technical rules and regulations governing the safe practice (design, materials, performance, construction standards, testing, measurements, workmanship, professional ethics, etc.) of reinforced concrete. It ensures that the methods and procedures employed in the design of reinforced concrete are guarded only by what has been tried, tested and subsequently approved to be safe and satisfactory in performance, by **legitimate authority.** This ensures that the art (of structural design) does not appear to be like a sheep without a Shepherd. The process of trying and testing are usually time-dependent. It is the result of such exercise, by those who are learned in the profession, usually in the form of recommendations/writing that are set forth in what is called Code or Standard, by those who carried out such exercise under the authority. They are the framework upon which the design and construction activities of reinforced concrete products are built and are usually a legal and imperial document. A reading of the preface to ACI and Eurocodes, though veiled in British Standard, suggests that issuing of Codes or Standards is the responsibility of the Government. Records of history supported this. For example, there was code of Hammurabi (about 1750BC) of the ancient Babylon enacted to guide against failure of buildings. Moses (about 1440B.C), the legislator of the ancient Israel included in his Law for Israel, a section that ensured a safe building construction. There are also sections that regulated building construction practices in the code enacted by Emperor Justinian (545AD) for the people of the ancient Byzantium Empire. Codes then, are instruments issued by government to for the purpose of producing safe and economic structures so that the public will be protected from poor or inadequate design and construction.

The preparation of code for reinforced concrete design and construction usually involves inputs from many organizations that are operators in the industry. For example, for the design reinforced concrete, over 10 of such organizations – professional and governmental - were involved in the preparation of the British Standard 8110 (1997) regulating it. These organizations were:

1. Association of Consulting Engineers
2. British Cement Association
3. British Precast Concrete Federation Ltd
4. Concrete Society
5. Department of the Environment (Building Research Establishment)
6. Department of the Environment (Property and Building Directorate)
7. Department of Transport (Highway Agency)
8. Federation of Civil Engineering Contractors
9. Institution of Civil Engineers
10. Institution of Structural Engineers
11. Steel Reinforcement Commission.

From the above, it can be observed that the BS 8110 (1997), representative of a typical national code, was based on **the collective opinion and corporate judgement** of experience relevant professionals. Although codes are regarded as unalterable, it is however subject to periodic reviews or updates as new systems occur, or new materials become available, or repeated failure of accepted systems occur.

The advantages of Codes are:

1) Ensuring that there is uniformity among the people of the same profession in the usage of things like symbols, words, definitions, design formulas, etc.
2) Ensuring that consumers of the professional services are protected and safeguarded against abuse, malpractices, misconduct, incompetence, etc. among professional members.
3) Bring about of a very stable environment under which both the professional and the client will operate.
4) Encourages the growth of the industry.

However, **the disadvantages of Code** are that:

1) it needs to be continually updated and modified as human experience dictates, and brilliant minds may not be around to carry out such revision.
2) It may be difficult to assemble the same set of minds that is capable of carrying out the revision of the codes.

Codes vary from one nation to the other. What is approved by one nation may not be approved in another nation. In America for example, they have their own Codes published by American Concrete Institute. India has its own which is being published by National Building Code. In Britain and Commonwealth countries, the Codes are published by British Standard Institute, and have thousands of codes on their stable, and particularly in relation to concrete. Here are some of them:

1. CP 110 - The Structural Use Concrete
2. CP 114 - The Structural Use of reinforced Concrete in Building
3. CP 115 - The Structural Use of Prestressed Concrete in Building
4. CP116 - The Structural Use of Precast Concrete in Building
5. BS 449 - The Use of Structural Steel in Building
6. BS 410 - Test Sieves
7. BS 4408 - Recommendations for Non-destructive Methods of Tests for Concrete
8. BS 5075 - Concrete Admixtures
9. BS 4550 - Methods of Testing Cement
10. BS 326 - Protection of Structures Against Lightning
11. BS 5337 - Liquid retaining Structures

1.7 Reinforced Concrete Design Practice in Nigeria – BS 8110 and the Eurocodes

In Nigeria, there used to be Nigerian Code of Practice NCP 1 for structural design and associated publications, but what has been in operation in Nigeria for structural design (for training and practice), are standards received from the United Kingdom of Great Britain, just like other British commonwealth countries. Thus, Nigeria has moved, just like the UK from CP 114 based on permissible stress method (or modular ratio or elastic method of design), to CP 110 and BS 8110, both of which are based on the limit state method of design. But the UK is just one of the nationals of the European Union (EU). Each nation has her own individual national/government standard governing reinforced concrete design practice. However, the idea to develop a set of harmonized and common structural design codes for European countries which started in 1974, as offshoot of the Treaty of Rome (in 1957) through the European Economic Community (EEC) gave birth to the Eurocodes – design tools for structural design in all European countries, including the UK. The common codes amongst European member states has been seen as advantageous particularly in lowering trade barriers between them and enables engineers, contractors and consultants from the member states to practice within all European countries (EC) and to compete fairly for works within Europe. It was also believed that the use of a common code will lead to a pooling of resources and sharing of expertise, thereby lowering the production costs. Although both the Eurocodes and the BS 8110 were based on the limit state of design principles, there are some differences in terminologies, coefficients, equations, safety factors for materials and loads, and many other design parameters. Some of these differences are presented in Tables 1.2 and 1.3.

Table 1: 2 Summary of Some Differences in Terminology Between BS 8110 and the Eurocodes

BS 8110	Eurocodes
Loads	Actions
Dead Loads	Permanent Actions
Imposed Loads	Variable Actions
Bending Moments	Internal Moments
Axial Forces	Internal Forces
Characteristics Dead Load	Characteristics Permanent Actions
Characteristics Imposed Load	Characteristics Variable Actions

Table 1.3: BS8110 and Eurocode2 basic span/effective depth ratios for rectangular beams

Support Conditions	BS 8110 – (1997)	Eurocode 2
Cantilever	7	7
Simply supported	20	18
Continuous	26	25
End Spans of Continuous Beams	-	23

In the European community, Eurocodes governs the practice of reinforced concrete design, and by implication, in Britain. The code includes the followings:

- EN 1990 Eurocode: Basis of Structural Design
- EN 1991 Eurocode 1: Actions on structures
- EN 1992 Eurocode 2: Design of concrete structures
- EN 1993 Eurocode 3: Design of steel structures
- EN 1994 Eurocode 4: Design of composite steel and concrete structures
- EN 1995 Eurocode 5: Design of timber structures
- EN 1996 Eurocode 6: Design of masonry structures
- EN 1997 Eurocode 7: Geotechnical design
- EN 1998 Eurocode 8: Design of structures for earthquake resistance
- EN 1999 Eurocode 9: Design of aluminium structures

Nevertheless, Nigeria is yet to officially follow UK to adopt the Eurocode, but BS 8110 is still in operation. With the recent decision by the UK to exit from the European Union, it is likely that in Nigeria, the BS 8110 will stay for a long time as the operating standard for structural design and for instructions in structural design of reinforced concrete. However, for designers and instructors who may want to adopt Eurocodes, understanding of some basics design procedures of the BS 8110 is very essential.

The limit state method of concrete design presented in this book is with respect to BS 8110.

Assignment

1. Discuss the concept of "structure" in relation to Civil Engineering.
2. What do you understand by the word "Structural Engineering"?
3. Differentiate between structural analysis and structural design
4. Write short notes on the followings
   a. Modular ratio method of design
   b. Elastic method of design
   c. Limit state method of design
5. What are codes
6. Why is code very important?
7. State the advantages and disadvantages of Codes

Chapter *2* - Structural Elements and Systems in Reinforced Concrete

2.1 Reinforced Concrete - Background

Concrete is a composite engineering material that has become the premier and the most favoured of all the construction materials for building and civil engineering construction. This is due to the following reasons:

1. Concrete can be produced in a variety of strengths, stiffnesses, unit weights, porosities and durability characteristics and properties, by using the same four basic components of cement, fine sand, coarse aggregates, and water.
2. In addition, it can also be produced in various shapes and sizes, and thus offers flexibility in forms. (see Section 2. 2 and 2.3 for details)
3. It allows other additives to be incorporated into the mix, so that the resulting concrete can be used for special application
4. the materials for its production are available and assessable
5. it can be produced at very low cost, when compared with other material like steel

Concrete has a long history that can be traced back to at least 9000 years. It has been used since ancient times, and was known to the ancient Egyptians, and even to earlier civilizations (Roberts et al; 2001). In those civilizations, concrete was employed, in one form or the other, for construction purposes. An analysis of ancient Egyptian pyramids has shown that concrete was employed in their construction. But it was the Romans who succeeded in putting their eternal imprint on concrete, because the name "concrete" was from the Latin, the language of the Romans.

The word **"concrete",** is from the Latin word "concretus" meaning "compact or condensed". Although, the Romans use a slightly different types and combination of materials for the production of concrete, they prepared the groundwork for the emergence of what we now called concrete. The use of concrete was widespread throughout the Romans Empire. Majority of the infrastructure that held their civilization together was of concrete construction. So skillful in the use of concrete were the Romans that they could depend only on the strength of concrete bonding to resist tension, without the use of tension reinforcement. Such innovation was used in the construction of notable works in Rome, like the gigantic Coliseum (Figure 2.1) - an amphitheatre built for sporting events, and the magnificent domed circular temple, the Pantheon (Figure 2.2).

Unlike many earlier amphitheaters, which had been dug into hillsides to provide adequate support, the Colosseum was a freestanding ellipsoidal structure constructed of stone and concrete. Constructed between 72 – 80 AD, Colosseum measuring some 190 long and 155 wide, was the largest amphitheater in the Roman world at the time. It had about 80 arched entrances allowing easy access to between 60 000 to 80,000 spectators.

Figure 2.1: The Coliseum

The dome of Pantheon was constructed without reinforcement in the concrete. The dome of the Pantheon is 43 meters high, and spans over a space of 43 metres in diameter, was built with unreinforced concrete. The dome is still standing, two thousand years after construction. Today, no structural engineer dare builds such a structure without reinforcement.

Figure 2.2: The Pantheons

These structures have not only survived to this present day, but also continue to be used (Delatte, 2001 and Neville, 2003). Moreover, with concrete, Romans could build canals of underground and elevated aqueducts without joints, by employing techniques which ensures that conduits could be impermeable to water without shrinkage and cracks. However, in spite of this long history, the use of concrete was in general restricted and severely constrained due to the non-availability of suitable binder and the inherent weakness of concrete in carrying tension.

The breakthrough came only in the nineteen centuries when Joseph Aspdin invented and patented cement in 1824, followed by the invention of what is today known as **reinforced concrete** by Joseph Loius Lambot in 1848. These presented the most significant events that later paved the way for extensive and widespread use of concrete in construction industry. Subsequent events, such as the introduction of air entraining agents in 1930 to increase the resistance to freeze/thaw damage, invention of superplasticiser in the early 1980's, went further into strengthening the position of concrete as primal construction material.

Concrete, to date, remains the most widely used man-made construction material for the construction of building and civil engineering projects like: pavements, myriads of architectural and building structures, foundations, motorways/roads, bridges/overpass, parking structures, dams, just to mention but few; because of its strength, durability, economy, and versatility. Concrete strength, according to Neville (2003), is an aggregation of contributions from three sources, namely:

a. The strength of the mortar (paste)
b. The bond between the mortar and the coarse aggregates
c. The strength of the coarse aggregates

This strength of concrete is a vital element of structural design, and it is specified for compliance purposes. The durability characteristics of concrete is seen from its ability to perform its intended functions without losing its strength and serviceability during the expected service life. The excellent condition of many structures that were built hundreds of years ago attest to its ability to withstand agents of degradation in the forms of actions that could be mechanical (abrasion, erosion, cavitations, impact, etc.), physical (thermal expansion of cement paste and aggregates, alternate freezing and thawing, etc.), and chemical (alkali-silica and alkali-carbonate reactions, chloride and sulfate attacks, etc). Concrete is also considerably fire resistant. With the exception of cement, the fact that the materials for its production like water, aggregates (sand, or gravel, or crushed stone), are cheap, and are readily available makes it more economical to use than other construction materials.

However, concrete, though strong in compression, it is weak in tension – which is about $\frac{1}{10}$ of the compressive strength. Plain concrete is therefore limited in load-carrying capacity because of the low tensile strength. This usually results in massive sections for relatively small loads. But steel is much stronger than concrete in both compression and tension, though more expensive and difficult to work with. When the tensile weakness of plain concrete is overcome by the provision of steel reinforcement in the tensile zone, reinforced concrete is produced. Figure 2. 3 shows a typical reinforced concrete beam. The steel reinforcement is placed near the bottom of the beam where they are most effective in resisting the tensile stresses due to bending. For this reason, reinforced concrete can be considered as a composite material.

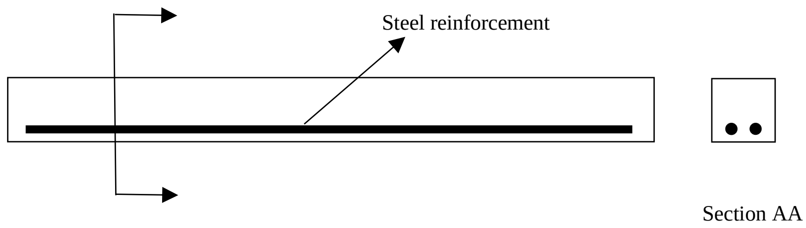


Figure 2.3: A typical reinforced concrete beam

2.2 Structural Concrete

Although, concrete has been defined generally, it is however necessary to distinguish two types of concrete. These are:

i. Concrete produced without adherence to any written national specification or/and standard. This is the type of concrete produced just by anybody
ii. Concrete produced to meet the recommendations and specifications of a national code or standard. This type of concrete is what is called **structural concrete.** It is assumed that the producer of structural concrete must have passed through relevant trainings in the nuances and interpretations of the national code or standard under which the concrete is to be produced. Thus, structural concrete can be defined as:

> "the concrete produced by qualified personnel, in accordance to the specifications of a national standard, in relation to the quality and approved properties of the ingredients, batching, mixing, placement, compaction and curing procedures".

Structural concrete will perform as expected during the whole service life of the structure as stipulated in Table 1.1.

2.3 Structural Elements

Structural elements represent the basic components used to form more complex structural systems. To perform its function of supporting a building in response to whatever loads that may be applied to it, structural elements must be such that they allow the building to meet the certain criteria. These are:

- Appropriateness. That is, it should be appropriate for the intended purpose
- Economy. That is, the appropriate arrangement of resources and of the site, as well as the moderation of expense in the work by means of thrifty reasoning to be achieved by not specifying what cannot be found or prepared at great expense
- Structural Adequacy
    - i.) State of Equilibrium
    - ii.) Stability

iii.) Adequate Strength

iv.) Rigidity (in relation to deformation/cracking)

- Maintainability.
- Sustainability. That is, good environmental impact.

2.4 Forms of Structural Elements in Reinforced Concrete

The forms of the structural elements that are used in reinforced concrete are many. Some of them are hereunder discussed.

2.4.1 Beam

This is a structural member loaded perpendicular to its longitudinal axis (Figure 2.4)

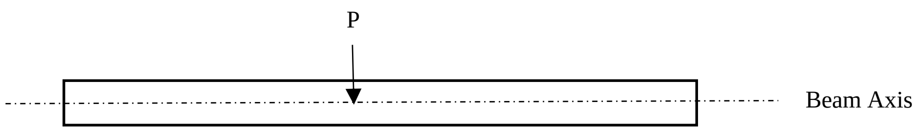


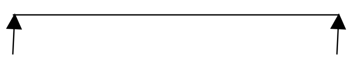
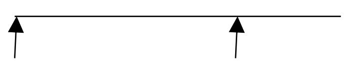
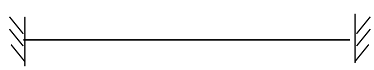
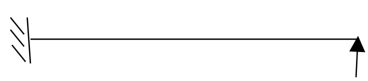
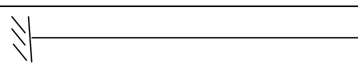
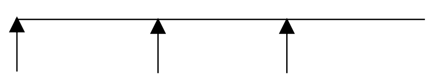
Figure 2.4: Mechanism of Load Transfer by a typical beam

Mechanisms of Load Transmission by beam is through the development of BM and SF at different sections. Beams can be classified in many ways.

1) **Classifications based on system of support**

The most common form of beams under this classification are shown Table 2.1.

Table 2.1: Common types of beam based on support systems

	Type of Beam	The Support Conditions
1	Simply supported Beam	
2	Simply supported Beam with overhang	
3	Fixed end Beam	
4	Propped Cantilever Beam	
5	Cantilever Beam	
6	Continuous Beam	

2)Classification based on the method of analysis

This is a function of the support systems. The support system will create either determinate or indeterminate structures. Analysis will be carried out accordingly. Analysis of determine beam will be carried out exclusively using the equation of static equilibrium. While methods for analyzing statically indeterminate beams include the consideration of:

i. Equations of static equilibrium
ii. Compatibility of deformations
iii. The material's stress/strain or constitutive relations

3)Classification based on design method

On the basis of this, a beam can be designed as:

- ✓ Singly reinforced – compression reinforcement is not considered as contributing to the load-resisting function of the beam (Figure 2.5)

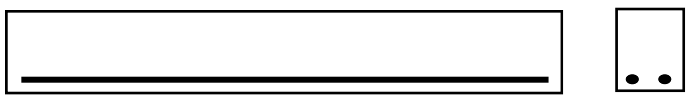


Figure 2. 5: Singly Reinforced Beam

- ✓ Doubly reinforced – compression reinforcement together with the tension reinforcement jointly responsible to the load resistance ability of the beam. The design is used when there is restriction to the depth of the beam. This is shown in Figure 2.6.

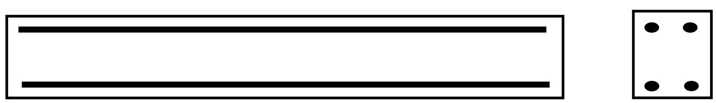


Figure 2.6: Doubly Reinforced Beam

4)Classification based on construction method

Beams are usually constructed monolithically with slabs. Depending on their positions, beams can either be in the form T-sections or L-sections as shown in figure 2.7. Along the perimeter of the building, are formed L-sections beams, while T-beams are formed at interiors.

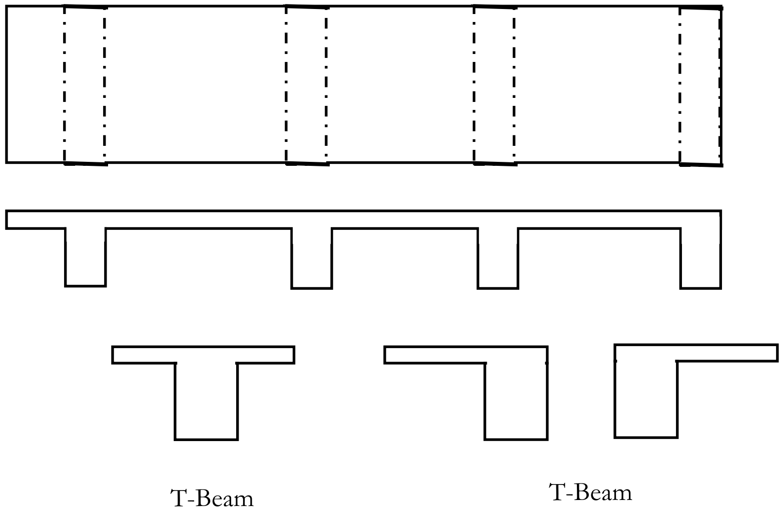


Figure 2.7: Typical T-Beam and L-Beam

5)Classification based on geometry

When beams are classified according to the ratio of span to height, shallow or deep beam results

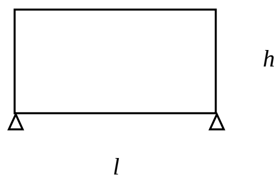


Figure 2.8: Deep Beam

$\frac{l}{h} < 2.0$ for simply supported deep beam and $\frac{l}{h} < 2.5$ for continuous deep beam (Figure 2. 8)

2.4.2 Column

Columns are usually a vertical member which carry load along its centroid axis (Figure 2. 9).

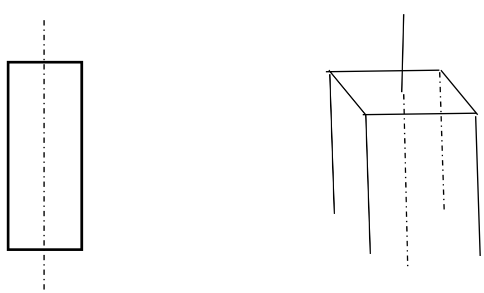


Figure 2.9: Column

In addition to axial load, columns could also transmit bending moment, if loads are not applied axially. Columns are divided into two on the bases of slenderness ratio into:

- Short column → failure is by crushing
- Long/Slender Column → failure is by buckling

2.4.3 Slab or Thick Plates or Walls

This is a planer member with small depth compared to its length and width.

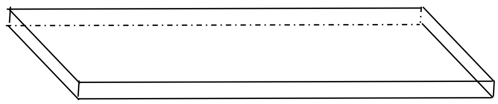


Figure 2.10: Slab

They are usually used as floors, roofs, bridge deck, wall for storage tanks, etc. Depending on the support, slabs can develop two modes of structural actions. These are one-way structural action and two-way structural action. Design approaches are different. In one-way structural action slab (figure 2. 10), supports are on the opposite edges and slab bends in one direction. Reinforcement is design in the principal direction.

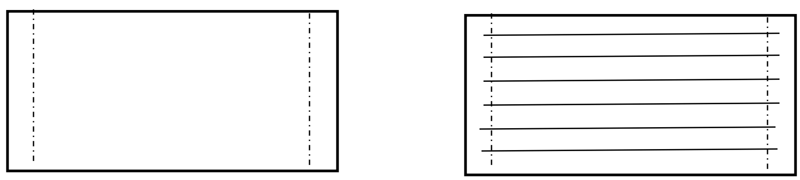


Figure 2.11: A typical 1-way slab

But in two-way structural action slab, support condition around the boundaries create double curvature. Reinforcement is thus designed for the two principal directions as shown in figure 2.12.

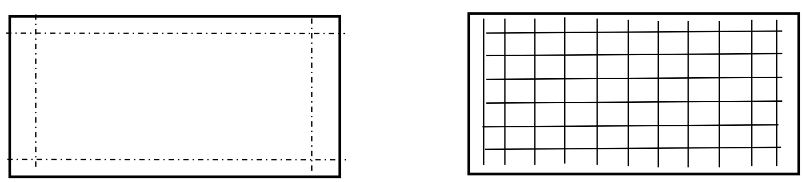


Figure 2.12: A typical 2-way slab

Depending on the forms of construction, slabs exist in many forms. These are:

1) **Solid Slab**

Solid slabs are the simplest to design and construct. The slab is of uniform thickness throughout its length. It is cast on flat formwork. Its design is similar to the design of a rectangular beam (Figure 2.13).

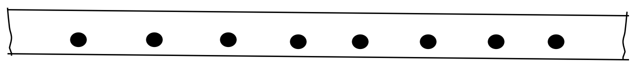


Figure 2.13: A solid Slab

In the design of solid slab, calculations are based on a width of one metre.

2) **Ribbed Slab**

When the span is in excess of about 4 m, the dead load, due to the self-weight of solid slab becomes significant. To reduce the dead weight, ribbed slab can be used as alternative and it is shown in figure 2.14.

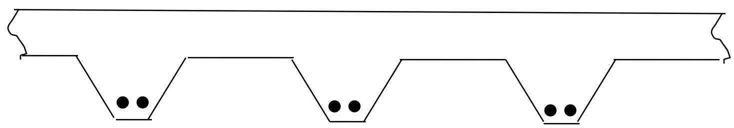


Figure 2.14: A ribbed slab

3) **Hollow block Slab**

The hollow block slab, as shown in figure 2.15, is another weight-saving device similar to ribbed slabs and are thus design in a similar way.

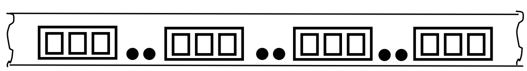


Figure 2.15: Hollow block slab

4) **Voided Slab**

Voided slabs (Figure 2.16) are also weight-saving devices whose design is also similar to ribbed slabs. They are used when slabs are thick and long in spans.

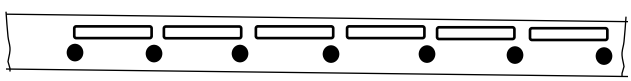


Figure 2.16: Voided Slab

5) **Flat Slabs (with drop panel and capital)**

Usually slabs are designed and constructed as a part of a structural system of slab-beam-column, in the loads from the slabs are transmitted to the columns by beams. In flat slabs, no such beam existed. The slabs are supported solely by the column. In order to reduce the shear and high bending at the columns, drop panels or column heads are provided. (Figure 2.17)

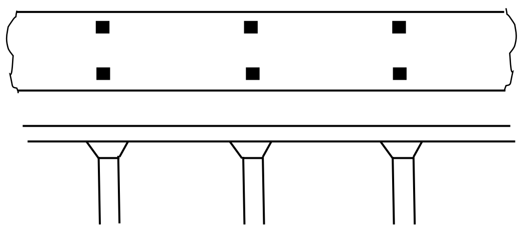


Figure 2.17: Flat slab

6) **Flat plate**

Flat plates are also beam-less slab. The slab loads are transmitted directly to the columns (Figure 2.18).

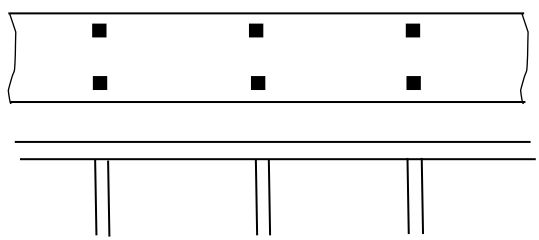


Figure 2.18: Flat plate

2.4.4 Foundation

This is a structural member responsible for the transmission and distribution of loads from the superstructure (through columns, concrete walls, block walls, etc.) to the ground without exceeding the bearing capacity of the soil nor exceeding the settlement limits. Foundation can be shallow or deep depending on its dimension. Example of shallow foundations are pad footing, strip footing, mat foundation, raft, etc. examples of deep foundations are pile foundation, piers, etc. (Figure 2.19)

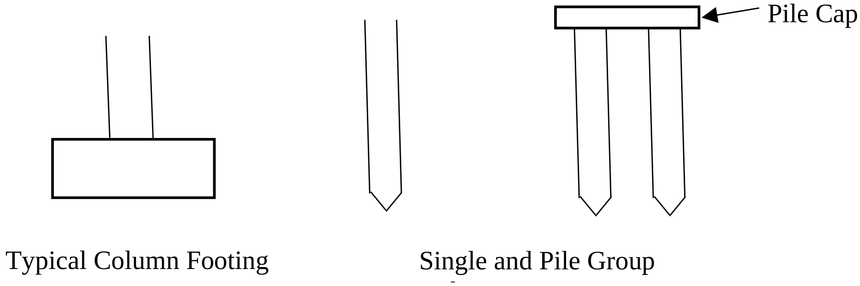


Figure 2.19: Some types of foundation in use

2.4.5 Stairs

Stairs are structural devices for vertical movement between floors in multi-storey buildings. They occur in many shapes and forms (Figure 2.20).

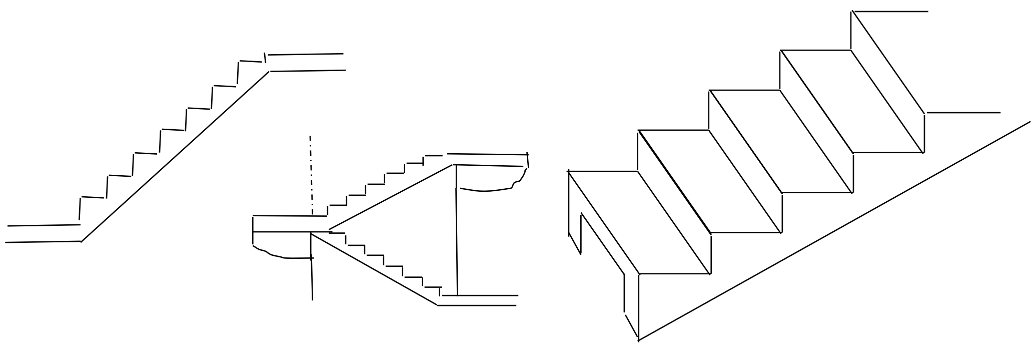


Figure 2.20: Typical stairs in use

2.4.6 Arch

Arch is a structural element curved in elevation (Figure 2.21) and stressed in direct compression that is used to span long openings or gaps (up to 600 m or more).

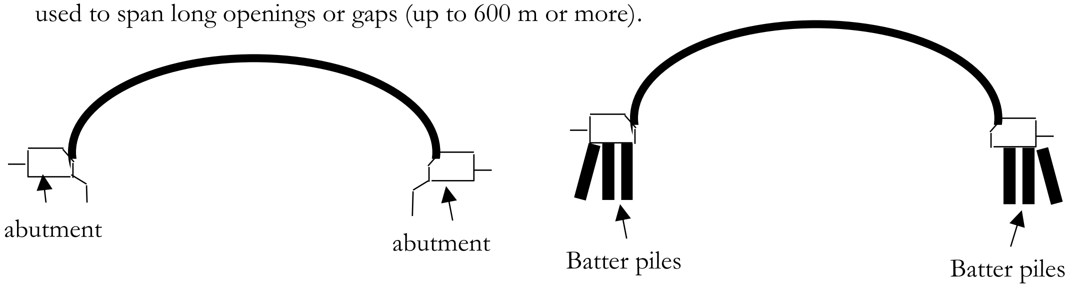


Figure 2.21: arches

For a given span and rise, only one shape of arch exists in which direct stress occur for a particular force system. For other loading conditions, bending moments develop and can produce large deflection in slender arches. Thus, selection of appropriate arch shape by early Roman builders reflect a rather sophisticated understanding of structural behavior. Arches transmit load by two mechanisms:

i. By direct forces only if the arche is shaped in such a way that the resultant of internal forces on each section passes through the centroid
ii. By compression and bending. The moments can produce deflection in slender arches

The supports for arches are provided by: (i) the construction of massive abutment, (ii) by tying the ends of the arch to a tension member, and (iii) by provision of piles to support the abutment

2.4.7 Shell

Shell is thin section and curved surface structural element that is used to cover large unobstructed areas. It can be cylindrical (singly-curved) or spherical (doubly-curved) in shape (Figure 2.22).

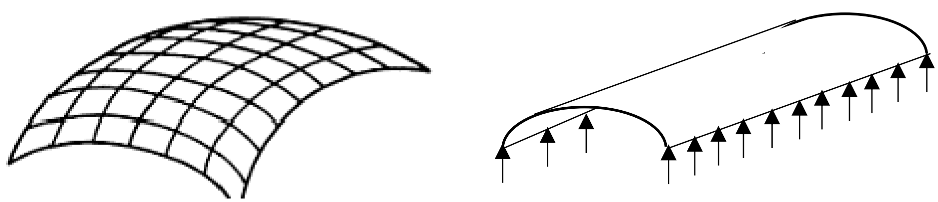


Figure 2.22: A shell member

Loads are transmitted by shell through development of in-plane stresses (called membrane stresses). Shell are used as domes in covering sporting arena, storage tanks, etc.

2.5 Classification of Concrete Structural Elements

Methods of classifications

1) According to geometry

i) One-dimensional or Linear elements

The length of the structure is large in comparison with other two dimensions. In structural modelling for analysis, only the large dimension is considered, and the other dimensions are ignored. The structural element can be straight, for example, beams, columns, etc. It can also appear as curved elements, for example, arches, etc.

ii) Two-dimensional or Surface elements

Length and breadth are large in comparison with the thickness. In analysis of such element, it is usual to model it as having to thickness. It can be planar, for example, slabs, walls, etc. it can also exist as curved (single or double) as in shell, domes, etc.

iii) Three-dimensional elements

All the dimensions are equally important. Examples are foundations, retaining walls, gravity dams, etc. All the dimensions are considered relevant in the analysis.

2) According to relationship between the applied load and the element

Two types of structural elements exist according to this classification, namely, Linear and non-linear. In Linear structural element, a linear relationship is assumed to exist between the applied loads and the resulting displacement in the member. This assumption rests on the conditions that (i) the structural material is elastic, and thus obeys Hooke's law throughout the range of loading considered, and (ii) that the changes in geometry in the structural element are so small that they can be neglected when stresses are being calculated.

In non-linear structural element, a non-linear relationship exists between the applied loads and displacements. This situation will exist under any of the following two conditions.

i) The material of the structural element is inelastic, that is, does not obey Hooke's law.
ii) The material is within elastic range, but the geometry of the element changes significantly during application of loads.

3) According to the materials of construction

Structures can be classified according to its predominant materials of construction. Common materials used for construction are:

i) Timber
ii) Concrete
   - In-situ vs. Precast
   - Reinforced Concrete
   - Prestressed
   - Etc.
iii) Steel
iv) Aluminum
v) Composite materials

4) According to Structural Efficiency

The structural efficiency (ε) is considered here in terms of the weight of material which has to be provided to carry a given amount of load. Efficiency in the use of material was also of a high priority partly in a genuine attempt to economize on material in order to save cost, but also as a consequence of the prevalence of the belief in the modernist ideal of 'rational' design. The efficiency of a structural element is regarded as high if the ratio of its strength to its weight is high. It can be expressed as:

$$\varepsilon = \frac{Srength\ of\ Structure}{Weight\ of\ Structure}$$

OR

$$= \frac{Load\ carried\ on\ a\ structural\ element}{Self\ weight\ per\ unit\ length\ of\ the\ Structural\ element}$$

When considering the efficiency of s structural member, the followings should be taken into consideration.

i. The volume and therefore the weight of material required for a structure is dependent principally on its overall form in relation to the pattern of applied load and on the shapes of the structural elements in both cross-section and longitudinal profile.
ii. The higher the efficiency of structural member, the more complex the form. Efficient structures are complex
iii. Some simple measures that can be taken to improve structural efficiency are:
   a. the use of I-shaped or box-shaped cross-sections for beams instead of solid rectangles,
   b. the use of triangulated internal geometry instead of a solid web for a girder.
iv. Efficient structures are more difficult to construct and maintain
v. The longer the span, the greater is the need for high efficiency. For example, a beam with long span must have a greater depth so as to have adequate strength. The self-weight of each beam is directly proportional to its depth and so the ratio of load carried to self-weight per unit length of beam (the structural efficiency) is low. Thus, the structural efficiency of the beam (its capacity to carry external load divided by its weight) would steadily diminish as the span increased.
vi. More efficient shapes will have to be used as the span is increased if a constant level of load to self-weight (efficiency) is to be maintained.

In summary, classification of structure on the basis of how effective the load-resisting functions of the building depends on whether appropriate efficient structural forms have been used or NOT.

The principal objective of engineering design is to provide an object which will function satisfactorily with maximum economy of means.

2.6 Structural Systems for Reinforced Concrete

An assemblage of structural elements results in structural system. The term **structural system** or **structural** frame in **structural** engineering refers to load-resisting **system** of a **structure**. The **structural system** transfers load through interconnected **structural** components or members. Designing a building requires that technical decisions be made concerning the suitable structural systems. These decisions are:

i. Selecting efficient, economical, and attractive structural form
ii. Evaluation of the safety of the selected structural forms with respect to strength and stiffness (sagging, vibration, sway, etc. under loads)
iii. Planning its erection under temporary loads.

The evolution of structural form, from the primitive **trial and error designs** of the Egyptians and Greeks to the **highly sophisticated configuration** of the present with Romans imprint, is related to many factors, which are:

i. The materials that are available. For example, the use of stones – brittle and low/variable in tensile strength - by Egyptians meant amongst others: (i) short beam spans to prevent bending failure, (ii) thick stone columns, (iii) low-storey buildings, etc. The result is "post and lintel" systems. But the discovery of cast iron and steel with high tensile and compressive strengths made possible the:
   - Design of shallow but strong beams
   - Design of columns with compact cross sections
   - Design of lighter structures with longer open spans
   - Allowed construction of taller structures, . . . and this led to skyscraper of today
   - Enabled construction of long-span bridges with steel cables
ii. The state of construction technology
   - Manual Vs Machines
   - Innovative construction of available materials. For example:
     - Combining steel with concrete to produce RC. This makes possible variety of forms to be constructed.
iii. The designers' knowledge of structural behavior and analysis
   - Analysis of Determinate and indeterminate structures
   - Use of Moment distribution by Hardy Cross for analyzing indeterminate structures/frames.
   - Introduction of computer permits designer to analyze complex structures like plate, shells, etc.
iv. The skill of the construction worker.
   - Compare the skill of Roman engineers with the Greek.
     - The use of efficient structural forms e.g. arch

- The use of selecting materials
- Good workmanship

2.7 System of Reinforced Concrete Building Structures

The advantages of reinforced concrete structural systems for building can be summarized as follows:

1) Versatility of Structural Properties. Concrete consists in four parts – cement, fine aggregate, coarse aggregate and water, that can be variously composed or proportioned or modified to give a wide range of concrete with different strengths, stiffness, durability, and all other desirable structural requirements.
2) Versatility of Form. Concrete is placed in the formwork in the fluidy state is readily adaptable to a wide range of geometric, architectural and functional requirements.
3) Durability. With proper concrete protection for the reinforcement, the building will have a long life even under a very severe climactic and environmental conditions
4) Fire resistance. Concrete has the best fire resistance of all the building materials. A structural system in reinforced concrete provides a best protection for the reinforcement.
5) Speed of construction. In terms of the entire period from the state of approval of the projects drawings to the date of completion, reinforced concrete buildings systems can be completed in a less time when compared with steel structures, although the field erection of steel is of course more rapid. But it must be preceded by steel supply process and fabrications of all the parts in the workshop.
6) Cost. The first cost of construction is comparable with the cost of similar building constructed in steel structural systems. But in almost every case, maintenance cost of reinforced concrete are less.
7) Availability of Labor and Materials. It is always possible to make use of local sources of materials and labor in reinforced concrete structures. For example, aggregates could be sourced from local environment or suitable alternatives can also be available, so that only cement and reinforcement are brought to the site.

These advantages are particularly noticeable in multi-storey buildings such as blocks of flats, public buildings, etc.

Reinforced concrete structural systems can be viewed as consisting in two parts

i. Horizontal parts. That is, floors, roofs, etc.
ii. Vertical parts. That is, columns, walls, partitions, etc.

All other types such as staircase or the underground parts are connected with the chosen type of horizontal and vertical structural systems.

2.8 Structural Systems for Low/Medium Storey Buildings

A building structure with less than 10 suspended floors is regarded as low to medium storey buildings. The structural systems, that is, composition and arrangement of structural elements that can be employed for such structure:

1) Load bearing walls
2) Portal frames
3) Rigid frames

The structural materials that can be used in the construction of the above-named structural systems can consist in the followings:

1) Reinforced Concrete structures
   a. In-situ concrete
   b. Precast concrete
   c. Prestressed Concrete
2) Timber structures
3) Steel framings
4) Innovative construction materials
   a. Lightweight concrete – Aerated concrete, foamed concrete, etc.
   b. Fibre-reinforced concrete
   c. Laterized concrete
5) Combination of two or more of the above

2.9 Structural Systems of Tall Buildings

When the numbers of the suspended floors in a building is ≥ 10, then lateral drift begins to control the design, and stiffness, rather than strength becomes the dominant factor. Structural system based on its relative effectiveness in resisting lateral loads. Readers are advised to consult relevant books on the subject.

Assignment

1. Explain some of the reasons that made concrete a dominant construction material.
2. Differentiate between structural element and structural system
3. What do you understand by structural efficiency
4. Discuss some of the factors responsible for the evolution of structural forms.

Chapter 3- Properties of Materials for Structural Concrete

3.1 Introduction

In order to determine the size of a member in a structure or the proportion of materials (concrete, steel, prestressing, etc.) in composite sections, thorough knowledge of the properties of materials is necessary. Such properties include:

a. Strength – which gives the overall picture of the quality of the concrete, and it is related to the structure of hardened cement paste
b. Elasticity – this a measure of weather strain appears or disappear immediately on application and removal of loads.
c. Creep – deformation or strain with time under constant loading
d. Durability – ability to withstand the environmental conditions for which it has been designed without deterioration over a period of years
e. Shrinkage – volumetric shortening of concrete by loss of moisture with time
f. Etc.

Of interest in the examination of materials properties are:

a. The stress (σ) or strain (Є) at which a material will fail
b. The stress-strain relationship determined from a simple test

3.2 Concrete

It has been found that not only do the properties vary within some particular batches of materials, but also the stress-strain relationship is complex. It was indeed found that the strength of samples taken from a batch of materials approximate to the normal distribution (Figure 3.1)

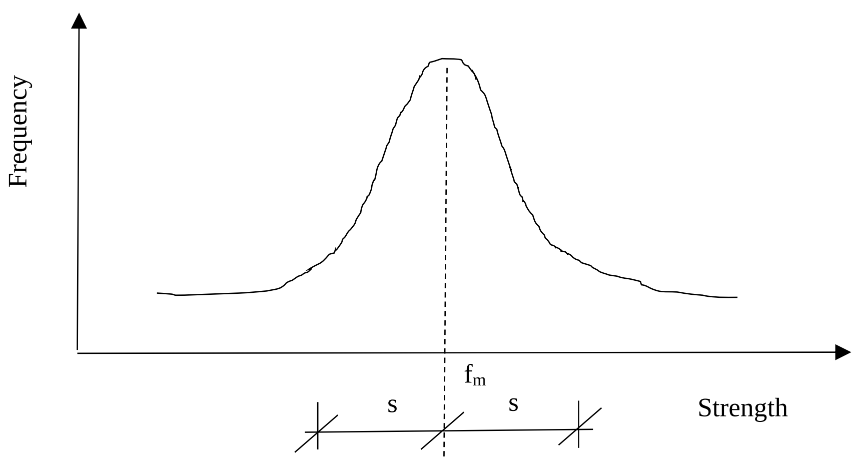


Fig. 3.1: Typical frequency distribution of strength of concrete mixes

Where:

f_s = strength of a sample

f_m = the arithmetic means of the sample strength

n= the number of samples

s =the standard deviation

$$= \sqrt{\frac{\sum(f_s - f_m)^2}{n-1}} \qquad \textbf{3.1}$$

The value of the standard deviation is an indicator of the quality control exercised during the production process. A publication by COREN (2017) recommends a standard deviation of 6MPa where quality control is low.

3.2.1 Characteristic Strength of Concrete (Clause 2.4.2.1)

a. Concrete

Concrete is a mixture of water, coarse aggregates, fine aggregates, and cementitious binder (usually Portland cement). Because of the difficulty to produce a homogenous mix, the strength of concrete and other properties varies considerably depending on factors like curing (methods and duration), compaction, and transportation. The compressive strength of concrete is usually determined by carrying out compressive tests on 28 days old cubes or cylinder. Carrying out many such test from the same mix and plotting the crushing strength against the frequency of occurrence will approximate to a normal distribution curve.

For design purpose, it is necessary to assume a unique value for the strength of the mix. A value that should not be too high or too low. This value is called the characteristic strength.

BS 8100 defined characteristic strength (f_c) of concrete as that value of the cube strength of concrete below which not more than 5% of the test would fall.

The characteristic and the mean strength are related by the expression

$$f_c = f_m - 1.64s \qquad \textbf{3.2}$$

Where:

f_c= characteristic strength

f_m= mean strength

s= standard deviation

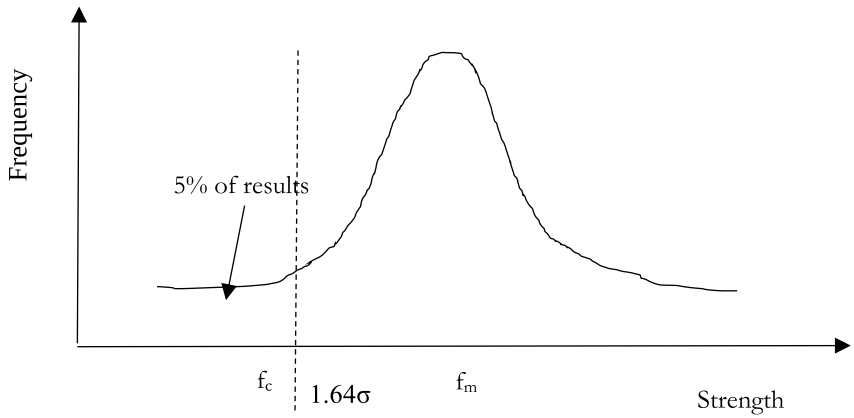


Figure 3.2: Characteristic strength of concrete

From figure 3.2, the characteristic strength f_c is:

Thus, assuming a mean strength of 50N/mm² and standard deviation of 5N/mm², the characteristic strength of the mix is:

$$= 50 - 1.64 * 5$$

$$= 41.8\text{N/mm}^2$$

Characteristic strength of concrete is also an indication of the "grade" of concrete. For example, a grade 30 concrete (C30) means a characteristic strength of 30N/mm². BS 8110 recommend that the lowest grade of concrete of C25. The Table 3.1 shows the characteristic strength of various grades of concrete normally specified for reinforced concrete. Concrete grade is specified in relation to cube strength or cylinder strength. In Table 3.1, the number in parenthesis is the cube compressive strength

Table 3.1: Characteristic Strength of Concrete

Concrete grade	Characteristic Strength fc (N/mm²)
C20 (25)	25
C25 (30)	30
C28 (35)	35
C32 (40)	40
C35 (45)	45
C40 (50)	50
C50 (60)	60

3.2.2 Concrete Mix Design

The manner a particular strength is achieved is through the process of **Mix Design.** Mix design is a process of selecting the relative proportions of cement, coarse aggregate, fine aggregates and water to achieve a particular strength**.** The results of process of mix design, at a particular water/cement ratio, is usually expressed in ratios as in this format: A: B: C.

Where

A = proportion of cement in the mix

B = proportion of fine aggregate in the mix

C = proportion of aggregate in the mix

For example, a concrete mix 1: 2: 4, means 1 part of cement to 2 parts of fine aggregate to 4 parts of coarse aggregate.

In the mix design, the most important requirements are:

1. The fresh concrete must be workable, or placeable at minimal effort without segregation.
2. The hardened concrete must be strong enough to carry the loads for which it has been designed
3. The hardened concrete must be able to withstand the conditions to which it will be exposed in service.
4. It must be capable of being produced economically.

Thus, requirements for a good concrete can be summarized as workability, strength, durability and economy.

In Nigeria, the commercial ordinary Portland cement are to be produced to meet NIS 444-1 (2014). Also, the Council for the Regulation of Engineering in Nigeria (COREN) has published a manual on the Concrete Mix Design (COREN, 2017) for use in Nigeria. The range of strength covered by the manual, using the cement produced in Nigeria, ranges from 10 – 115 MPa.

3.2.3 Elastic Modulus for Concrete (E_c)

When no information is available, short term elastic modulus may be taken from Table 2

Table 3. 2: Elastic Modulus of Different Characteristic Strength

Cube Strength of Concrete (N/mm^2)	Modulus of Elasticity of Concrete Ec (kN/mm^2)
20	25
25	26
30	28
40	31
50	34
60	36

In calculating the weight of the structure, density of concrete should be taken as 2400kg/m^3 (or 24KN/m^3)

3.2.4 Design Strength of Concrete

The tests to determine the characteristic strength of concrete and reinforcement are carried out on near perfect specimens, prepared under controlled laboratory conditions. This condition will not occur in practice. For example, concrete test cubes are usually taken at the mixer and compacted, cured under ideal conditions. But the strength of the structure may be reduced by:

a) Segregation in transit
b) Improper casting conditions
c) Bad curing conditions due to the effect of heat, rain, frost, wind, etc
d) Inadequate compaction.

Steel strength may vary as a result of corrosion of steel and the difference between the assumed and the actual cross-section. To take into account these differences, the characteristic strength is divided by a partial factor of safety (γ_m) (Table 3.3). The resultant strength is called the design strength given as:

$$\text{Design Strength} = \frac{f_c}{\gamma}$$

Table 3.3 Factor of Safety (Clause 2.4.4.1)

Limit State	Material	
	Concrete	Steel
Ultimate		
Flexure	1.50	1.15
Shear	1.25	1.15
Bond	1.40	-
Serviceability	1.00	1.00

3.2.5 Stress-Strain Relationship for Concrete (clause 2.4.2.3)

The stress-strain (σ-ε) curve of concrete are influenced by a number of factors such as:

a) Creep – continuous deformation under sustained load
b) Type of aggregate
c) Strength of concrete
d) Age of concrete
e) Curing of concrete

The typical stress-strain curve for concrete is shown (Figure 3.3) for a concrete cylinder under a uniaxial compression.

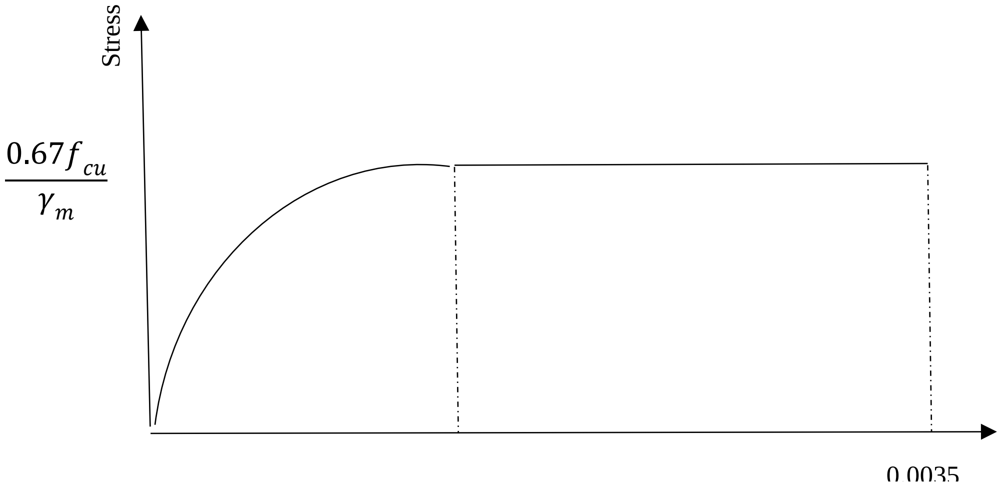


Figure 3. 3: Stress- Strain Curve for normal weight Concrete

Note

i. The stress-strain behavior is not linear
ii. The maximum compressive stress at failure is about 0.8 of characteristic strength (i.e. $0.8f_c$).
iii. BS 8100 assumes 0.67 (instead of $0.8f_c$) because of difficulty in obtaining a mathematically model for its behavior using fig. 3.3.

Thus, the design compressive strength is:

$$\frac{0.67f_c}{\gamma_m} = 0.45f_c \qquad \textbf{3.3}$$

Assignment

Concrete samples are taken from three batches (A, B, and C) of concrete and cubes are cast from them. The crushing strength of the cubes are as follows:

A. 30, 34, 25, 34, 34, 45, 50, 40 N/mm^2
B. 35, 30, 34, 50, 40, 45, 35, 25 N/mm^2
C. 40, 35, 45, 40, 35, 35, 40, 35 N/mm^2

For each of the batches, determine the

i) The average strength
ii) The characteristic strength*

Ans: A (36.5, 23.21N/mm^2), B (36.75, 23.58N/mm^2), (38.13, 32.03N/mm^2)

3.3 Reinforcement Steel

Concrete is strong on compression but weak in tension. Because of this, it is the normal practice to provide steel reinforcement in those areas where tensile stresses in the concrete are likely to develop. Thus, it is the tensile strength of the reinforcement that the designer is concerned with.

3.3.1 Stress-Strain Relationship for Steel

The ordinary real diagrams are not suitable, for use. The Code recommend the approximate tri-linear diagram for all grades of reinforcement and the same value for modulus of elasticity (E_s) of 200KN/mm^2 **(Figure 3.4)**

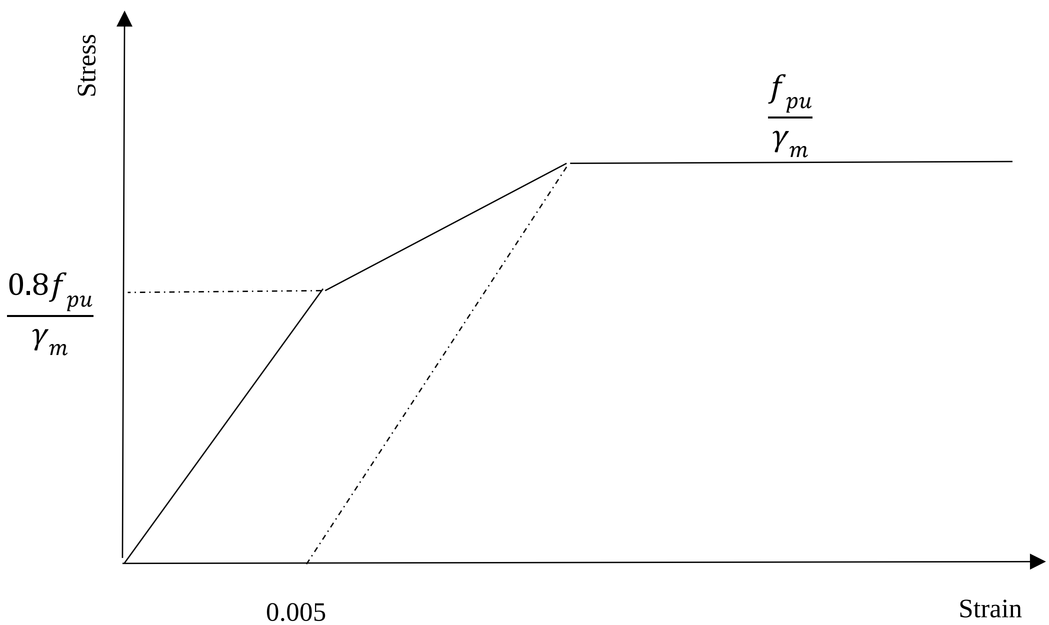


Fig. 3.4: Tri-linear Stress-Strain Diagram for Steel (2.4.2.3)

3.3.2 Characteristic Strength of Reinforcement Steel

BS 8100 defined characteristic strength (f_c) of steel as that value of the yield stress of steel (reinforcement) below which not more than 5% of the test would fall

The typical values of characteristic strength of steel (f_y), for mild steel and high yield reinforcement, according to BS 8110 recommendation, that are available in Nigeria, are shown in Table 3.4. But in Practice, high yield steel is used, most of the times, as reinforcement in Nigeria.

Table 3.4: Steel used as reinforcement in Nigeria

Designations	Characteristic Strength (fy)	Sizes
Mild Steel	250 N/mm2	≥ 8 mm
High yield	460 N/mm2	≥ 8 mm

3.4 Loading

In addition to material properties, the type and the magnitude of the loadings to which the structure may be subjected to during its design life must be known by the designer. Loadings on structure are divided into three:

i) Dead load

These are loads which are due to the effects of gravity, i.e. the self-weight of all permanent constructions such as beams, columns, walls, roofs, finishes. If the position of permanent partition walls is known, their weight can be assessed and included in the dead load.

ii) Imposed load

These are loads due to variable effects such as the movement of the people, furniture, equipment and traffic. Their values are adopted based on observations and measurement

iii) Wind load

This is an environmental loading in which source is beyond human control. Some of the main features which influence the wind loading imposed on a structure are:

- Geographical location
- Physical location – open country,
- Topography – exposed hill top, escarpment
- Altitude – height above mean sea level
- Building shape -

iv) Others

In some circumstances other lading types may be considered such as settlement, fatigue, temperature effects, dynamic loadings, or impact effects, bomb blast, etc. In most design cases, the design consideration combining these is most appropriate.

3.4.1 Characteristic Load

Classifications of loads on structure are:

1. Dead loads – weight of the structure (Gk)
2. Imposed loads – due to furniture, occupant, machinery, vehicles (Qk)
3. Wind load - (Wk)

Because of scarcity of information necessary to determine characteristic Dead Load (G_k), Imposed load (Q_k), and Wind load (W_k), designers are recommended to use the values in the following documents:

i) Reynolds and Steedman and Threlfall – Reinforced Concrete Designers Handbook

ii) BS 648 – Schedule of Weight for Building Materials (extract in Table 3.5)

iii) BS 6399 – Design loading for Building (extracts in Table 3.6),

- Part I: Code of Practice for Dead and Imposed Loads
- Part II: Code of Practice for Wind loads.

The characteristic load selected should be that which produces the worst effect.

The "characteristic load" is defined as the load above which not more than 5% of the load falls when all loads likely to be applied to the structure during its working life are considered.

Table 3.5: Some selected unit masses of Structural materials (based on BS 648)

Asphalt	Roofing 2 layers, 19 mm thick	42kg/m²
	Damp-proofing, 19 mm thick	41 kg/m²
	Roads and footpaths, 19 mm thick	44 kg/m²
Bitumen roofing felts	Mineral surfaced bitumen	3.5 kg/m²
Blockwork	Solid per 25 mm thick, stone aggregate	55 kg/m²
Brickwork	Clay, solid per 25 mm thick medium density	55 kg/m²
Concrete	Natural aggregates	2400 kg/m³
Plaster	Two coats gypsum, 13 mm thick	22 kg/m²
Plastics sheeting (corrugated)		4.5 kg/m²
Plywood	per mm thick	0.7 kg/m²
Reinforced concrete		2400 kg/m³
Rendering	Rendering Cement: sand (1:3), 13 mm thick	30 kg/m²
Screeding	Cement: sand (1:3), 13 mm thick	30 kg/m²
Terrazzo	25 mm thick	54 kg/m²

Table 3.6: Typical Imposed load (BS 6399, Part 2)

		Intensity of distributed load KN/m²	Concentrated Load
Type 1	Self-contained dwelling unit	1.5	1.4
Type 2	Apartment houses, boarding houses, lodging houses, guest houses, hostels, residential clubs and communal areas in blocks of flats Boiler rooms, motor rooms, fan rooms and the like including the weight of machines	7.5	4.5
	Communal kitchens, laundries	3.0	4.5
	Dining rooms, lounges, billiard rooms	2.0	2.7
	Bedrooms, dormitories	1.5	1.8
	Corridors, hallways, stairs, landings, footbridges, etc.	3.0	4.5
Type 3. Hotels and motels	Boiler rooms, motor rooms, fan rooms and the like, including the weight of machinery	7.5	4.5
	Corridors, hallways, stairs, landings, footbridges, etc.	4.0	4.5
	Kitchens, laundries	3.0	4.5
	Dining rooms, lounges, billiard rooms	2.0	2.7
	Bedrooms	2.0	1.8

3.4.2 Design Load (F)

Variations in the characteristics loads, usually statistically determined, often arise as a result of a number of reasons like:

i) Inaccuracies in the assessment of the loading and stress distribution within the structure

ii) Constructional inaccuracies

iii) Errors in analysis and design

iv) Unusual increase in load beyond those envisaged when deriving characteristic loads.

In order to account for these effects, the characteristic loads F are multiplied by the appropriate partial safety factor for loads ($\mathbf{g}_g$). Each characteristic load (dead, imposed, wind) is multiplied by a partial safety factor and the load are added to give the design load. That is:

$$F = \mathbf{g}_g G_k + \mathbf{q}_k Q_k + \mathbf{w}_k W_k \qquad \textbf{3.4}$$

The values of the partial safety of factor were given by code for the different limit state are as follows:

A. Values for **the ultimate limit state** (cl 2.4.3.1) for different load combination are:

1. Dead and Imposed load
    a) Design dead load = $1.4G_k$
    b) Design imposed load = $1.6\ Q_k$
2. Dead and Wind Load
    a) Design dead load = $0.9G_k$
    b) Design wind load = $1.4W_k$
3. Dead, Imposed and Wind Load
    a) Design dead load = $1.2G_k$
    b) Design imposed load = $1.2Q_k$
    c) Design wind load = $1.2W_k$

B. Values for **the serviceability limit state** (cl 2.4.3.1) for different load combination are:

1. Dead and Imposed Load
    a) Design dead load = $1.0G_k$
    b) Design imposed load = $1.0Q_k$
2. Dead and Wind Load
    a) Design dead load = $1.0G_k$
    b) Design wind load = $1.0W_k$
3. Dead, Imposed and Wind Load
    a) Design dead load = $1.0G_k$
    b) Design imposed load = $0.8Q_k$
    c) Design wind load = $0.8W_k$

Critical (Vertical) Loading Combinations (Cl. 3.2.1.2.2)

The design loads are described as either:

i. Maximum design load = $1.4G_k + 1.6Q_k$ (Figure 3.5)
ii. Minimum design load = $1.0G_k$ (Figure 3.4)

1) The arrangement of load considered should be that which causes the most severe condition of stress, stability, deflection, etc.

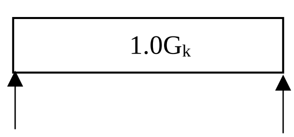


Figure 3.5: Minimum design load

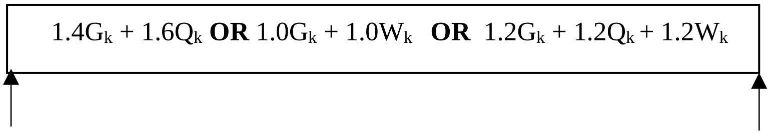


Figure 3.6: Maximum design load

2) When considering dead and imposed loads on a continuous beam with overhang system, the worst stresses at the center of span will occur when alternate load are loaded with dead load only ($1.0G_k$), and full combined load ($1.4G_k + 1.6Q_k$).

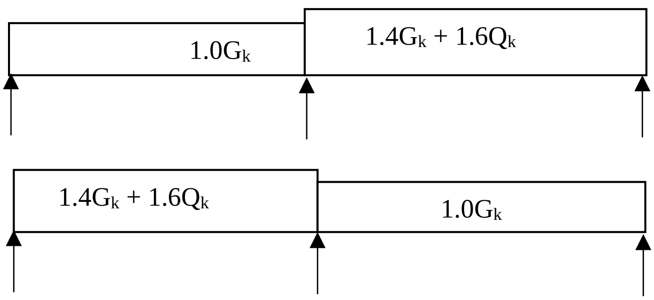


Figure 3.7 Alternate combination of loads

In order to obtain the loading conditions that give the maximum bending moments and shear forces, it is necessary to considered varieties of load combinations between spans (Figure 3.5). Analysis is done for each case, and the results of analysis of all the cases considered are plotted on the same graph. This graph is what is called **Bending Moment and Shear Force envelope**. In carrying out the analysis, the self-weight of the structural element constitutes part of the DEAD LOAD. In estimating the dead load of the concrete member, the unit weight of the concrete is taken to be 2400Kg/m^3 (or 24KN/m^3)

Assignment

For the beam shown in Figure 3.8, draw the envelope for SF and BM for all possible load combinations at the ultimate limit state according to clause 3.2.1.2.2. Determine the maximum BM and SF at the supports and the span. (Take q_k = 10KN/m and g_k = 8KN/m).

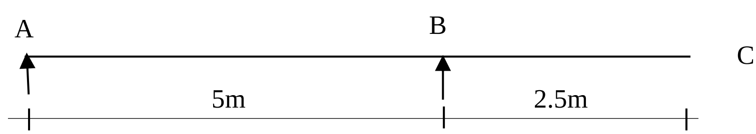


Figure 3.8

Answer: Support A: SF = 63 KN; Support B: SF = - 85.00 KN, at C: SF = 0.00KN.

Span AB: BM = 47.80 KN.m, Support B: BM = -85.00 KN.m

3.5 Load Estimation in Practice

In the design of concrete elements, loads are transferred from one structural element to another. It is very important that the loads be traced accurately and also properly assessed. A method of determining how much load is transferred from the slab to be beam is presented below to determine the design load for beam B. The slab in Figure 3.9 is a one-way (of 2 panels X and Y) slab with total depth of 150 mm.

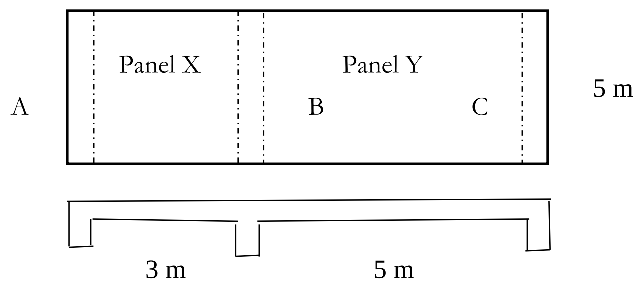


Figure 3.9

Giving the following Slab Loadings

gk = 5.00 kN/m² qk = 4.00 kN/m²

Density of concrete = 2400 kg/m ³

Take the clear dimension of the beam as b_w = 225 mm, h = 400 mm

Slab

Dead load on slab = 5.00Kn/m²

Slab self-weight = 0.15 × 2400 (x 10) = 3600 N/m² = 3.6 kN/m²

Slab imposed load (*q*k) = 4. 0 kN/m²

Total dead load = dead load + self-weight = 5.00 + 3.6 = 8.60KN/m²

Design Load (at ultimate load) = 1.4*g*k + 1.6*q*k = 1.4 x 8.6 + 1.6 × 4.0 = 12.04 + 6.4

= 18.44 kN/m ²

Beam

for a one-way slab, the load from the slab will be shared half way on either side (total width of 4m), as shown in Figure 3.10.

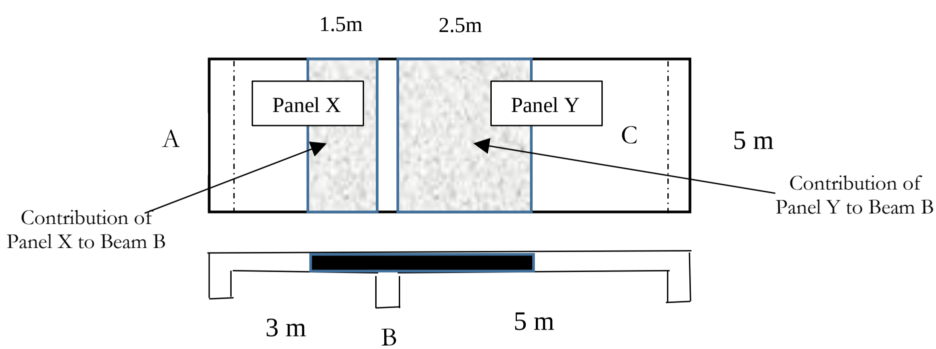


Figure 3.10: Load estimation for Beam B

Self-weight of the beam = 2400 x 0.225 x 0.4 (x 10)= 2160 N/m = 2.16 KN/m

Self-weight of the Slab on the beam = 3.6 x 4 = 14.4KN/m

Dead load from the slab = 5 x 4 = 20KN/m

Imposed load from the slab = 4 x 4 = 16KN/m

Design Load for the beam B

= 1.4gk + 1.6qk = 1.4 (2.16 + 14.4 + 20) + 1.6 (16)

= 51.18 + 25.60

= 76.78KN/m

Assignment

1) The concrete structure in Figures 3.11 has dead load gk = 4Kn/m^2 and live load qk = 5kn/m^2

If the depth of the slab = 200mm, total depth of the beam = 600mm, the width of the beam = 225mm, and the density of concrete to be 2400kg/m^3, Calculate the design loads on the beams A, B, C, D, and E

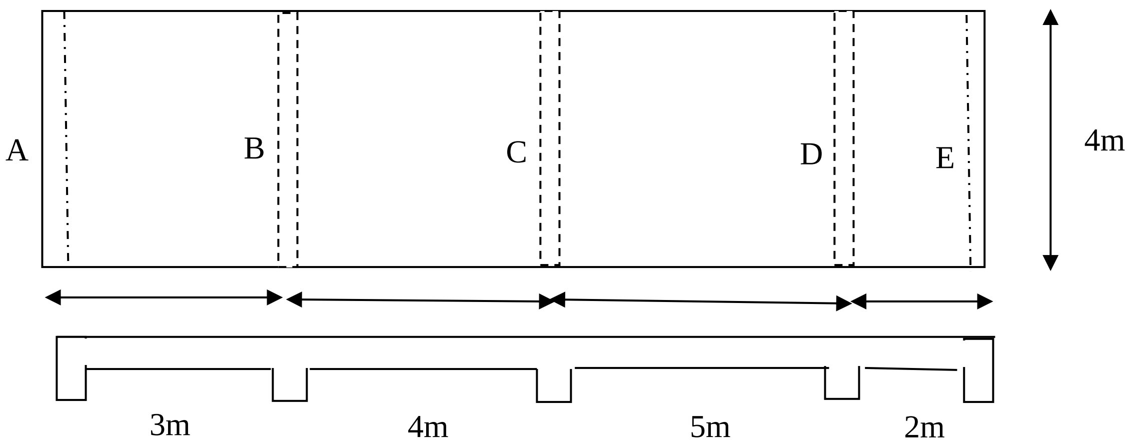


Figure 3.11

3) Calculate the design loads on the beams A, B, C, and D for a typical two-way slab with 2 Panels shown in Figure 3.12a. Assume the loads are transferred to the beams as shown Figure 3.12b.

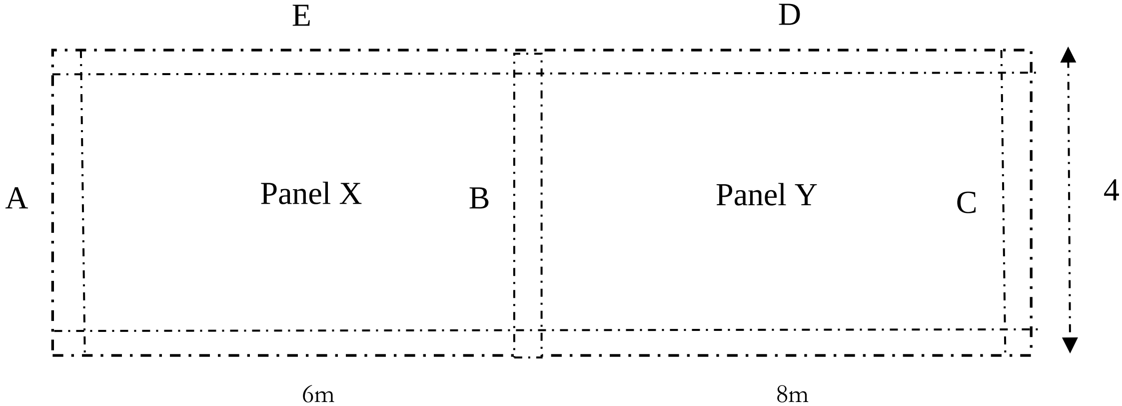


Figure 3.12a

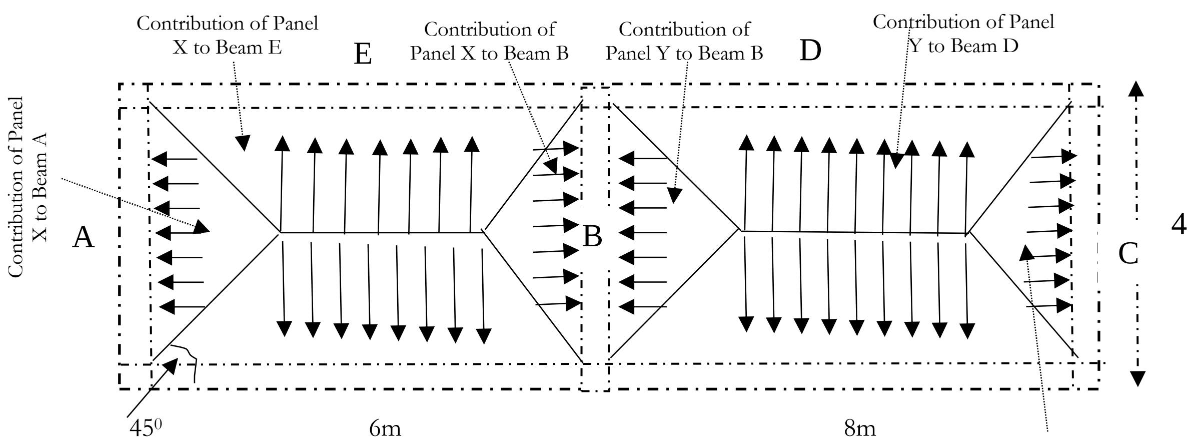


Figure 3.12b

Answers

1) **Beam A (33.50 KN/m), Beam B (74.14 KN/m), Beam C (94. 46 KN/m), Beam D (74.14 KN/m), and Beam E (23.34 KN/m)**
2) **Beam A (23.34 KN/m), Beam B (43.66 KN/m), Beam C (23.34 KN/m) Beam D (33.50 KN/m), Beam E (30.04 KN/m)**

Comments on the Assignment

1. Figure 3.10 is the approach adopted to estimate the loads on beams supporting a one-way slab. Figure 3.11 is similar to Figure 3.10, and it is to be treated as such.
2. On the other hand, Figure 3.12 is the method followed for the estimation of loads from a two-way slab coming to the beams.
3. The design of these types of slabs will be dealt with in later chapters.

Chapter 4- Analysis of Reinforced Concrete Structures

4.1 Introduction

Elastic analysis is usually used to determine internal forces and moments for both the ultimate limits states and serviceability limit state. The consequent of this is that:

1. Section sizes must be assumed so that stiffness of each member can be calculated.
2. The structure is often too large for analysis as a complete structure, even by computer, it is thus usually necessary to break down the structure into suitable sub-frames for the purpose of analysis
3. In order to ensure that the section is designed for the worst condition of loading which can be reasonably expected, a number of different load combination must be considered for each sub-frame.

4.2 Stiffness of a Member

All methods of elastic analysis involved the determination of the member stiffness (k), which is defined as:

$$\boldsymbol{k = \frac{EI}{L}} \qquad \textbf{4.1}$$

Where:

E = Youngs Modulus of elasticity of the material,

I = second moment of area of the cross section about the centroidal axis

L = actual length of the member between the joint centres

The term EI is termed as the "flexural rigidity", and it should be based on the actual properties of the material and the actual cross section of the member.

Flexural Rigidity (EI) for Concrete

For concrete, both E and I are dependent on the state of stress.

i. E is assumed to have a constant value associated with its characteristic strength.
ii. I, the second moment of area, is related to the geometry of the section (figure 4.1)

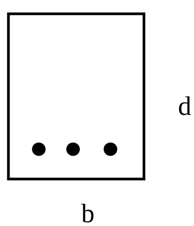


Figure 4.1 Typical geometry of cross section

The value of I may be based on any of the following sections

a. The full concrete section – entire concrete section ignoring the reinforcement

$$I = \frac{bh^3}{12} \quad \text{(for a rectangular cross section)} \qquad \textbf{4.2}$$

This is simple to calculate, and it is normally used.

b. The gross section – the entire cross section including the reinforcement on the basis of the modular ratio, n (n = $\frac{E_S}{E_C}$)

c. The transformed section – the compression area of the concrete section combined with the reinforcement on the basis of the modular ratio, n (n = $\frac{E_S}{E_C}$).

But the cross section described in (a) is simpler to calculate, and thus would normally be chosen.

4.3 Structural Analysis

Once the basic form of a structure and the external loads have been defined, stress analysis can be made to determine internal forces (bending moments, shear forces, etc.) in various members of the structure and the displacement at some controlling points. This is process is termed structural analysis of the structure and it is governed by the principles of the Theory of Structure. Design processes are usually preceded by the Structural analysis of the structure. For the purpose of analysis, structures are divided into determinate and indeterminate structures.

4.3.1 Determinate Structures - Beams and Slabs

Hand calculations are suitable for analyzing statically determinate structures such as simply supported beams and slabs. Analysis being solely based on the application of equations of static equilibrium. A typical analysis of determinate structure is shown in Example 4. 1.

Example 4.1

Calculate and draw the shear force and the bending moment diagram for Figure 4. 2

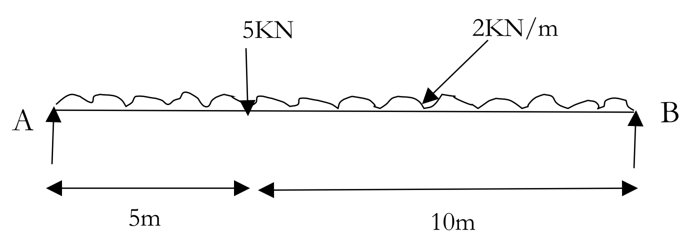


Figure 4. 2: A simply supported beam/slab carrying a uniformly distributed and point loads

Solution

The structure shown all reactions is shown in figure 4. 3

The reactions at supports A and B are R_A and R_B.

$R_A + R_B = 5 + 2 \times 15 = 35KN$ **4.3**

Moment about A, $15RB = 5 \times 5 + 2 \times 15 \times \frac{15}{2} = 250KN$

$R_B = \frac{250}{15} = 16.67KN$ and $R_A = 18.33KN$ (from equation 4.3)

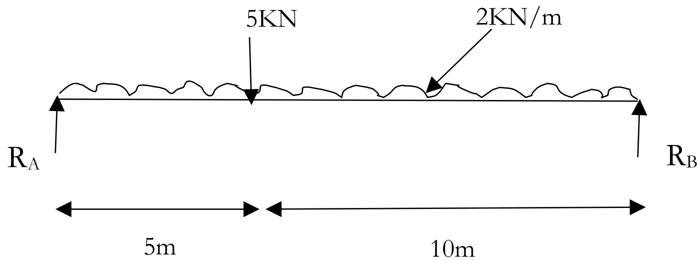


Figure 4. 3: The beam/slab showing all reactions and loads

For the section $0 \le X \le 5$ m (Figure 4.4)

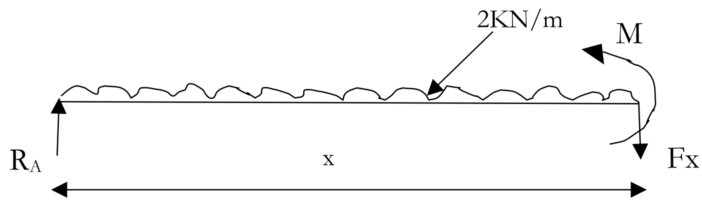


Figure 4. 4

Shear Force (from figure 4.4),

$Fx = R_A - 2x \quad = 18.33 - 2x$ **4.4**

At $x = 0$ $\rightarrow$ $Fx = 18.33KN$

At $x = 5$ $\rightarrow$ $Fx = 8.33KN$

Bending Moment (from figure 4.4),

$Mx = R_Ax - x^2 \quad = 18.33x - x^2$ **4.5**

At $x = 0$ $\rightarrow$ $Mx = 0KN.m$

At $x = 5m$ $\rightarrow$ $Mx = 66.65KN.m$

For the section $5 \le X \le 15$ m (figure 4.5)

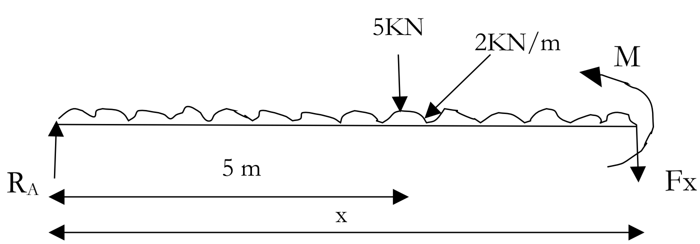


Figure 4. 5

Shear Force (from figure 4.5),

$$F_x = R_A - 2x - 5 \quad = 18.33 - 2x - 5$$

$$= 13.33 - 2x \qquad \textbf{4.6}$$

At $x = 5m$ $\rightarrow F_x = 3.33KN$

At $x = 15m$ $\rightarrow F_x = -16.67KN$

The shear force has changed sign. Thus, at the point where shear force changes sign (cross the axis), the shear force is zero. That is, setting equation 4.6 to zero:

$$0 = 13.33 - 2x$$

$x = 6.67$ m (from support A)

Bending Moment (from figure 4.5),

$$M_x = R_A x - 2.\, x. \frac{x}{2} - 5(x - 5) = 18.33x - x^2 - 5x + 25$$

$$= -x^2 + 13.33x + 25 \qquad \textbf{4.7}$$

At $x = 5m$ $\rightarrow M_x = 66.65KN.m$

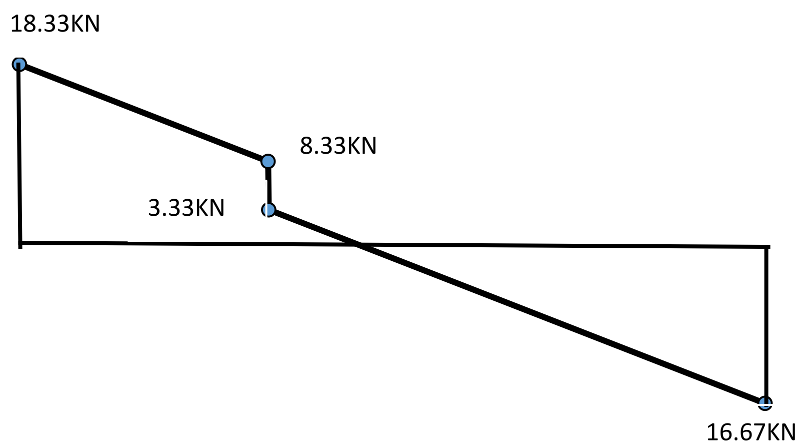
At $x = 15m$ $\rightarrow M_x = 0KN.m$

But the maximum Bending moment occurs where the shear force is zero. That is, at x = 6.67m.

Therefore, substituting for x = 6.67 in equation 4.7

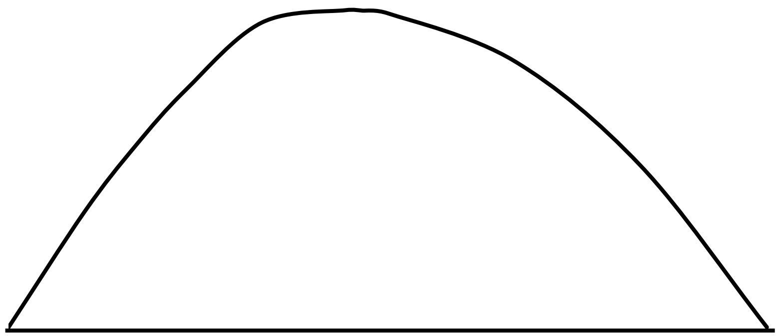
M (maximum) = 69. 42KN.m

The shear force and the bending moment diagrams are presented in figure 4.6.



a. Shear Force Diagram

69.42KN.m



b. Bending Moment Diagram

Figure 4.6: The Shear force and the Bending Moment Diagrams

4.3.2 Indeterminate Structures - Beams and Slabs

Indeterminate structures are analyses through consideration of three fundamentals which are:

i. Equilibrium as required by the equation of statics
ii. Compatibility (of displacement, not only along an element but also at the joints where elements intersect)
iii. Constitutive or Stress-Strain relations → Material Laws peculiar to each type of structure (Member stiffness)

In the case of statically determinate structures, the equilibrium equations can be solved independently of the compatibility and constitutive relations to obtain the reactions and member forces. In the analysis of statically indeterminate structures, however, the equilibrium equations alone are not sufficient for determining the reactions and member forces. Therefore, it becomes necessary to satisfy simultaneously the three types of fundamental relationships (i.e., equilibrium, compatibility, and constitutive relations) to determine the structural response.

Variety of methods for analyzing statically indeterminate structures, and include:

i. moment distribution,
ii. slope deflection
iii. virtual work,
iv. etc.

You are referred to your notes and any standard work on this subject to refresh your memory. A typical analysis of indeterminate structure using moment distribution method is presented in Example 4.2.

Example 4.2

Figure 4. 7 is a 2-span continuous beam.

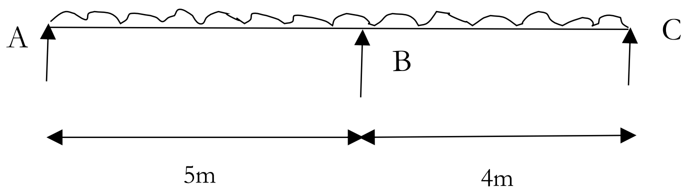


Figure 4.7: Example 4.2

If the Design load at the ultimate is:

Maximum Design Load $= 15.86\ kN/m$

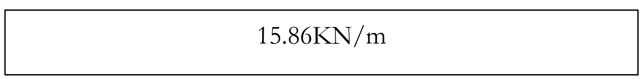
Minimum Design Load $= \ 11.06kN/m$

Use suitable method to obtain the maximum shear force and bending moment.

Solution

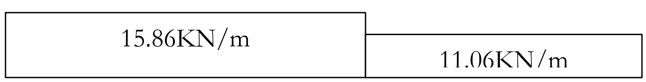
Io order to obtain the maximum shear force and bending moments, four critical load combinations will be evaluated. This is shown in Figure 4.8.

Case 1 – Maximum loadings on Spans



Case 2 – Minimum Loadings on Spans

Case 3 – Maximum on first Span and Minimum on second Span



Case 4 – Minimum on first Span and Maximum on Second Span

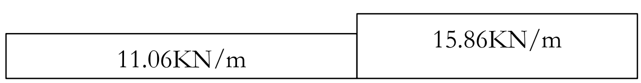


Figure 4.8: Loading combinations for maximum shear force and bending moment.

Although, many methods can be sued to analyzed this structure, moment distribution will be used here for the analysis.

Stiffness Factor

For hinged beams

$$K = \frac{3}{4}\left(\frac{1}{L}\right)$$

For span AB

$$K = \frac{3}{4}\left(\frac{1}{5}\right) = \frac{3}{20}$$

For span BC

$$K = \frac{3}{4}\left(\frac{1}{4}\right) = \frac{3}{16}$$

Distribution Factor

$$\text{Distribution factor AB} = \frac{K_{AB}}{K_{AB}} = \frac{\frac{3}{20}}{\frac{3}{20}} = 1$$

$$\text{Distribution factor BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{\frac{3}{20}}{\frac{3}{20}+\frac{3}{16}} = \frac{4}{9}$$

$$\text{Distribution factor BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = \frac{\frac{3}{16}}{\frac{3}{20}+\frac{3}{16}} = \frac{5}{9}$$

$$\text{Distribution factor CB} = \frac{K_{CB}}{K_{CB}} = \frac{\frac{3}{20}}{\frac{3}{20}} = 1$$

Fixed Ends Moments

$$M_{AB} = \frac{-\omega L^2}{12} = \frac{-15.86(5)^2}{12} = -33.\ 042\text{KN.m}$$

$$M_{BA} = \frac{\omega L^2}{12} = \frac{15.86(5)^2}{12} = 33.\ 042\text{KN.m}$$

$$M_{BC} = \frac{-\omega L^2}{12} = \frac{-15.86(4)^2}{12} = -21.\ 147\text{KN.m}$$

$$M_{CB} = \frac{\omega L^2}{12} = \frac{15.86(4)^2}{12} = 21.\ 147\text{KN.m}$$

Table 4. 1: Moments distribution table for Loading Case 1

	A		B			C
DF	1.0		0.44	0.56		1.0
FEM	- 33.042		+ 33.042	- 21.147		+ 21.147
BAL	+33.042		- 5.287	- 6.608		- 21.147
CO			+16.521	- 10.574		
	0		+ 44.276	- 38.329		0
BAL			- 2.643	- 3.304		
∑KN.m	0		+41.633	- 41.633		0

Span AB

$\Sigma M_A = 0$;

$15.86 \text{ x } 5 \text{ x } 2.5 + 41.633 – 5R_B = 0$

$198.25 + 41.633 = 5R_B$

$R_B = 47.98$ KN

$\Sigma V = 0$;

$R_A + R_B = 15.86 \text{ x } 5$

$R_A + R_B = 79.3$

$R_A = 79.3 – 47.977$

$R_A = 31.32$ KN

Span BC

$\Sigma MB = 0$;

$15.86 \times 4 \times 2 = 41.633 + 4R_C$

$126.88 - 41.633 = 4R_C$

$85.247/4 = R_C$

$R_C = 21.312$ KN

$\Sigma V = 0$;

$R_B + R_C = 15.86 \times 4$

$R_B + R_C = 63.44$

$R_B = 63.44 - 21.312$

$R_B = 42.128$ KN

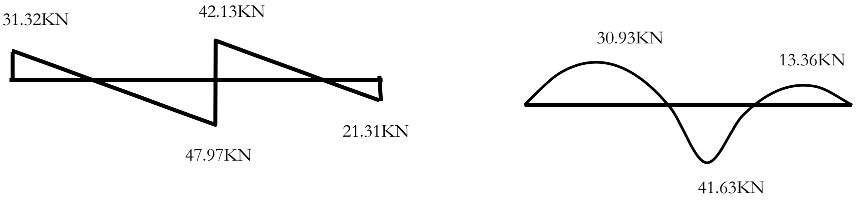
Thus;

$R_A = 31.323$ KN

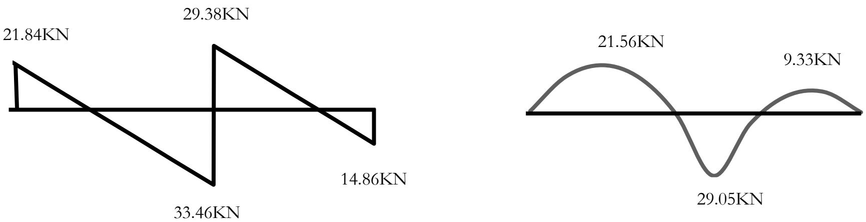
$R_B = 47.977 + 42.128 = 90.105$ KN

$R_C = 21.312$ KN

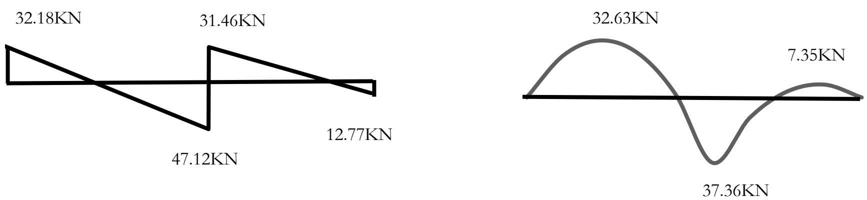
The shear forces and the bending moments are similarly calculated for the remaining three cases. The diagrams for shear forces, bending moments, and the envelopes are shown in Figure 4 9. The maximum values were shown in the envelope.



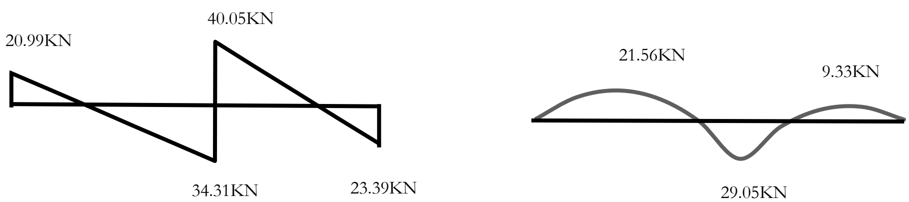
Case 1: Shear Force and Bending Moment



Case 2: Shear Force and Bending Moment



Case 3: Shear Force and Bending Moment



Case 4: Shear Force and Bending Moment

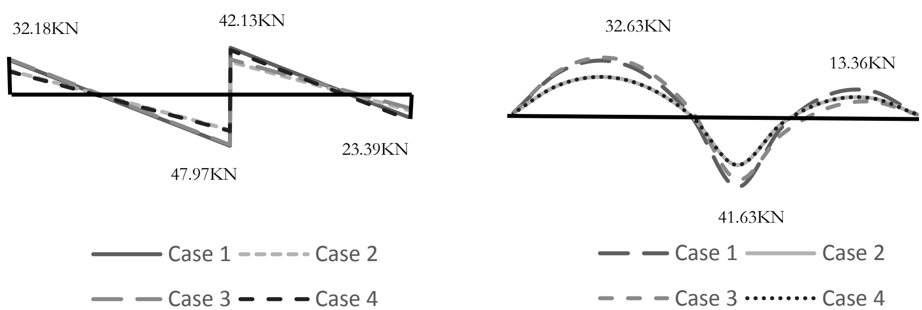
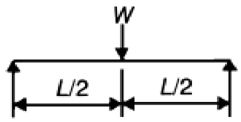
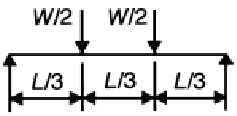
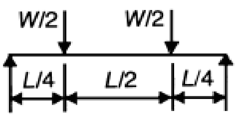
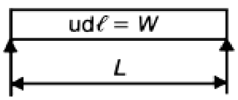
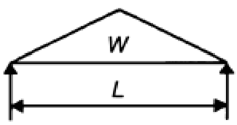
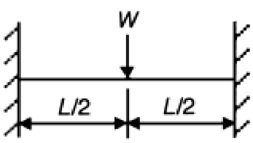
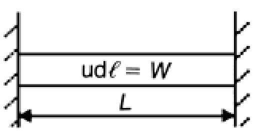
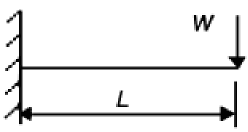
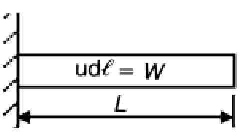


Figure 4. 9. The Shear Forces, Bending Moments and the Envelopes Diagrams

For various standard load cases, formulae for calculating the maximum bending moments, shear forces and deflections are available which can be used to rapidly analyses beams and slabs (Table 4. 2).

Table 4.2: Bending moments, shear forces and deflections for various standard load cases

Loading	*Maximum bending moment*	*Maximum shearing force*	*Maximum deflection*
	$\frac{WL}{4}$	$\frac{W}{2}$	$\frac{WL^3}{48EI}$
	$\frac{WL}{6}$	$\frac{W}{2}$	$\frac{23WL^3}{1296EI}$
	$\frac{WL}{8}$	$\frac{W}{2}$	$\frac{11WL^3}{768EI}$
	$\frac{WL}{8}$	$\frac{W}{2}$	$\frac{5WL^3}{384EI}$
	$\frac{WL}{6}$	$\frac{W}{2}$	$\frac{WL^3}{60EI}$
	$\frac{WL}{8}$ (at supports and at midspan)	$\frac{W}{2}$	$\frac{WL^3}{192EI}$
	$\frac{WL}{12}$ at supports $\frac{WL}{24}$ at midspan	$\frac{W}{2}$	$\frac{WL^3}{384EI}$
	WL	W	$\frac{WL^3}{3EI}$
	$\frac{WL}{2}$	W	$\frac{WL^3}{8EI}$

However, there will be many instances in practical design where the use of standard formulae is not convenient and the designers will use any other appropriate method of analysis. Alternatively, the designer may resort to using various commercially available **computer packages.**

4.3.3 Analysis of Framed Structures

Reinforced concrete structures behaves like a 3-dimesional rigid frame. Usually, for all structures that are 3-dimensional, quite a large number of joints can result each with six degrees of freedom (Figure 4.10). The number of simultaneous equations to be formed and solved is therefore 6*j* and this could be tedious, even with computer.

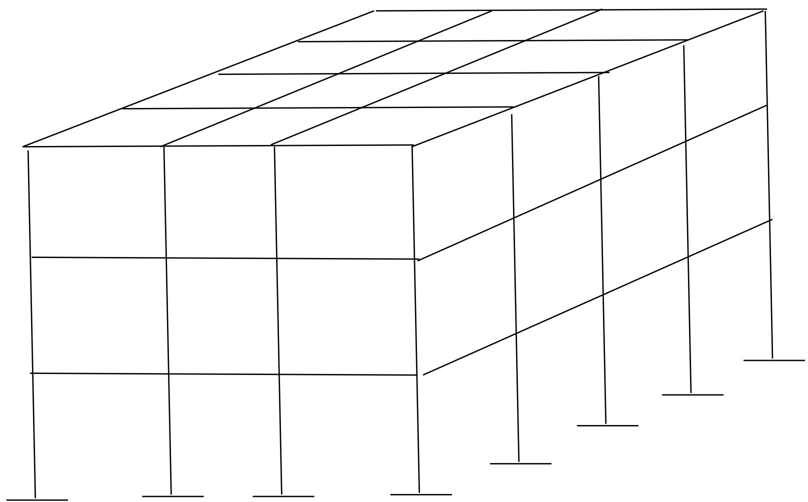


Figure 4.10: 3-Dimensional Structure

It is thus usual to adopt some simplification to reduce the work. The most common simplification is to divide the structure into a series of plane frames (Figure 4.11). The number of joints is now reduced to 3*j*. this can be analyzed by computer without difficulty.

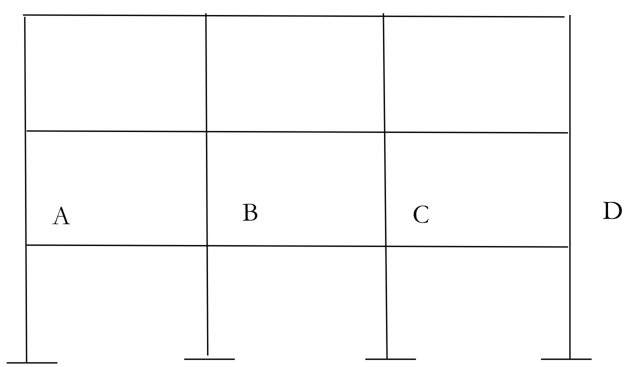


Figure 4.11: Plane Frame Simplification of 3-Dimensional Structure in Figure 4.10

However, if hand calculation is to be used, the plane frame is too large and a further simplification is necessary. This is achieved by subdividing the plane frame into subframes, to facilitate hand calculation. In this case:

i. slabs can be analyzed as continuous, 1-way or 2-way spanning and supported by beams or structural walls
ii. columns and main beams can be considered as series of rigid plane frame supporting:
    a. vertical loads
    b. vertical and lateral loads

For the purpose of analysis, frame structure is considered either braced or unbraced.

Braced frame structure are structures designed to resist vertical loads only. In such structures, some mechanisms for resisting lateral loads and sidesway are, in some ways, incorporated into the frame. Such mechanisms include the provision of bearing walls, shear walls, shear cores, truss, tubular systems. All these are regarded as stiffening systems.

On the other hand, the **unbraced frame** structure has none of the stiffening systems and are thus to be designed to resist both vertical and lateral loads.

4.3.3.1 Analysis of Braced Framed Structure

For a rigid braced frame designed to resist vertical loading only, incorporating in it the mechanisms for resisting lateral loading and sideway, the BS 8110 allows the moments, loads, and shears in the individual column and beams to be derived from an elastic analysis of a series of subframes. Such analysis is simplified by subdividing the structure into subframes. Three ways of doing this are:

a. One Floor Level Subframe (BS 8110: Clause 3.2.1.2.1)
b. Two Free Joint Subframe Method (BS 8110: Clause 3.2.1.2.3)
c. Continuous beam Method (BS 8110: Clause 3.2.1.2.4)

4.3.3.1.1 One Floor Level Subframe Method

Each subframe is taken to consist in of the beams at one level together with the columns above and below.

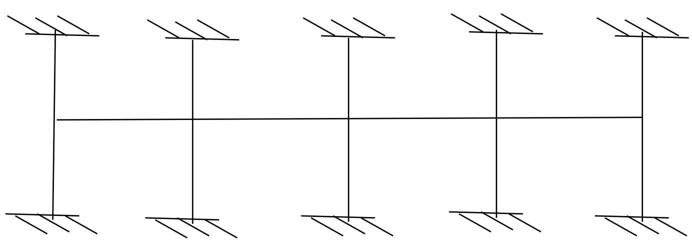


Figure 4.12: One floor level Subframe Method

Features

i. A continuous beam at one level together with all the columns joining it both above and below. This allows the shear forces and the bending moments in both the beams and the column to be obtained from the same set of calculations and analysis.

ii. The ends of the columns are assumed to be fixed unless the assumption of a pinned end is clearly appropriate.

iii. The full stiffness of the members is used.

4.3.3.1.2 Individual Beam and Associated Columns Method

This allows the moments and forces in individual beam to be found by considering a subframe consisting:

i. Only of that beam

ii. The columns attached to the ends of that beam

iii. The beams on each side$(\frac{1}{2})\frac{I}{l}$

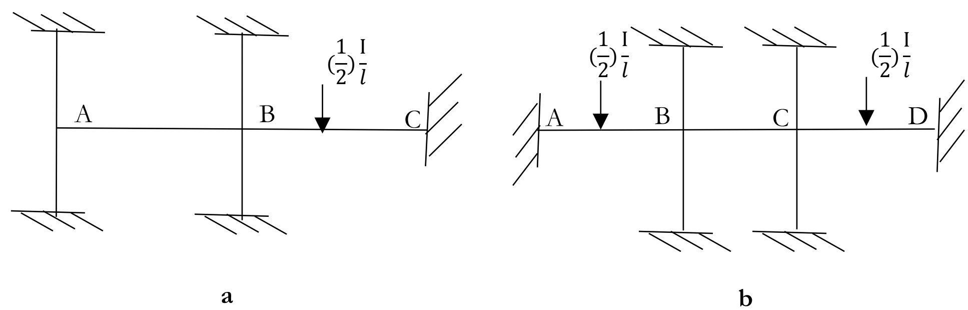


Figure 4.13: Individual Beam and associated Columns Method

Features

i. The columns and beams end remote from the beam under consideration may be assumed to be fixed

ii. The stiffness of the beams on each side of the beam under consideration is taken as half of their actual values, if they are considered as fixed at their outer ends

iii. Two joints free to rotate with all columns fixed at their remote ends.

iv. The structure is analyzed using moment distribution. But the two free joints may make the calculation lengthy in order to reach the degree of accuracy.

The moment in the individual column can also be obtained, provided that the sub-frame has its central beam as the longest of the two spans framing into the column under consideration. For example, if beam AB is longer than beam BC in Figures 4. 11 and 4.13, then the subframe in Figure 4.13a should be used. But if BC is longer than AB, then the subframe in Figure 4.13b should be used.

4.3.3. 1.3 Continuous Beam Simplification Method

In the continuous beam simplification method, the moments and shear forces in the beam at one level can be obtained by considering the beams as a continuous beam over supports providing no restraint to rotation. Thus, the beams at the level ABCD in the frame in Figure 4.11 may be analyzed as a continuous beam on simple supports as shown in Figure 4.14a. The loading arrangements to be considered are the same for the subframe described in one floor level subframe method.

The column moments can be obtained by simple moment distribution procedure, on the assumption that:

i. the column and beam ends remote from the junction under consideration are fixed, and
ii. that the beams possess half their actual stiffness

The loading arrangement should be such as to cause the maximum moment in the column. This approach leads to conservative design

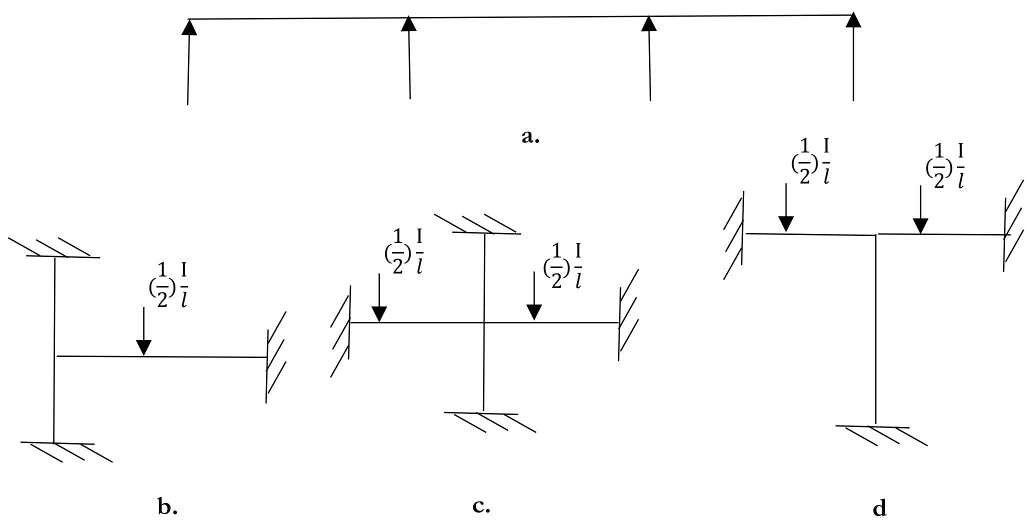


Figure 4.14: Continuous Beam Method

Example 4.3 is an example of a typical frame solved through one floor level subframe method

Example 4.3

Figure 4. 15 shows a braced Frame.

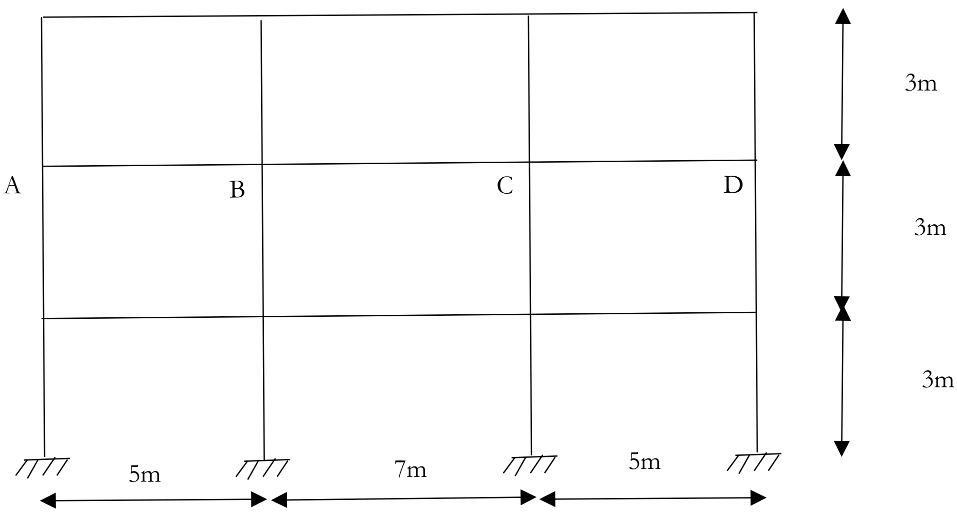


Figure 4. 15: Braced frame Structure for Example 4:3

Using any of the subframe methods of analysis, calculate the shear forces and bending moments in Beam ABCD together with the moments in the adjacent columns. The dead load (including the self-weight of the beam) on the beam is 30KN/m, while the live load is 15KN/m. of the beam Other data includes: (i) The Beams cross section, b = 300 mm and h = 550 mm, (ii) The Columns cross section, b = 300 mm, h = 400 mm

Solution

Using one floor level subframe. The structure is shown in figure 4.

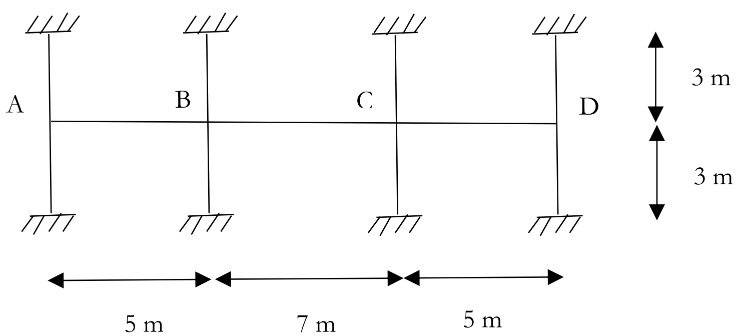


Figure 4. 16: The one floor level for analysis

Stiffness

For Beams

$I = \frac{bh^3}{12} = \frac{300\ x\ 550^3}{12} = 4.16 \times 10^9 \text{ mm}^4$

Spans AB and CD

$k_{AB} = k_{CD} = \frac{EI}{L} = \frac{4.16\ x\ 10^9}{5000} = 8.3 \times 10^5 E \text{ mm}^3$

Span BC

$k_{BC} = \frac{EI}{L} = \frac{4.16\ x\ 10^9}{7000} = 5.94 \times 10^5 E \text{ mm}^3$

Columns

$I = \frac{bh^3}{12} = \frac{300\ x\ 400^3}{12} = 1.60 \times 10^9 \text{ mm}^4$

$K_{lower} = k_{upper} = \frac{EI}{L} = \frac{1.6\ x\ 10^9}{3000} = 5.33 \times 10^5 E \text{ mm}^3$

$K_{lower\ +}\ k_{upper} = 10.66 \times 10^5 E \text{ mm}^3$

Distribution Factors (DF)

At joints A and D

$\Sigma k = 8.3 \times 10^5 E \text{ mm}^3 + 10.66 \times 10^5 E \text{ mm}^3 = 18.96\ 10^5 E \text{ mm}^3$

$DF_{AB} = DF_{DC} = \frac{8.3}{18.96} = 0.44$

$DF_{columns} = \frac{10.66}{18.96} = 0.56$

At joints B and C

$\Sigma k = 8.3 \times 10^5 E + 5.94 \times 10^5 E + 10.66 \times 10^5 E = 24.90 \times 10^5 E$

$DF_{BA} = DF_{CD} = \frac{8.3}{24.90} = 0.33$

$DF_{BC} = DF_{CB} = \frac{5.94}{24.90} = 0.24$

$DF_{columns} = \frac{10.66}{24.90} = 0.43$

Design Load

Maximum design Load, w = 1.4gk + 1.6qk = 30 x 1.4 + 1.6 x 15 = 66KN/m

Minimum Load, w = 1.0gk = 1.0 x 30 = 30KN/m

Critical Loading cases to be evaluated so as to obtain maximum shear force and bending moments are shown in Figure 4:17.

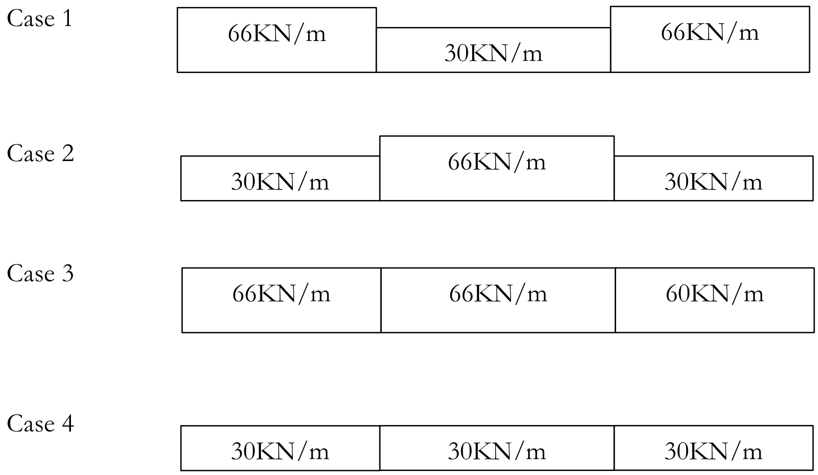


Figure 4.17: Load cases to be evaluated to obtain maximum shear force and bending moments

Case 1

Fixed End Moments

$FEM_{AB} = FEM_{BA} = FEM_{CD} = FEM_{DC} = \frac{wl^2}{12} = \frac{66\ x\ 5^2}{12} = 137.50$ KN.m

$FEM_{BC} = FEM_{CB} = \frac{wl^2}{12} = \frac{30\ x\ 7^2}{12} = 122.5$ KN.m

Using moment distribution method, the results are presented in Table 4. 3.

Table 4. 3: Moments distribution for Loading Case 1

	A		**B**			**C**			**D**	
	∑Cols	AB	BA	**∑Cols**	BC	CB	**∑Cols**	CD	DC	**∑Cols**
DF	0.56	0.44	0.330	0.430	0.240	0.240	0.430	0.330	0.44	0.56
FEM	0.00	- 137.50	+ 137.50	0.00	- 122.50	+ 122.50	0.00	- 137.50	+ 137.50	0.00
OBM*	-137.50			+ 15.00			- 15.00			+ 137.50
Balance	+ 77.00	+ 60.50	- 4.95	- 6.45	- 3.60	+ 3.60	+ 6.45	+4.95	- 60.50	- 77.00
CO		- 2.48	+ 30.25		+ 1.80	- 1.80		- 30.25	+ 2.48	
OBM*	- 2.48			+ 32.05			- 32.05			+ 2.48
Balance	+ 1.39	+ 1.09	- 10.58	- 13.78	- 7.69	+ 7.69	+ 13.78	+ 10.58	- 1.09	- 1.39
CO		- 5.29	+ 0.55		+ 3.85	- 3.85		- 0.55	+ 5.29	
OBM*	- 5.29			+ 4.40			- 4.40			+ 5.29
Balance	+ 2.96	+ 2.33	- 1.45	- 1.89	- 1.06	+ 1.06	+ 1.89	+ 1.45	- 2.33	- 2.96
∑KN.m	+81.35	- 81.35	+151.32	- 22.12	- 129.20	+129.20	+ 22.12	- 151.32	+81.35	- 81.35

OBM* = Out of balance Moments

Moment in individual column M_{col} can now be calculated by dividing the column moments in Table 4 in the ratio of their stiffness. That is:

$$M_{col} = \sum M_{col}\ x\ \frac{k_{col}}{\Sigma k_{col}} \qquad \textbf{4. 8}$$

Column Moments

$$M_{AUpper} = 81.35 \text{ x } \frac{5.33}{\Sigma 10.66} = 40.68 \text{ KN.m}$$

$$M_{ALower} = 81.35 \text{ x } \frac{5.33}{\Sigma 10.66} = 40.68 \text{ KN.m}$$

The moments in upper and lower columns at joints B, C, and D are similarly calculated. The plot is shown in Figure 4.18.

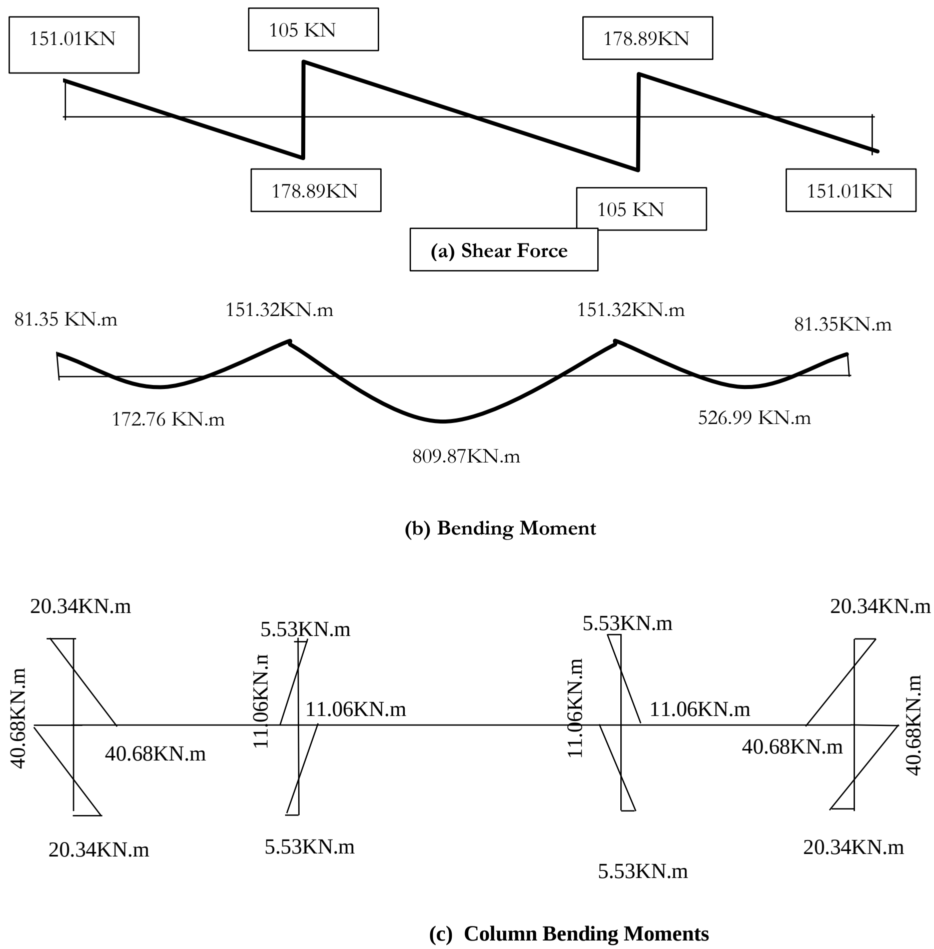


Figure 4.18: Beam Shear and Bending Moment, and the Column Moments

Each of the remaining loading case is similarly analyzed and plotted. They are then combined to obtained the envelopes.

Assignment

1. Re-do Example 4.3, using continuous beam simplification method.
2. With reference to Figure 4.15, calculate the bending moments in Beam AB.

4.3.3.2 Analysis of Unbraced Framed Structure

When the frame structure is unbraced, consideration of lateral loads comes into play. The lateral load can be caused by wind pressures, or retained earth, or by seismic forces. The BS 8110 (Clause 3.2.1.3.2) states that for such frame, the design moments for the individual members can be obtained from either (a) or (b) below, depending on which one gave the larger values:

a. Those obtained by simplified analysis of the types given in section **4.3.3. 1**

b. The sum of the effects of:

   i. Single storey analysis, idealized as in Figure 4.19, but loaded with 1.2 (g_k + q_k); and

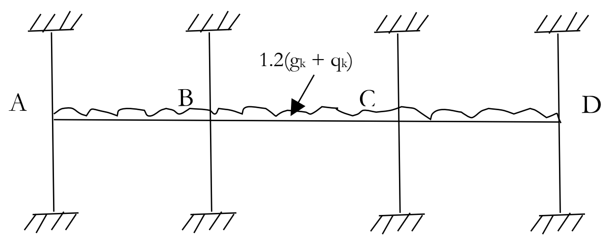


Figure 4.19: Single Storey Analysis

   ii. The analysis of the complete frame loaded with 1.2w_k only, and assuming that the point of contraflexures occur at the centers of all beams and columns (Figure 4.20).

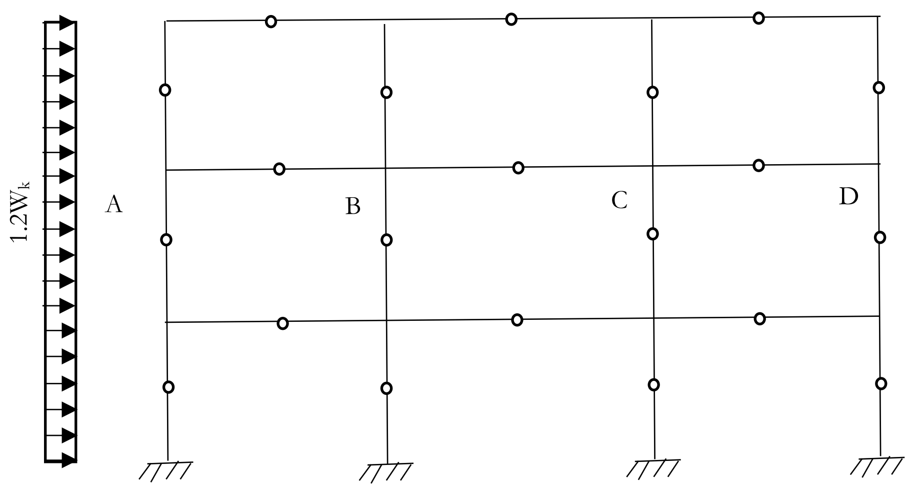


Figure 4.20: Frame Analysis for Wind loading only

The single storey analysis under a uniform loading of 1.2 (g_k + q_k) is similar to that given in Example 4.3.

If the wind load is not given, then it can be assessed in accordance with CP 3 Chapter 5: Part 2 (5), which gives two methods for the estimation of the forces to be taken as acting on the structure.

Method 1 – Procedure

1. Determine the basic wind speed V (m/s), appropriate for the locality under consideration. For example, Figure 4. 21 is a typical plot showing the wind speed in some parts of Nigeria.

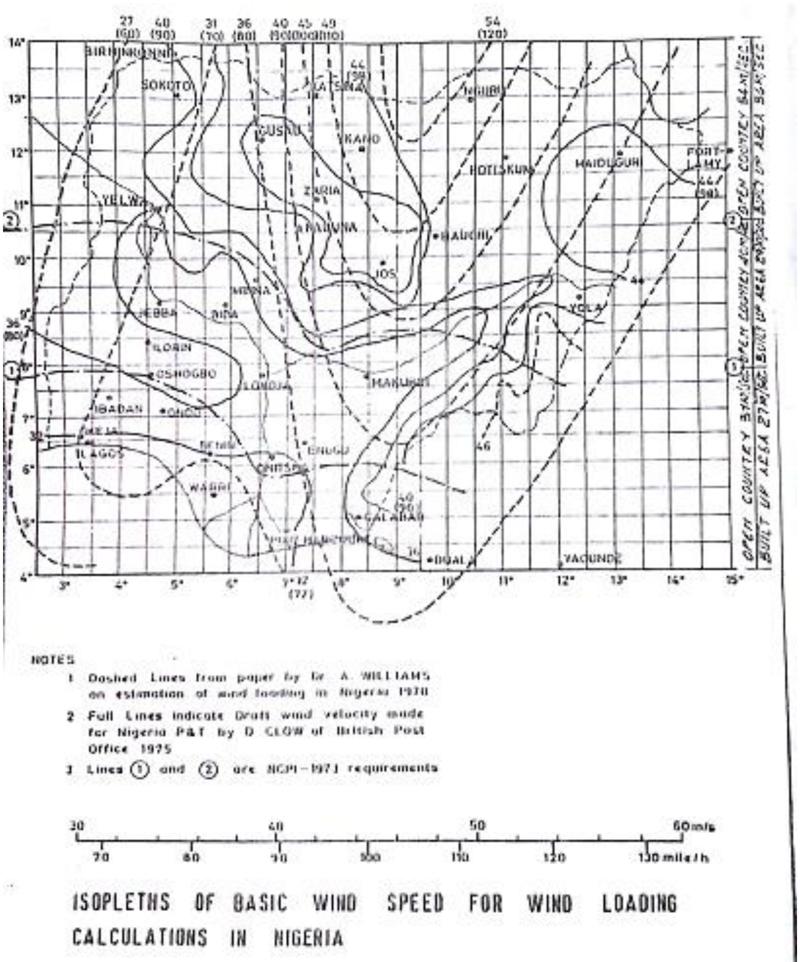


Figure 4.21: Map of Nigeria Showing basic Wind Speed (m/s) (Ove Arup & Partners, Nigeria)

2. Determine values for three factors from Tables
   a. S1 = topography factor (usually 1)
   b. S2 = ground roughness, building size and height above ground factor. This varies with various parameter values (e. g. open country, town, city); and has a range from 0.47 – 1.27

c. Statistical factor. This depends upon the degree of security required and the number of years that the structure is expected to be exposed to the wind. The normal value is given as 1.0.

3. Calculate the design wind speed Vs (m/s)

$$Vs = V \times S1 \times S2 \times S3 \text{ (m/s)} \qquad \textbf{4.9}$$

4. The design wind speed is converted to the dynamic pressure q (N/m^2), where

$$q = 0.613 V_s^2 \text{ (N/m}^2\text{)} \qquad \textbf{4.10}$$

5. Determination of force F on the building, where F:

$$F = qA(C_{pe} - C_{pi}) \qquad \textbf{4.11}$$

where

A = surface area of the element or structure being considered.

C_{pe} = external pressure coefficient. The values depend on:

a. the building aspect ratio (height: width; length: width)

b. the wind direction

c. the surface being considered

when the wind is normal to the windward face, the value of C_{pe} for that face is generally, +0.7, whereas the leeward face then has a value varying between – 0.2 and – 0.4, depending on the plan dimensions.

C_{pi} = internal pressure coefficient.

Method 2 - Procedure

The second procedure is only applicable to a limited range of rectangular (in plan) building shapes. It gives the force F directly as:

$$F = C_t A_c q \qquad \textbf{4.12}$$

Where

C_t = the force coefficient obtained from tabulated values which range from 0.7 to 1.6, depending on the aspect ratio

A_c = the effective frontal area of the structure

q = as in Method 1 above.

Having obtained the force F from any of the two methods above, the force is then applied at the roof and floor levels.

The basic method of introducing pinned joints into the frame leads to two alternative methods of analysis., namely, the cantilever method and the portal method. The assumption that forms the basis of the **Cantilever method of analysis** is that the axial force in a column, due to the wind loading, is proportional to its distance from the center of gravity of all the columns in that frame. The calculation will be straight forward when the system is symmetrical in both size and position. On the other hand, **the Portal Frame Method**. This assumes that the members at a given level of a frame, can be split into a series of portal frames, and that the lateral loading on each of these frames is in proportion to its horizontal span. Example 4.4 demonstrate the application of the two methods.

Example 4.4

For the unbraced frame shown in Figure 4.22, draw the bending moment diagram for the applied wind load.

a. Cantilever Method
b. Portal frame Method

Take $g_k = 35KN/m$ $q_k = 40KN/m$ $w_k = 7.5KN/m$

Assume the columns are of the same cross-section

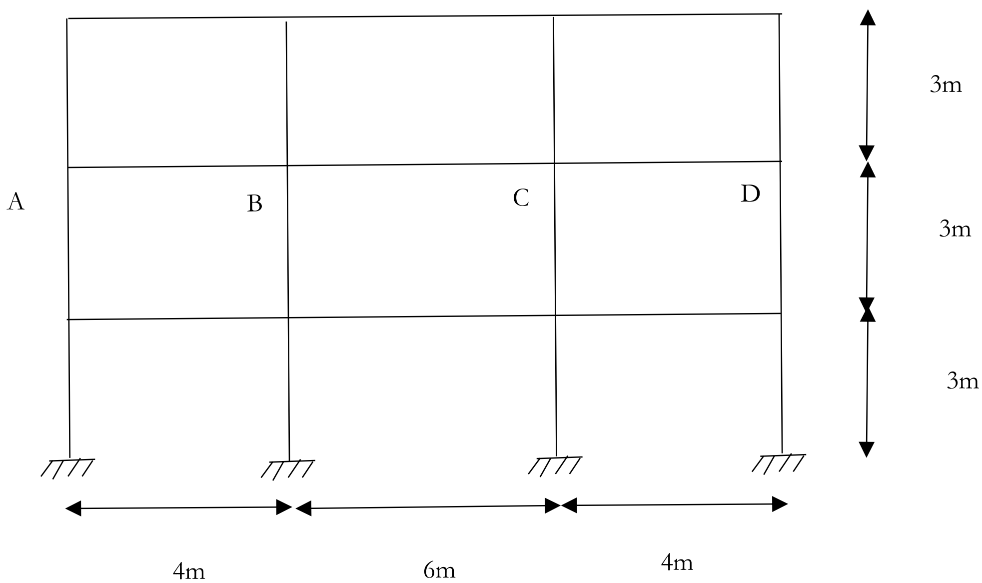


Figure 4.22: Unbraced frame for Example 4.4

a. Cantilever Method

Solution

The uniformly distributed wind load is applied as shown in Figure 4.23.

The uniformly distributed wind load is now converted to point load as follows:

The horizontal force due to the wind load at the roof level = $1.2 \times 7.5 \times \frac{3}{2} = 13.50$ KN

The horizontal force due to the wind load at the floor levels = $1.2 \times 7.5 \times 3 = 27.0$ KN

The horizontal force due to the wind load at the ground level = 1.2 x 7.5 x $\frac{3}{2}$ = 13.50 KN

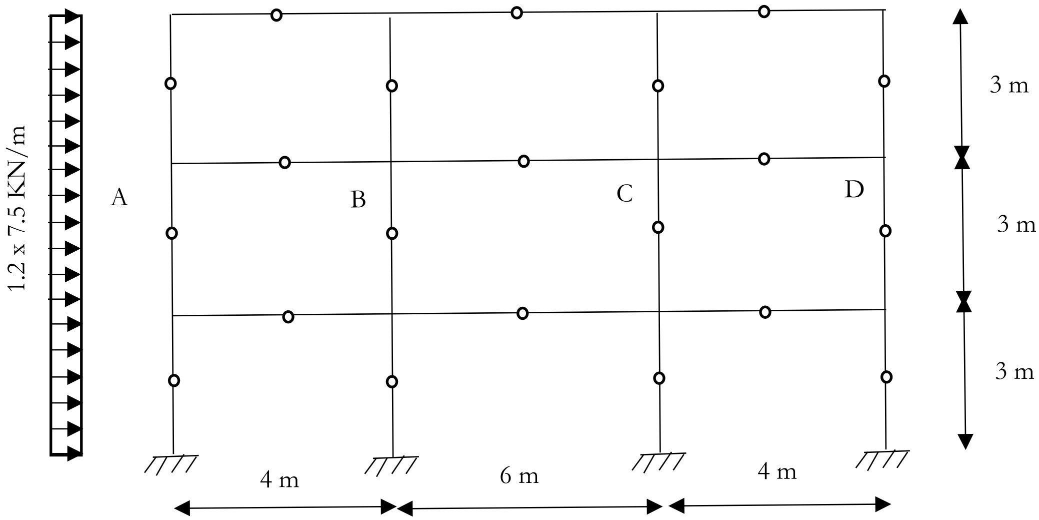


Figure 4.23: The Uniformly distributed load as applied to the Frame

The equivalent point loads due to the wind are applied at the floor levels as shown in the labelled frame of Figure 4.24.

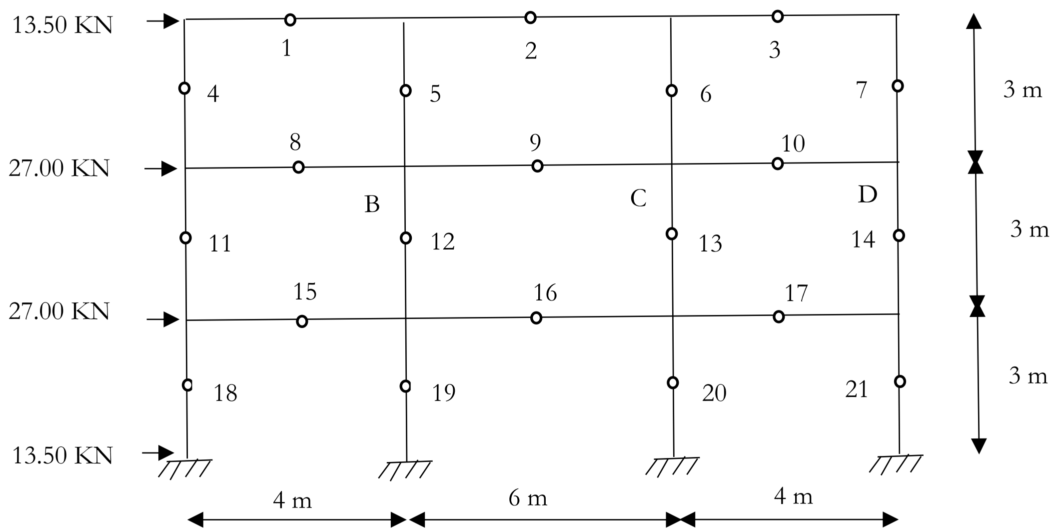


Figure 4.24: The Wind loads applied as point loads at floor levels

The cantilever method of analysis is based on the assumption that the axial force in a column, due to the wind loading, is proportional to its distance from the center of gravity of all the columns in that frame. Since the columns are of the same cross section, the axial force in the columns will be proportional to their distance from the center of the group

The Floor Level

Figure 4.25 shows the applied forces at the roof level. It is evident that V_1 and V_2 are of opposite sign as V_3 and V_4.

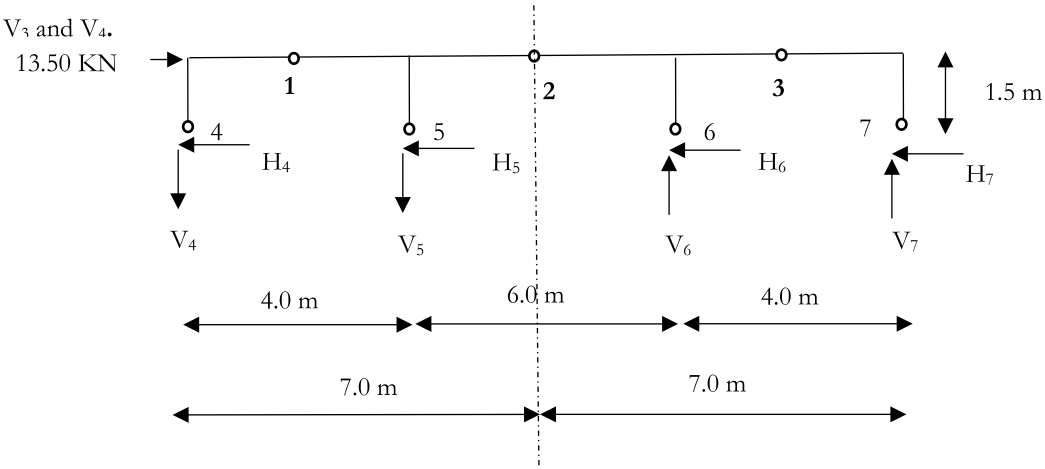


Figure 4. 25: Wind Loading for the Roof level – Cantilever Method

The analysis of the resulting statically determinate systems now follows. The vertical reactions at the pins in Figure 4.24 have the following relationship:

$$\frac{V_4}{7} = \frac{V_5}{3} = \frac{V_6}{3} = \frac{V_7}{7} \qquad \textbf{4.13}$$

Therefore

$$V_4 = 2.333V_5 = 2.333V_6 = V_7 \qquad \textbf{4.14}$$

Taking moments about point 7 for the whole structure as shown in Figure 4.24,

$$13.5 \text{ x } 1.5 - 14V_4 - 10V_5 + 4V_6 = 0$$

$$14V_4 + 10V_5 = 20.25 + V_6$$

$$14\,(2.333V_5) + 10V_5 = 20.25 + 2.33V_5$$

$$32.62V_5 + 10\,V_5 - 2.333V_5 = 20.25$$

$$V_5 = 0.503\text{KN}$$

From equation 4.14

V4 = 2.333 x 0.5 = 1.17KN

V7 = 1.17KN

V5 = 0.503KN

V6 = 0.503KN

Taking moments about pin 1 for forces to the left of pin 1

$1.5H_4 - 2V_4 = 0$

$1.5H_4 = 2 \times 1.17$

$H_4 = 1.56KN$

Also, taking moment about pin 2

$- 7V_4 - 3V_5 + 1.5H_4 + 1.5H_5 = 0$

$- 7 (1.17) - 3 (0.503) + 1.5 (1.56) + 1.5H_5 = 0$

$- 8.19 - 1.51 + 2.34 + 1.5 H_5 = 0$

$H_5 = 4.91KN$

Due to symmetry

$H_6 = 4.91KN$

$H_7 = 1.56KN$

A round-off error of 0.5 is apparent. The resulting bending moment diagram for the roof is shown in Figure 4.26

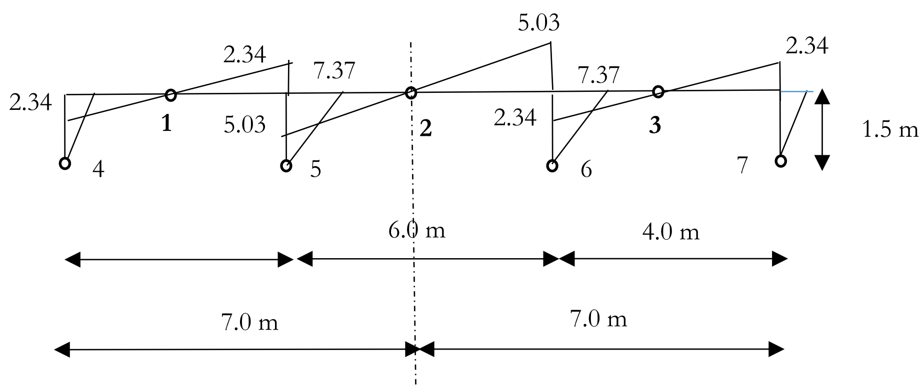


Figure 4.26: Roof Bending Moments due to Wind loading (Cantilever Method)

Moving down the structure to the second floor, as shown in Figure 4.27

As before, dividing the vertical loading

$$\frac{V_{11}}{7} = \frac{V_{12}}{3} = \frac{V_{13}}{3} = \frac{V_{14}}{7} \quad \textbf{4.15}$$

$$V_{11} = 2.333V_{12} = 2.333V_{13} = V_{14} \quad \textbf{4.16}$$

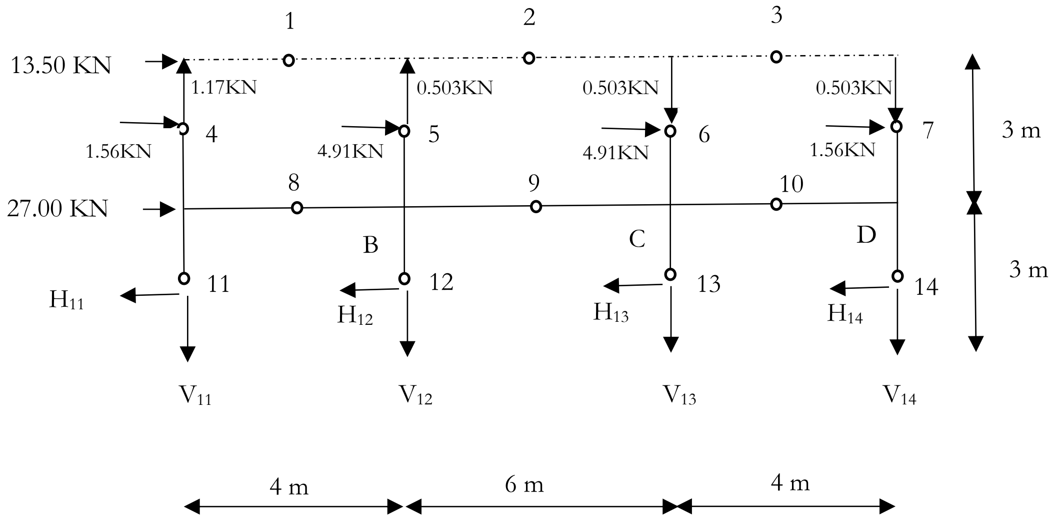


Figure 4.27: Wind Loading for the Second Floor – Cantilever Method

Taking moment about point 14 for the whole structure, we have:

$- 14V_{11} - 10V_{12} - 4V_{13} + 27 \times 1.5 + 13.5 \times 4.5 = 0$

$- 14 \times 2.333V_{12} – 10V_{12} – 4V_{12} + 101.25 = 0$

$V_{12} = 2.17KN$

$V_{13} = 2.17KN$

$V_{11} = 2.333 \times 2.17 = 5.06KN$

V14 = 5.06KN

Taking moment about point 8 for the left portion

$(H_{11} + 1.56) \times 1.5 - (5.06 - 1.17) \times 4 = 0$

$H_{11} = 8.83$

Using the principles of symmetry

$H_{14} = H_{11} = 8.83KN$

$H_{12} = H_{13} = \frac{(13.5+27)- (8.83 \ x\ 2)}{2} = 11.42KN.$

The bending moment and shear force diagrams can now be drawn for this floor, and the process continues with the next floor.

b. Portal Frame Method of Analysis

The portal frame method assumes that the members at a given level of a frame, can be split into a series of portal frames, and that the lateral loading on each of these frames is in proportion to its horizontal span.

The roof floor level, with columns below in Figure 4.23 are separated from the rest of the frame as shown in Figure 4. 28, with the 13.5KN force applied as shown.

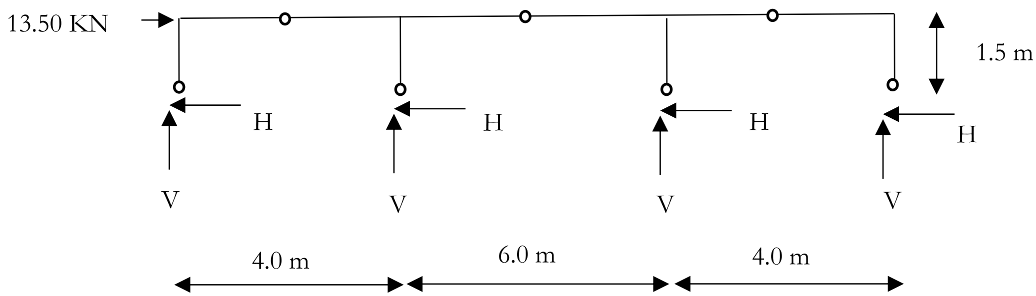


Figure 4.28: The Roof level for Portal frame analysis

In order to simplify the analysis, the 3-bay frame is split into three separate portals, and the horizontal force of 13.5KN is divided between the portals in the proportion of their spans. That is:

For 4 m span

$$\text{Horizontal force} = \frac{4}{4+6+4} \times 13.5 = 3.86\text{KN}$$

For 6 m span

$$\text{Horizontal force} = \frac{6}{4+6+4} \times 13.5 = 5.79\text{KN}$$

Total Horizontal Force = 3.86 + 5.79 + 3.86 = 13.51KN = the total lateral force of 13.5KN (neglecting the rounding-off error)

This is shown in Figure 4.29.

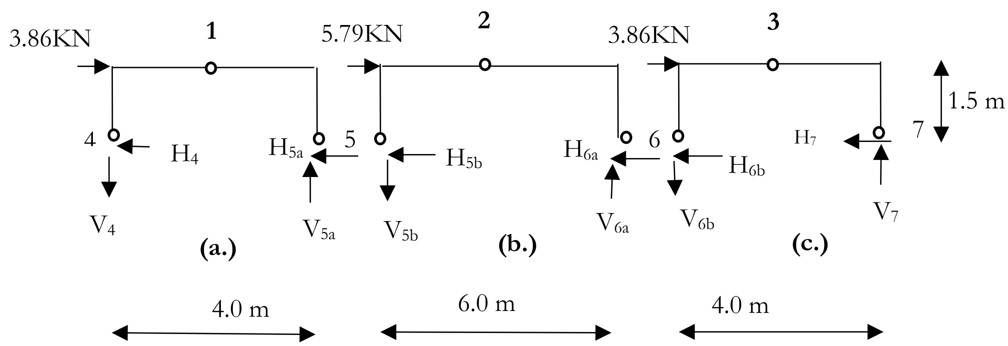


Figure 4.29: Wind loading – Portal Frame Method

Form Figure 4.29, it can be observed that each of the portal has three pins, and therefore statically determinate. Now consider the frame in Figure 4:29a

Taking moment about joint 5,

$$-4V_4 + 3.86 \times 1.5 = 0$$

$V_4 = 1.45KN$

Taking moment about joint 1, for the left hand of the portal

$- 2V_4 + 1.5H_4 = 0$, by substituting for V_4

$H_4 = 1.93KN$

By symmetry

$H_{5a} = 1.93KN$

And taking moment about point 1 for the right hand of the portal

$V_{5a}= 1.45KN$

For the Figure 4.29b, taking moments about point 6,

$5.79 \times 1.5 – 6V_{5b} = 0$

$V_{5b} = 1.45KN$

Taking moment about point 2 for the left hand

$- 3V_{5b} + 1.5H_{5b} = 0$

$H_{5b} = 2.9KN$

$H_{6a} = 2.9KN$ (from symmetry)

Taking moment about point 2 for the right side of the frame

$V_{6a} = 1.45KN$

The results of the analysis are shown in Figure 4.30

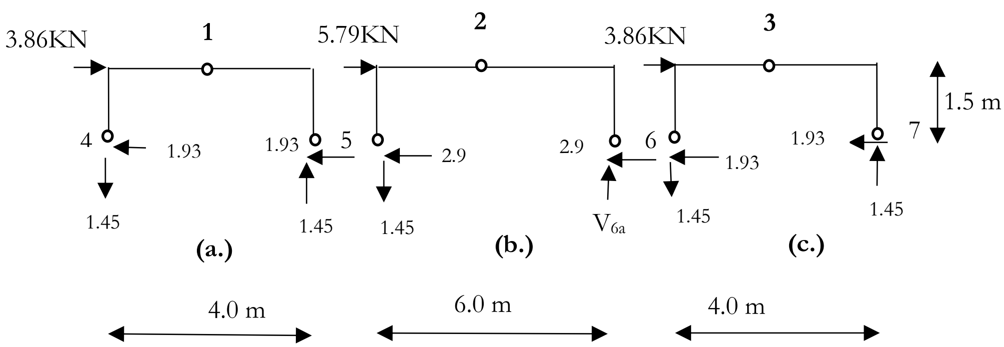


Figure 4.30: The forces acting on the separate portals

The separate forces are now re-combined and the forces acting on the three bays are as shown in Figure 4.31

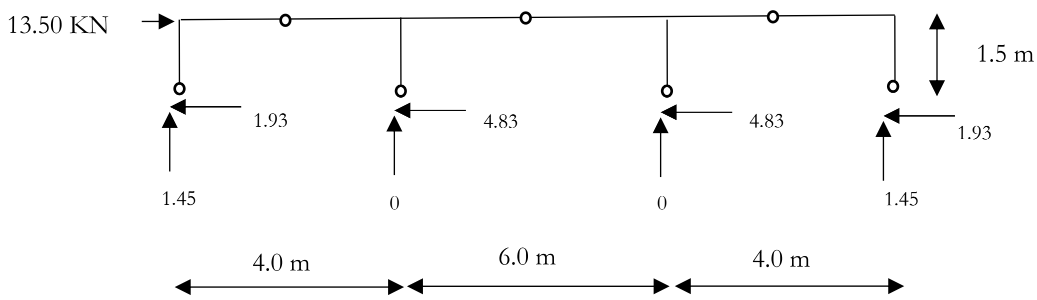


Figure 4.31: The combined forces acting on the Frame

The bending moment diagram obtained from the combined forces of Figure 4.31 is shown in Figure 4.32.

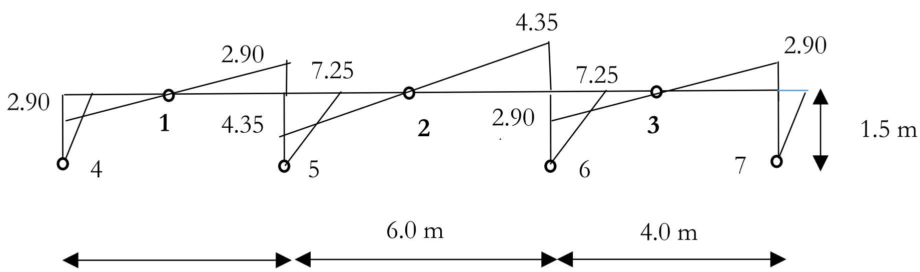


Figure 4.32: The bending Moment for the roof floor (Portal Frame Method)

The next lower floor can be analyzed in the similar way. Analysis is also simplified by assuming that the horizontal force at this level (27KN) is divided between the separate frames in the proportion of their spans

At the end of analysis, a decision has to be made to determine which of the loading combination gave the larger values for bending moments, shear forces, etc. as explained at the beginning of this section.

A brief analysis of braced and unbraced frames has been attempted in this section. Bearing in mind that the designer aims for safe, robust and durable structures, Kong and Evans (1987) advised that unbraced frames should be avoided if possible. Lateral stability in two orthogonal directions should be provided by a system of strong points within the structure so as to produce a braced structure. For robustness, all members of the structure should be effectively held together with **ties** in the longitudinal, transverse and vertical directions. Detailed provisions are given in the BS 8110 (Clause 3.1.4).

Chapter 5 - Limit State Design Method

5.1 Introduction

According to BS 8110, when a structure reached a condition or a state that it becomes unserviceable, it is said to have reached a "limit state". Thus, the goal of limit state is to achieve acceptability that the structure will not become "unserviceable" (that is, reached a limit state) in its lifetime. The conditions examined in the limit state design method are:

1) The Ultimate Limit State

   Ensures that neither the whole nor part of the structure should collapse under foreseeable load.

2) The Serviceability Limit States:

   a) of Deflection

      That the deflection of the structure should not adversely affect the appearance of the structure

   b) of Cracking

      That the cracking of the concrete should not adversely affect the appearance or the durability of the structure. For example, excessive cracks allow ingress of water which can cause corrosion of steel and frost damage.

   c) of Vibration

      That the vibration should not be such as to cause alarm or discomfort especially in industrial buildings.

   d) Durability

      This is considered in relation to the life span of the structure and its conditions of exposure

   e) Fatigue

      This must be considered if cyclic loading is likely

   f) Fire Resistance

      This is considered in terms of resistance to collapse, flame penetration and heat transfer

   g) Special Circumstances

      This is a special circumstance not covered by the common limit states such as earthquake and seismic resistance.

5.2 Limit State Requirement

All the important limit state has to be considered in design. In the RC structures, the three most important for design calculations are:

1) Ultimate limit state
2) Serviceability limit state of deflection
3) Serviceability limit state of cracking

Durability and fire resistance will be combined with grade of concrete, cement content, cover to reinforcement, all of which will be decided before calculation begins. The usual approach will be to design on the basis of most critical Limit State, and then check that the remaining Limit State will not be reached. After the calculation, the deflection should be checked to establish that it is satisfactory and also the cracking is satisfactory.

The criteria we have to comply for the various Limit state are as follows:

1) Ultimate Limit State – the strength of the structure should be sufficient to withstand the design "combination of load"
2) Serviceability Limit State of Deflection – the designer must be satisfied that deflection are not excessive and with regard to a particular structure, reasonable limit is:
   i. Deflection = $\frac{l}{250}$, for normal structure
   ii. Deflection = $\frac{l}{250}$or 20mm where partition and finishes are likely to be affected
   iii. Deflection = $\frac{H}{500}$, where tall slender structure is under consideration
3) Serviceability Limit State of Cracking – the estimated surface width of crack should not in general exceed 0.3mm. For an aggressive environment, the crack width at point nearest to the main Grater control in calculation is exercised almost exclusively by limitation of the spacing of reinforcement, diameter and amount of concrete cover to the steel

Serviceability Limit State of Vibration – and other limit state on RC structure should be considered separately as there is no recommendation in code. Reference should be made to specialist literature,

The Ultimate limit state models the behavior of structural elements at failure due to varieties of mechanisms including bending, shear, compression, and tension. On the other hand, the Serviceability limit state models the behavior of the member at working load. And it is usually connected with the limits state of deflection and cracking, within the context of reinforced concrete.

The design process involves basing the design on the most critical one and then checking for the remaining limit states. This requires the understanding of materials properties and loadings. Important notations for a rectangular section (in tension) are shown in Figure 5.1.

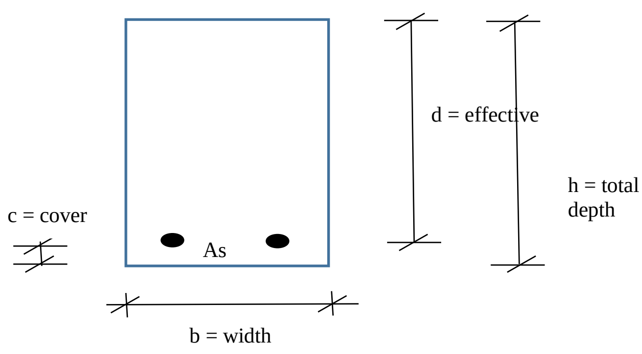


Figure 5.1: Important notations for a rectangular section.

5.3 Serviceability Limit State and Reinforcement Details

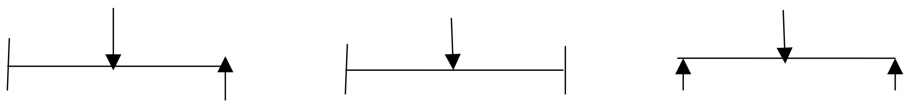
5.3.1 Serviceability Limit State

In RC elements, the serviceability state of deflection and cracking control important dimension. The requirement for fire resistant (though not SLS is included because it also controls size of beams and spacing of reinforcement), influence the dimension of members. The leading dimensions of a structure (i.e. span, height, etc.) are determined by user and aesthetic requirement and consideration.

4.3.2 Deflection

Excessive deflection may produce cracks in non-load bearing elements and finishes, discomfort the user of the structure, and effect the appearance. If a member behaves in a linear elastic manner, the deflection is of the form:

$$\delta = \frac{kwl^3}{EI}$$



k $= \frac{1}{185}$ *or* $\frac{7}{768}$ $\qquad \frac{1}{384}$ *or* $\frac{1}{192}$ $\qquad \frac{5}{384}$ *or* $\frac{1}{48}$

Figure 5.2: Deflection configurations for some standard structure

But for concrete, stress-strain relationship is non-linear and thus E varies with stress. The second moment of area I is for a composite section which may or may not crack. Since E and I cannot be determined with accuracy for a RC concrete member, Code adopted a simple approach based on limiting span/effective depth ratios.

Effective span (clause 3.4.1.2, BS 8110).

a. Simply supported beam

Effect span should be taken as the lesser of:

b. the distance between centres of bearings (A)

c. the clear distance between supports (B) plus the effective depth, d, of the beam

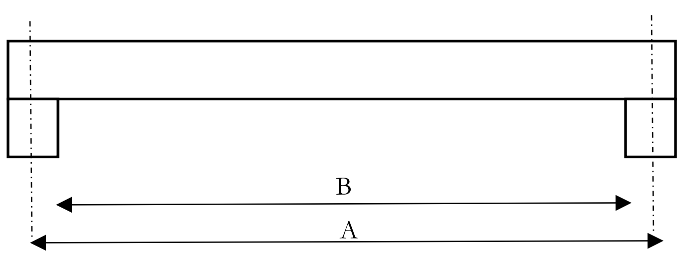


Figure 5.3: Effective Span of Beams

b. For a continuous beam the effective span should normally be taken as the distance between the centres of supports.

The code expects that all calculations relating to beam design should be based on the effective span of the beam. On the basis of the effective span, the defection criteria is satisfied if the span/effective depth ratio does not exceed the appropriate limiting values given in Table 3. 9 (BS 8110) reproduced below as Table 5.1.

Effective depth, d = $\frac{span}{basic\ ratio\ \ (Table\ 3.9)\ X\ modification\ factor\ (Table\ 3.10\ or\ 3.11)}$

Table 5.1: Minimum effective depth ratio (Table 3.9, BS 8110)

Support Conditions	Rectangular Section	Flanged Beams $b_w/b < 0.3$
Cantilever	7	5.6
Simply Supported	20	16.0
Continuous	26	20.8

Using the values in Table 3. 9 (BS 8110) ensures:

1. that the total deflection is limited to $\frac{span}{250}$ and
2. the part of deflection occurring after construction of finishes and partitions will be limited to $\frac{span}{500}$ or 20 mm whichever is lesser

NOTE

1. Table 3.9 is applicable if deflection of $\frac{span}{250}$ is acceptable and the span is not greater than 10m. For span exceeding 10m Table 3.9 should be modified as suggested (Cl. 3.4.6.4)
2. Deflection is influenced by the amount of tension reinforcement and its stress. Therefore, span/effective depth ratio should be multiplied according to the area of reinforcement provided. Values of span/effective depth ratio obtained from Table 3.10 (for tension reinforcement) or Table 3.11 (for compression reinforcement) should therefore be multiplied by the appropriate factor obtained from Table 3.10.
3. A value of between 1.05 - 1.40 can be assumed to make initial estimate of effective depth, and then checked the appropriateness or otherwise of the assumed value later.

Assignment

1) By assuming a modification factor of 1.25, determine the effective depth of a rectangular:
   a. 8m simply supported beam
   b. 8m Span continuous beam
   c. 8m span cantilever beam
2) What conclusions can be made from the answers you obtained from Question 1?

Answer: (a) 320 mm (b) 246.15 mm (c) 914.29 mm

5.3.3 Cracking, Durability and Fire Resistance

Apart from the need to ensure that the design is structurally sound, the designer must also ensure proper performance of the structure in service. This involves two limit states of:

i) Cracking
ii) Durability
iii) Fire

Cracking control is through limitations on the spacing of reinforcement and the amount of concrete cover to the steel.

Durability is achieved by controls on the strength of the concrete and the thickness of the cover to the reinforcement. Tables 5.2 (BS 8110, 1997), reproduced below, gives various exposure conditions.

Table 5.2: Classification of Exposure conditions (Table 3.2, BS 8110)

Environment	Exposure conditions
Mild	Concrete surfaces protected against weather or aggressive conditions
Moderate	Exposed concrete surfaces but sheltered from severe rain or freezing whilst wet Concrete surfaces continuously under non-aggressive water Concrete in contact with non-aggressive soil (see sulfate class 1 of Table 7a in BS 5328-1:1997) Concrete subject to condensation
Severe	Concrete surfaces exposed to severe rain, alternate wetting and drying or occasional freezing or severe condensation
Very severe	Concrete surfaces occasionally exposed to sea water spray or de-icing salts (directly or indirectly) Concrete surfaces exposed to corrosive fumes or severe freezing conditions whilst wet
Most severe	Concrete surfaces frequently exposed to sea water spray or de-icing salts (directly or indirectly) Concrete in sea water tidal zone down to 1 m below lowest low water
Abrasive[a]	Concrete surfaces exposed to abrasive action, e.g. machinery, metal tyred vehicles or water carrying solids

NOTE 1 For aggressive soil and water conditions see **5.3.4** of BS 5328-1:1997.
NOTE 2 For marine conditions see also BS 6349.

[a] For flooring see BS 8204.

The nominal cover to reinforcement for each class of exposure condition is given in Table 3.3 (BS 8110, 1997), and it is reproduced in Table 5.3.

Table 5.3: Nominal cover to all reinforcement (Including links) to meet durability requirement

Conditions of exposure (see **3.3.4**)	Nominal cover Dimensions in millimetres				
Mild	25	20	20[a]	20[a]	20[a]
Moderate	—	35	30	25	20
Severe	—	—	40	30	25
Very severe	—	—	50[b]	40[b]	30
Most severe	—	—	—	—	50
Abrasive	—	—	—	See NOTE 3	See NOTE 3
Maximum free water/cement ratio	0.65	0.60	0.55	0.50	0.45
Minimum cement content (kg/m^3)	275	300	325	350	400
Lowest grade of concrete	C30	C35	C40	C45	C50

NOTE 1 This table relates to normal-weight aggregate of 20 mm nominal size. Adjustments to minimum cement contents for aggregates other than 20 mm nominal maximum size are detailed in Table 8 of BS 5328-1:1997.
NOTE 2 Use of sulfate resisting cement conforming to BS 4027. These cements have lower resistance to chloride ion migration. If they are used in reinforced concrete in very severe or most severe exposure conditions, the covers in Table 3.3 should be increased by 10 mm.
NOTE 3 Cover should be not less than the nominal value corresponding to the relevant environmental category plus any allowance for loss of cover due to abrasion.

[a] These covers may be reduced to 15 mm provided that the nominal maximum size of aggregate does not exceed 15 mm.
[b] Where concrete is subject to freezing whilst wet, air-entrainment should be used (see **5.3.3** of BS 5328-1:1997) and the strength grade may be reduced by 5.

Fire resistance is obtained by ensuring that cross sections are of the minimum thickness and that the cover to reinforcement to reinforcement is sufficient according to Table 3.4 (BS 8110, 1997), reproduced here in Table 5.4.

Table 5.4: Nominal cover to all reinforcement (Including links) to meet specified periods of fire resistance.

Fire resistance *h*	Nominal cover						
	Beams[a]		Floors		Ribs		Columns[a] mm
	Simply supported mm	Continuous mm	Simply supported mm	Continuous mm	Simply supported mm	Continuous mm	
0.5	20[b]	20[b]	20[b]	20[b]	20[b]	20[b]	20[b]
1	20[b]	20[b]	20	20	20	20[b]	20[b]
1.5	20	20[b]	25	20	35	20	20
2	40	30	35	25	45	35	25
3	60	40	45	35	55	45	25
4	70	50	55	45	65	55	25

NOTE 1 The nominal covers given relate specifically to the minimum member dimensions given in Figure 3.2. Guidance on increased covers necessary if smaller members are used is given in section **4** of BS 8110-2:1985.
NOTE 2 Cases that lie below the bold line require attention to the additional measures necessary to reduce the risks of spalling (see section **4** of BS 8110-2:1985).

[a] For the purposes of assessing a nominal cover for beams and columns, the cover to main bars which would have been obtained from Tables 4.2 and 4.3 of BS 8110-2:1985 has been reduced by a notional allowance for stirrups of 10 mm to cover the range 8 mm to 12 mm (see also **3.3.6**).
[b] These covers may be reduced to 15 mm provided that the nominal maximum size of aggregate does not exceed 15 mm (see **3.3.1.3**).

5.3.3.1 Minimum Distance between bars

BS 8110 specifies minimum and maximum distances between reinforcement. The minimum spacing is based on the need to achieve good compaction. And the maximum spacing arise from the need to ensure that cracking does not exceed 0.3mm.

Minimum Spacing

The minimum spacing is governed by the desire to:

i. To allow the aggregate to move within the bars in other to obtain proper compaction and good bond. Thus, the distance must be higher than the aggregate size (Figure 5.4).
ii. Achieve good transmission of force from bar to bar in concrete. Subsequently, the space should not be less than the bar diameter.
iii. Allow the use of immersion poker type of vibrator. For poker of 40mm, the space should be at least 50mm.

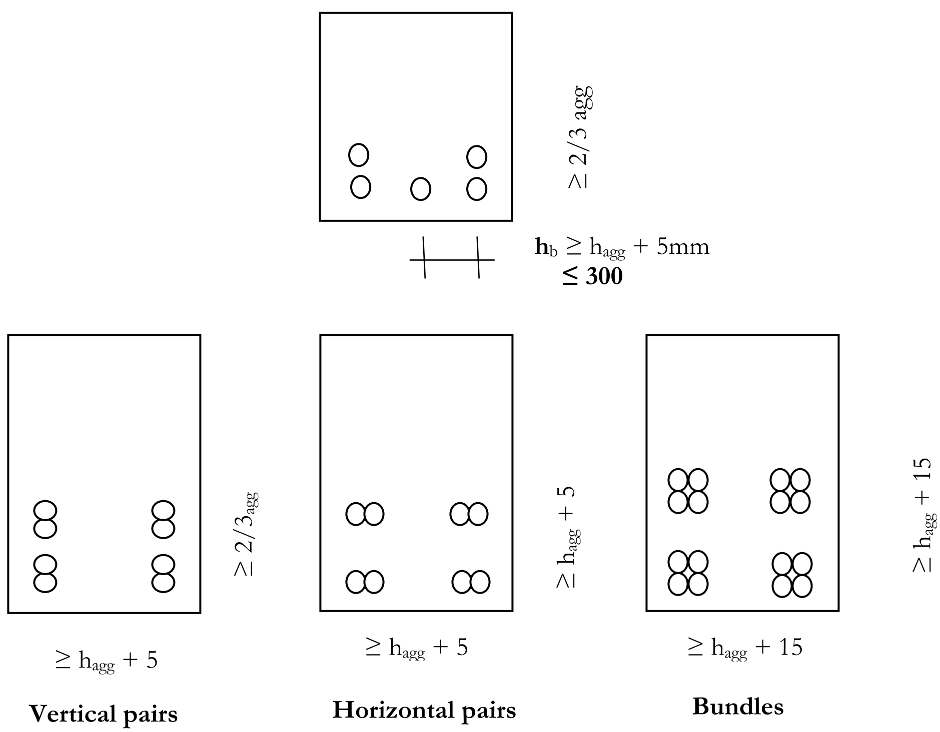


Figure 5.4: Minimum Spacing of reinforcement for different arrangement

The Maximum Spacing

The maximum spacing is restricted in other to control cracking. BS 8110 recommends spacing on the basis of the steel grade (Clause 3.12.11.1)

Table 5.5: Maximum Spacing of bars for control of cracks

f_y	Maximum Spacing
250	**300**
460	**160**

The larger the yield stress, the smaller the spacing.

5.3.3.2 Cover to Reinforcement

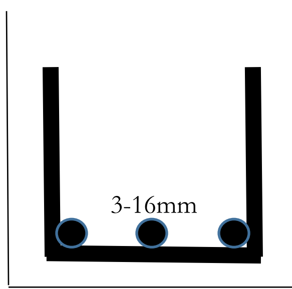
Minimum Cover to Reinforcement

i. Cover to reinforcement is necessary to ensure the bond of the steel with the concrete so that both steel and concrete are effective in resisting the applied force.
ii. Cover is also necessary to prevent corrosion of steel reinforcement and to resist damage by fire.

Also, the requirement of fire resistance which is given in Tables 3.2 – 3.4 BS 8110, as well as the need to ensure that concrete is placed and compacted well between bars and shutter are used to determine cover. The example below will demonstrate how the cover to reinforcement is determined.

Example 5.1

Determine the cover of the cross section of the beam in Figure 5.5 (assume simply supported)



Link = 8mm
Exposure = moderate
Fire Resistance = 3 hrs.
Aggregate size = 20mm
f_{cu} = 35N/mm^2

Figure 5.5

Solution

1) Durability requirement (Table 3.3 BS 8110 or Table 5.3 in this Book)
   For the moderate condition of exposure for the concrete grade given
   C = 35mm (to main reinforcement)
   Nominal C = 35 – 8 (link) = 27 mm
2) Fire Resistance Requirement (Table 3.4 BS 8110 or Table 5.4 in this Book)
   C = 60mm (to main reinforcement)
   C = 60 – 8

C = 52 mm

3) Concreting Requirement

Recall

Minimum spacing between reinforcement (Figure 5.4) = 20 + 5 = 25

C = 25 – 8 (to main reinforcement)

C = 17mm

From the three possible values, the largest value obtained must be used, which is 35mm

THUS

C = 52 mm

5.3.3.3 Area of Reinforcement

BS 8110 recommends that the maximum crack width should not exceed 0.3mm in order to avoid corrosion of steel. This requirement will be met by observing the detailing rules given in BS 8110 with regard to:

i. Minimum reinforcement area

ii. Minimum and Maximum clear spacing between Reinforcement

5.3.3.3.1 Minimum area of reinforcement

In design, reinforcement calculations usually end with an area of steel (mm^2 for beams and mm^2/m width for slabs). For a rectangular section of height h and width b (Figure 5.6), the area of tension reinforcement As, should lie within the following limits;

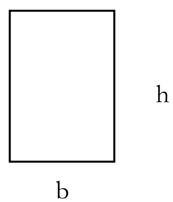


Figure 5.6: Gross section for area of reinforcement

for low yield, when $f_y = 250N/mm^2$

$$0.24\%bh < As < 4\%bh$$

For high yield, when $f_y = 460N/mm^2$

$$0.13\%bh < As < 4\%bh$$

In practice however, the areas of steel obtained from calculations must be converted to an actual number of bars of a certain size for beam, and in case of slab, a size of a bar at a certain pitch (Tables 5.6 and 5.7). For example, if an area of reinforcement obtained from calculation is **2100mm²,** possible choices are:

Table 5.6: Cross Sectional areas for Beam

Bar size (mm)	Number of bars									
	1	2	3	4	5	6	7	8	9	10
6	28.3	56.6	84.9	113	142	170	198	226	255	283
8	50.3	101	151	201	252	302	352	402	453	503
10	78.5	157	236	314	393	471	550	628	707	785
12	113	226	339	452	566	679	792	905	1020	1130
16	201	402	603	804	1010	1210	1410	1610	1810	2010
20	314	628	943	1260	1570	1890	2200	2510	2830	3140
25	491	982	1470	1960	2450	2950	3440	3930	4420	4910
32	804	1610	2410	3220	4020	4830	5630	6430	7240	8040
40	1260	2510	3770	5030	6280	7540	8800	10100	11300	12600

For beams are:

i. 3Nos 32 (2410mm2),
ii. 8Nos of 20mm (2510mm^2),
iii. 2Nos of 40mm (2510mm^2), etc.

Table 5.7: Cross-sectional area per metre width for various bar spacing (mm^2)

Bar size (mm)	Spacing of bars								
	50	75	100	125	150	175	200	250	300
6	566	377	283	226	189	162	142	113	94.3
8	1010	671	503	402	335	287	252	201	168
10	1570	1050	785	628	523	449	393	314	262
12	2260	1510	1130	905	754	646	566	452	377
16	4020	2680	2010	1610	1340	1150	1010	804	670
20	6280	4190	3140	2510	2090	1800	1570	1260	1050
25	9820	6550	4910	3930	3270	2810	2450	1960	1640
32	16100	10700	8040	6430	5360	4600	4020	3220	2680
40	25100	16800	12600	10100	8380	7180	6280	5030	4190

For slab are:

i. 12@50mm (2260mm^2),
ii. 20@125mm (2510mm^2),
iii. 25@200mm (2450mm^2), etc.

Alternatively, the conversion of areas of reinforcement to numbers and spacing can be obtained approximately through the following procedures.

i. For Beam

$$\text{The number of reinforcement} = \frac{Calculated\ area\ of\ reinforocement}{Area\ of\ the\ chosen\ diameter\ of\ reinforcemet}$$

For example, if 32mm diameter is chosen (area = 804 mm^2), using the same area of 2100mm^2

Then

$$\text{The number of reinforcement} = \frac{Calculated\ area\ of\ reinfrocement}{Area\ of\ the\ chosen\ diameter\ of\ reinforcemet} = \frac{2100}{804}$$

$$= 2.62$$

Approximately = 3 numbers

ii. For slab

$$\text{The spacing of reinforcement} = \frac{area\ of\ the\ chosen\ diameter\ of\ reinfrocement}{calculated\ area\ of\ freinforcemet} \text{x } 1000$$

For example, for a 25mm diameter bar (area = 491 mm^2)

$$= \frac{491}{2100} x1000$$

$$= 233.81\text{mm}$$

Approximately = 200 mm

NOTE

But it should be noted that:

i. A beam should not contain less than 2 bars as tension reinforcement

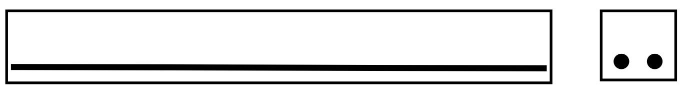


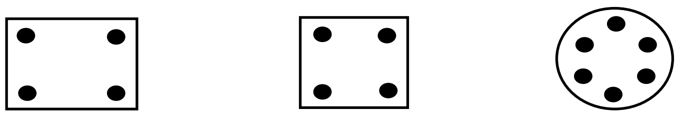
Figure 5.7: Minimum number of reinforcement in beams

ii. The number chosen, should be, as much as possible, symmetrical in arrangement

Symmetrical   Asymmetrical (not allowed)

Figure 5.8: Symmetrical and Unsymmetrical arrangement of reinforcement in beams

iii. A rectangular (or square) column should contain minimum of 4 bars, and circular columns, minimum of 6 bars



rectangle   square   circle

Figure 5.9: Minimum number of reinforcement for different sections

5.3.3.2 Secondary Reinforcement

The code recommendations for secondary reinforcement (in one-way slabs) are:

a. High Yield = 0.12% expressed as % of gross sectional area

b. Mild Steel = 0.15% expressed as % of gross sectional area

5.3.3.3 Minimum Area of Links

Links are provided in beams to resist the shear forces and to prevent premature buckling failure of the longitudinal bars in columns. The minimum area is given by the Code as follows:

a. Mild steel links

$$\frac{A_{sv}}{S_v} = 0.002b$$

b. High harvest steel link

$$\frac{A_{sv}}{S_v} = 0.0012b$$

Where:

A_{sv} = cross-section area of 2 leg of a link

S_v = spacing of links

b_t = breadth of the beams at level of tension reinforcement

The smallest practical bar diameter for link is 6mm. The spacing should not be less than 75mm, and must not exceed 0.75d when resisting shear. In practice, maximum spacing does not exceed 300mm.

Assignment

1. Obtain the number of 12mm diameter reinforcement required in a beam for calculated areas of steel of 1500mm^2
2. If the structural member in "No. 1" is a slab, obtain the spacing required using the same diameter.
3. Re-do questions 1 and 2 using approximate method.

Chapter *6* - Design of Reinforced Concrete Beams

6.1 Introduction

Beam is a horizontal structural member which can be made of materials like steel, timber, or concrete provided it has the strength to resist the tension, compression, and shear force induced in it by the application of load. From cost consideration, and the fact that it can be shaped as desired, plain concrete is a good material for beam construction. It is very strong in compression but weak in tensile strength. Because of its weakness in tension, it is also weak in bending, shear and torsion. This makes plain concrete to have limited application when resistance to tensile stress is necessary.

Steel is another material that is very strong in tension and compression, but it is very expensive and difficult to work. If, however steel is imbedded in the tensile zone of plain concrete, the plain concrete is said to be reinforced, thus the name "reinforced concrete".

The consequence of overcoming tensile weakness in plain concrete with steel (which is strong in tension) is the production of a new material called **reinforced concrete**.

Types of Beam

In most buildings, beams form part of a monolithic floor system (Figure 6.1), and the reinforced concrete floor provide a flange for the rectangular section of the beam.

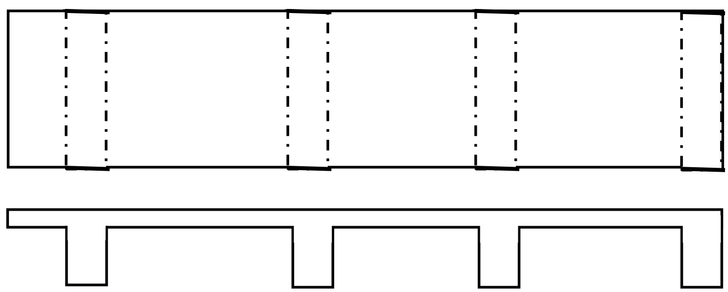


Figure 6.1: Beams as part of beam-slab systems

Beams in reinforced concrete structures can be defined according to the following classifications:

1) Classification according to the Cross section

a) T-Section – these are internal beams in beam-slab floor system.

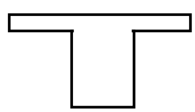


Figure 6.2. A Typical T-section beam

b) L-Section. These are perimeter beams in beam-slab floor system.

Figure 6.3: A typical L-section Beam

2) Classification according to Design considerations

a) Singly reinforced → these are beams reinforced with tension steel only.

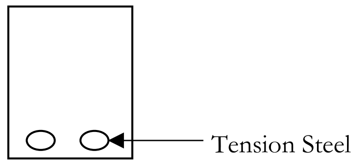


Figure 6.4: A Singly-reinforced concrete beam

b) Doubly reinforced

When a beam is reinforced with tension and compression steel, it is termed doubly reinforced. Inclusion of compression steel will increase the moment capacity of the beam and hence allow more slender sections to be used. Thus, doubly reinforced beams are used in preference to singly reinforced beams when there is some restriction on the construction depth of the section tension reinforcement at the bottom and compression reinforcement at the top

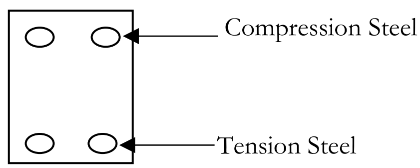


Figure 6.5: A Doubly-reinforced concrete beam

In the simplest form, reinforced concrete beam consists of a concrete beam which contains two or more reinforcement placed at the tension zone. This is usually at the bottom of the beam where the steel is effective in resisting the tensile stresses due to bending.

In the design, the span and the ultimate load for which it must be designed are already known. The section dimensions like breadth, depth, cross sectional areas of steel reinforcement have to be determined. For a given bending moment, there is no unique section which will satisfy the requirement. There are as many possible combination of breadth, depth, steel area, etc. as there are number of designers.

Beam Dimensions

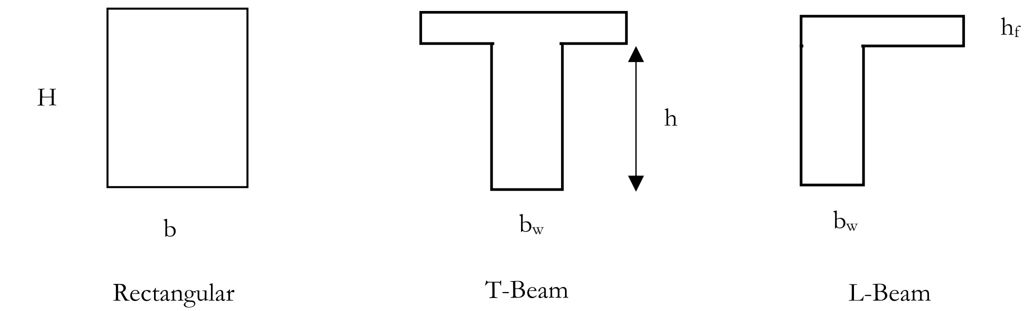


Figure 6.6: Beam dimensions

where:

H = overall depth
h = depth up to the soffit of the slab for flanged beam
b_w = breath of the web
h_f = height of the slab (flange)

6.2 Design of Singly-Reinforced Beams (Ultimate Strength of Section in Flexure/Bending)

Consider simply supported beam (Figure 6.7) and the stress-strain diagrams (Figure 6.8)

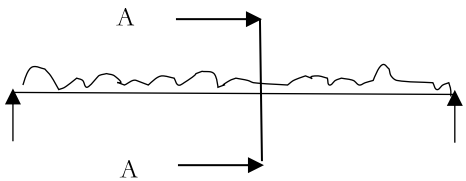


Figure 6.7: A simply supported beam with uniformly distributed load

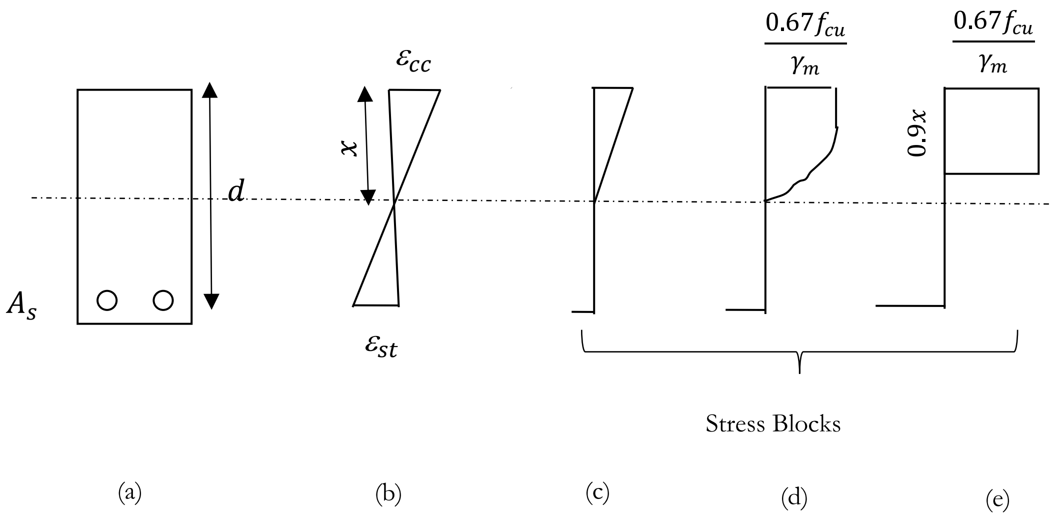


Figure 6.8: The stress-strain diagram of the beam section AA: (a) = sections, (b) = strains, (c) = triangular (Low strain), **(**d) = rectangular parabolic (large strain), and e = equivalent rectangle).

Consider the simply supported beam loaded with a uniformly distributed load as shown (Figure 6.8). The load will cause the beam to defect as downward putting the top portion in compression and the bottom portion in tension. The will make the bottom portion to crack. At a distance below the compression face x, there is neither compression nor tension. Thus, the strain at this point is zero. The axis through this point is called the neutral axis NA.

From the diagrams above, the following observations can be made.

1) At small loads, the stress distribution will be triangular assuming plane section remain plane. This is the elastic region and Hooke's Law applies (b).
2) Above the NA the stress distribution is initially triangular (since stress and strain are directly proportional), and the stress in the concrete below the NA is zero, since the concrete is unable to resist any tensile stress, having cracked. All the tensile stress is assumed to be resisted by the steel reinforcement. (diagram c)
3) As loading increases, and the mid-span moment also increases, the stress distribution changes to "d".
4) At failure, the stress distribution will depend on whether:
   a) The section is under-reinforced
   b) The section is over-reinforced

If the section is over-reinforced, the failure mechanism will be by crushing of concrete due to the fact that the compressive capacity is exceeded, and it is sudden without warning, since the steel does not yield. Not only that steel is expensive, thus over-reinforcing will lead to uneconomical design.

However, if the section is under-reinforced, the steel yields first, and the beam will show considerable deflection that will be accompanied by severe cracks and spalling from the tension face. This will provide an ample warning. This is an economic design as the greater portion of the steel is utilized.

Thus, in under-reinforced section, the tensile force at the ultimate limit state is (from Figure 6.8):

$$F_{st} = \text{design stress x area} \quad \textbf{6.1}$$

$$= \frac{f_y\, A_S}{\gamma_m} \quad \textbf{6.2}$$

Where f_y is the yield stress of steel, A_s the area of reinforcement, and γ_m the factor of safety for reinforcement (= 1.15). However, for concrete, the Code replaces the diagram in "d" with the rectangular stress block in "e" in order to obtain the compressive force in concrete. Also, the rectangular stress block is used in the development of expression for the moment of resistance in singly-reinforced concrete beam section.

Ultimate Moment of Resistance M_u for Singly Reinforced Beam

Consider the singly reinforced beam in Figure 6.9, which is extract from Figures 6.7 and 6.8.

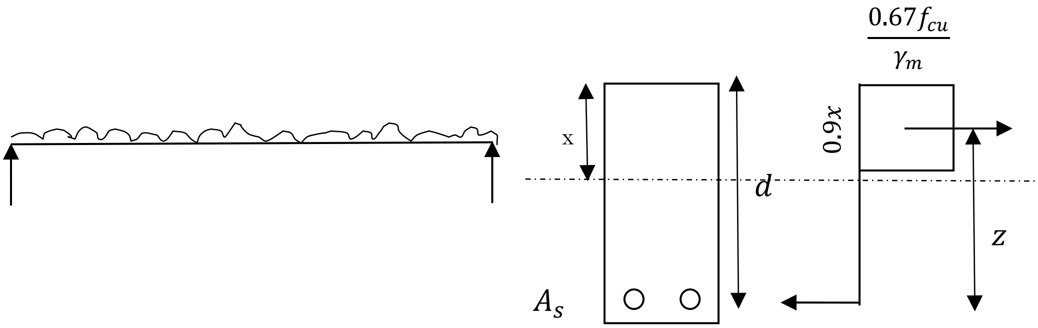


Figure 6.9: Ultimate moment of resistance for singly-reinforced section

NOTE

1) The loading on the beam gives rise to maximum moment at the mid span, which is called **ultimate design moment M**.
2) The resulting curvature of the beam produces a compressive force F_{cc} in the concrete and a tensile force F_{st} in the reinforcement.

   Since equilibrium condition exists,

$$F_{cc} = F_{st} \qquad 6.3$$

These two forces are separated by a distance "z", called lever arm now forms a couple. The moment of the couple is called **ultimate moment of resistance Mu**, which opposes the ultimate design moment M.

For structural stability, Mu $\geq$ M

By multiplying the compressive force by the lever arm z, compressive Moment is obtained, and ditto for tensile moment

$$Mu = F_{cc}\, z = F_{st}\, z \qquad 6.4$$

From the stress block

$$Fcc = \text{stress x area} = \frac{0.67f_{cu}}{\gamma_m} \text{ x } 0.9 \text{ x b} \qquad 6.5$$

Also

$$z = d - \frac{0.9x}{2} \qquad 6.6$$

Now the depth of the neutral axis x is limited by Code to a maximum of 0.5d for under-reinforced. That is:

$$x \leq 0.5d \qquad 6.7$$

Combining equations 6.4 – 6.7, and putting γ = 1.5, the ultimate moment of resistance Mu is given by:

$$\mathbf{Mu = 0.156fcubd^2} \qquad 6.8$$

It is to be noted that:

1) Mu depends only on the properties of the concrete, and not the steel reinforcement.
2) Provided Mu > M, a beam whose section is singly-reinforced will be sufficient to resist the design moment M.

Calculation of Area of Reinforcement

From equation 6.4

$$M = F_{st}\, z = \frac{f_y A_s}{\gamma_m} z \qquad \textbf{6.9}$$

By re-arranging, and putting $\gamma_m = 1.15$ gives

$$\mathbf{As} = \frac{M}{0.87 f_y z} \qquad \textbf{6.10}$$

Equation 6.10 requires the determination of z, which can be done graphically or mathematically. The Code however expressed it as:

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}] \qquad \textbf{6.11}$$

Once "z" has been determined, the area of tension reinforcement can then be calculated using equation 10

NOTE

Clause 3.4.4.1 of BS 8110 stipulated that "z" should not exceed 0.95d in order to give reasonable concrete area in compression.

Equation 6.10 can be used provided M ≤ Mu, or K ≤ K' where:

$$K = \frac{M}{f_{cu} b d^2} \qquad \textbf{6.12}$$

$$K' = \frac{M_u}{f_{cu} b d^2} \qquad \textbf{6.13}$$

In summary, to design for bending requires the calculation of maximum design moment M and the ultimate moment of resistance of the section Mu.

Case 1: IF Mu > M, only tension reinforcement is required and the area of the reinforcement is calculated from equation 10.

Case 2 IF Mu < M, then the designer can:

i. either increase the section sizes until Mu > M), or
ii. design as doubly-reinforced section.

Example 6. 1

A 6m simply supported rectangular beam carries characteristic dead (including self-weight of beam), gk, and imposed, *q*k, loads of 10 kN/m and 8 kN/m respectively. If the beam dimensions are breadth, *b*, 250 mm and effective depth, *d*, 450 mm, calculate the area of reinforcement required. Take f_y = 460N/mm^2 and f_{cu} = 30N/mm^2, concrete cover = 35 mm, and maximum aggregate = 20 mm.

Solution

The beam is drawn in the figure 6.10

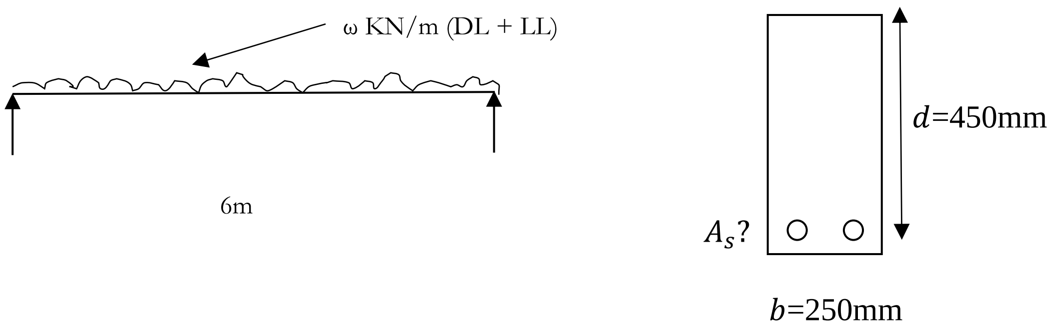


Figure 6.10: Example 6.1

Design Load = 1.4gk + 1.6qk = 1.4 x 10 + 1.6 x 8 = 14.0 + 12.8 = 26.8 KN/m

For a simply supported beam, the maximum mid-span moment M

$$M = \frac{\omega l^2}{8} = \frac{26.8 \ x \ 6^2}{8} = 120.\ 6\text{KN.m}$$

The ultimate moment of resistance Mu

Mu = **0.156fcubd²**

= 0.156 x 30 x 250 x (450)² = 236. 93KN.m

Since Mu is greater than M

Then design the beam as singly reinforced.

$$K = \frac{M}{f_{cu}bd^2} = \frac{120.6 \ x \ 1000000}{30 \ x \ 250 \ x \ 450 \ x \ 450} = 0.08$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

$= d[0.5 + \sqrt{(0.25 – 0.089)}] = 0.90d < 0.95d$ O.K

= 0.90 x 450 = 405 mm

Thus

$$\mathbf{As} = \frac{M}{0.87 f_y z} = \frac{120.6 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 405}$$

= 744.07 mm^2

For the purpose of structural detailing, the area of steel has to be converted into a certain number of bars of a given diameter. This is usually achieved using steel area tables similar to that shown in Table 5.3 and 5.4. The area of the number of bars chosen should be such that:

- **i.** it is not less than the calculated value
- **ii.** it is not more than 10% of the calculated value.

From this Table, the followings are possible numbers of bar configuration that can be used:

- **i.** 2Nos of 25mm (982mm^2)
- **ii.** 3Nos of 20mm (943mm^2)
- **iii.** 4Nos of 16mm (804mm^2)

The final choice will be determined by

- i. Availability in the market in sufficient quantities
- ii. The need for adequate spacing for compaction
- iii. The ease of construction

Also, in structural detailing, mild steel is differentiated from high yield steel by designation. In Nigeria, the designations are:

Mild steel is represented by **R**

High yield steel is represented by **Y**

From the calculation above, based on the fact that 20 mm diameter is more common in Nigeria, option "ii" will be used.

Therefore

Use 3Y20 (943mm^2)

Checks

Some checks will have to be carried out to ensure compliance with certain provisions of the codes.

i. Spacing

The minimum spacing between bars = h_{agg} + 5mm = 25 mm (Assuming 20 mm maximum aggregate)

The width (b) requited for the reinforcement arrangement

b = 2 x cover + 2 x link diameter + 3 x bar diameter + 2 x spacing between bars

for cover = 35 mm and link diameter = 10mm,

then,

b = 2 x 35 + 2 x 10 + 3 x 20 + 2 x 25 = 70 + 20 + 60 + 50 = 190 mm << 250 hence OK.

(Clause 3.12.11.2.6)

ii. Deflection

In order to check that deflection is within limits set by the Codes, two span/effective depth ratios are calculated and compared:

i) Based on the amount of reinforcement provided using the Tables 3.9 and 3.10 in the Standard and service stress f_s calculated by using equation 8 of the code (fs = $2fyA_{sr}/3A_{sp}\beta$). β = 1 for simply supported beam. This is termed $\frac{permissible\ span}{deph}$ ratio.

ii) Based on the actual configuration, the length and effective depth of the beam. This is termed $\frac{actual\ \ span}{depth}$ ratio.

For the deflection to be satisfactory, the $\frac{permissible\ span}{depth}$ ratio must be higher.

From the previous example,

M = 120.6KN.m

b = 250mm

d = 450mm

As (required) = 829.93 mm^2

As (provided = 943mm^2

Therefore

$$\frac{M}{bd^2} = \frac{120.6\ x\ 1000000}{250\ x\ 450\ x\ 450} = 2.382$$

And equation 8 of Table 3. 16 of BS 8110, the service stress f_s

$$f_s = \frac{2fyA_{s\,reuired}}{3_{As\ provided}} \text{ x } \frac{1}{\beta} \quad (\beta = 1\ for \text{ simply supported beam/slab}$$

$$= \frac{2\ x\ 460\ x\ \ 829.93}{3\ x\ 943} \text{ x } \frac{1}{1} \quad = \ 269.90 \text{ N/mm}^2.$$

From Table 3.10, the modification factor is 0.90

And for simply supported Beam, the span/effective depth ratio is 20. Thus, by multiplying this value by the modification factor, the permissible span/depth ratio is obtained. That is:

$$\frac{permissible\ span}{depth} = 20 \text{ x } 0.9 \ = 18$$

But,

$$\frac{actual\ span}{depth} \text{ ratio} = \frac{6000}{450} \ = 13.33$$

Since the $\frac{permissible\ span}{depth}$ ratio (18) is higher than the $\frac{actual\ span}{depth}$ ratio (13.33).

The deflection is OK.

The beam can be sketched as in the figure 6.11

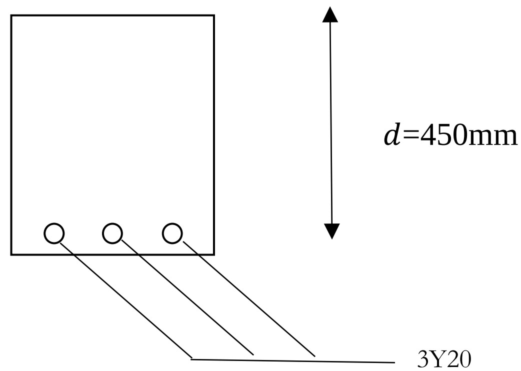


Figure 6.11: A sketch of reinforcement arrangement

Example 6.2
A rectangular simply supported beam with overhang is to be designed according to the BS 8110.

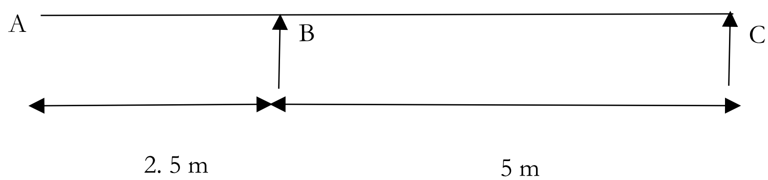


Figure 6.12: A Sketch for Example 6.2

The data for the beam is as follows:

(i.) **Dead load** (including the self-weight of the beam) = 8 KN/m (ii) **Live load** = 10 KN/m

(iii) **Exposure condition** = moderate, (iv) $\mathbf{f_y}$ = 460N/mm^2 (v) $\mathbf{f_c}$ = 30N/mm^2

(vi) **b** = 250 mm (vii) **d** = 400 mm, (viii) Main bar = 12 - 20 mm, and

(ix) link diameter = 8 mm, Fire resistance = 3 hrs

Solution

The Maximum design load = 1.4gk + 1.6qk = 1.4 x 8 + 1.6 x 10

= 27.2 KN/m

The Minimum design load = 1.0gk

= 1. X 8 = 8KN/m

The loading cases considered are shown in Figure 6.13. Analyzing the beam, considering alternating loading cases, the Bending moment and Shear force envelopes in Figure 6.14 were obtained. Only the peaks values were shown.

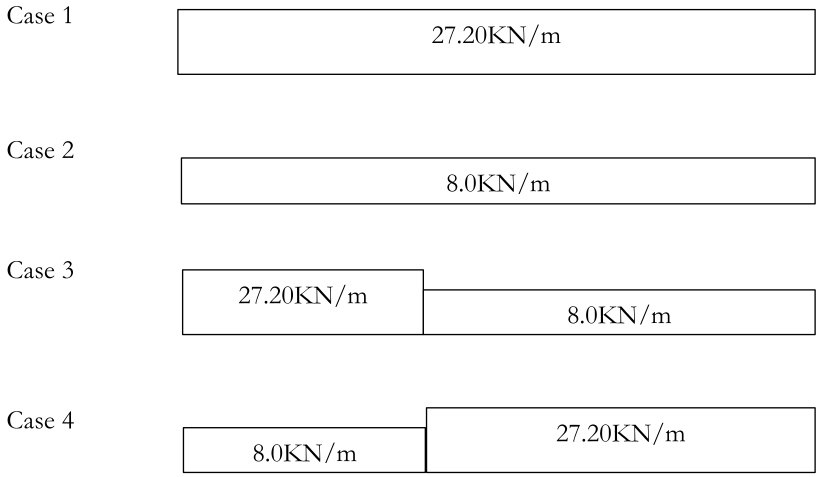


Figure 6.13: The Loading cases

From the bending moments envelope (Figure 6.14), the maximum BM are:

Support B = 85 KN.m

Span BC = 73.01 KN.m

Design of Support Reinforcement at B

M = 85 KN.m

The ultimate moment of resistance Mu

Mu = **0.156fcubd²**

= 0.156 x 30 x 250 x $(400)^2$ = 187. 20KN.m

Since Mu is greater than M

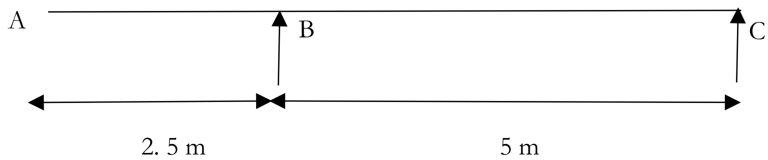
Then design the beam as singly reinforced.

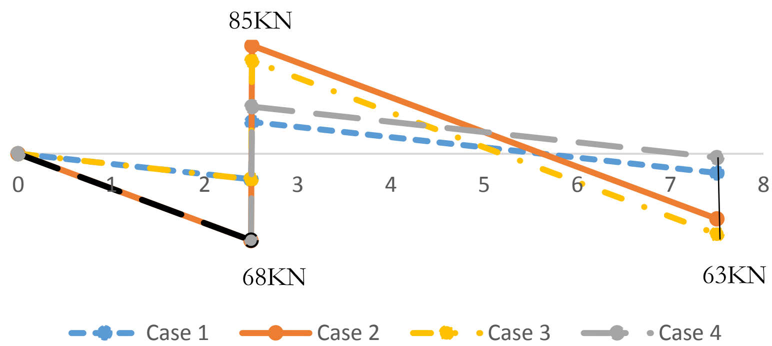
$$K = \frac{M}{f_{cu}bd^2} = \frac{85\ x\ 1000000}{30\ x\ 250\ x\ 400\ x\ 400} = 0.071$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

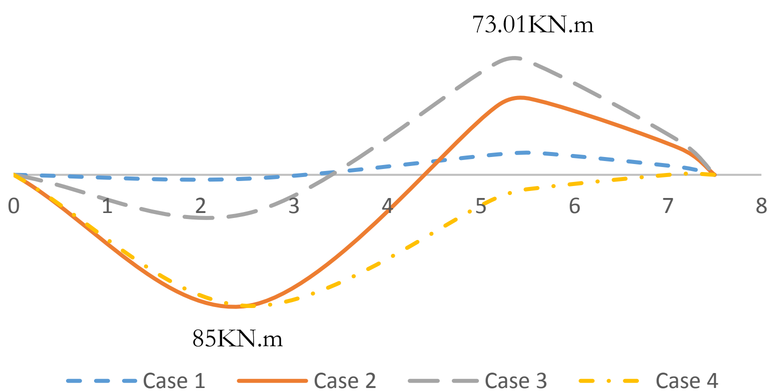
= d [0.5 + √(0.25 – 0.079)] = d[0.5 + 0.42] = 0.92d < 0.95d O.K

= 0.92 x 400 = 368 mm





c. Shear Force Diagram



d. Bending Moment Diagram

Figure 6.14: The Bending Moments and Shear Force Envelopes for the Beam

Thus,

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{85 \; x \; 1000000}{0.87 \; x \; 460 \; x \; 368}$$

$= 577.16 \text{ mm}^2$

Possible configuration of reinforcement is:

i. 3Y16 (603 mm^2)

ii. 2Y20 (628mm^2)

Any of the above will be safe, adequate and sufficient. But for the purpose of this example, we shall use the 1st option, that is **3Y16**

Design of Span BC Reinforcement

M = 73.01 KN.m

Mu = $\mathbf{0.156 f_{cu} b d^2}$

= 0.156 x 30 x 250 x $(400)^2$ = 187. 20KN.m

Since Mu is greater than M

Then design the beam as singly reinforced.

$$K = \frac{M}{f_{cu} b d^2} = \frac{73.01 \ x \ 1000000}{30 \ x \ 250 \ x \ 400 \ x \ 400} = 0.061$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

= d[0.5 + √(0.25 – 0.079)] = d[0.5 + 0.43] = 0.93d < 0.95d O.K

= 0.93 x 400 = 372 mm

Thus

$$\mathbf{As} = \frac{M}{0.87 f_y z} = \frac{73.1 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 372}$$

= 491.02 mm^2

Possible reinforcement configurations are:

i. 5Y12 (565 mm^2)
ii. 3Y16 (603 mm^2)
iii. 2Y20 (628 mm^2)

The second option seems visible in Nigeria environment, if other requirements (like bar arrangement) are met. So, use

3Y16 (603 mm^2)

Checks

Some checks will have to be carried out to ensure compliance with certain provisions of the codes.

i. **Spacing/arrangement of bars**

The minimum spacing between bars = h_{agg} + 5mm = 25 mm (Assuming 20 mm maximum aggregate)

The width (b) requited for the reinforcement arrangement

b = 2 x cover + 2 x link diameter + 3 x bar diameter + 2 x spacing

We have to determine the amount of cover from three conditions

- Durability requirements (Table 3.3)

  Cover, C = 35 mm (to main reinforcement)

  Nominal C = 35 – 8 (link diameter) = 27 mm

- Fire Resistant requirement (Table 3.4)

  Cover, C = 60 mm (to main reinforcement)

  Nominal C = 80 – 8 (link diameter) = 52

- Concreting requirement

  Spacing between aggregate = 20 (i. e. maximum aggregate size) + 5 = 25 mm

Therefore, the suitable nominal cover C = 52 mm.

Thus,

b = 2 x cover + 2 x link diameter + 3 x bar diameter + 2 x spacing

= 2 x 52 + 2 x 8 + 3 x 16 + 2 x 25 = 218 mm << 250 mm hence OK.

ii. Deflection

In order to check that deflection is within limits set by the Codes, two span/effective depth ratios are calculated and compared:

i) Based on the amount of reinforcement provided using the Tables 6.1 and 6.2 (Tables 3.10 and 3.11 BS 8110) in the Standard and service stress f_s calculated by using equation 8 of the code ($fs = 2fyA_{sr}/3A_{sp}\beta$). β = 1 for simply supported beam. This is termed $\frac{permissible\ span}{depth}$ ratio.

ii) Based on the actual configuration, the length and effective depth of the beam. This is termed $\frac{actual\ \ span}{depth}$ ratio.

For the deflection to be satisfactory, the $\frac{permissible\ span}{depth}$ ratio must be higher.

Table 6.1: Modification factor for Tension reinforcement

Service stress	M/bd^2								
	0.50	**0.75**	**1.00**	**1.50**	**2.00**	**3.00**	**4.00**	**5.00**	**6.00**
100	2.00	2.00	2.00	1.86	1.63	1.36	1.19	1.08	1.01
150	2.00	2.00	1.98	1.69	1.49	1.25	1.11	1.01	0.94
(f_y = 250) 167	2.00	2.00	1.91	1.63	1.44	1.21	1.08	0.99	0.92
200	2.00	1.95	1.76	1.51	1.35	1.14	1.02	0.94	0.88
250	1.90	1.70	1.55	1.34	1.20	1.04	0.94	0.87	0.82
300	1.60	1.44	1.33	1.16	1.06	0.93	0.85	0.80	0.76
(f_y = 460) 307	1.56	1.41	1.30	1.14	1.04	0.91	0.84	0.79	0.76

NOTE 1 The values in the table derive from the equation:

$$\text{Modification factor} = 0.55 + \frac{(477 - f_s)}{120\left(0.9 + \frac{M}{bd^2}\right)} \leq 2.0 \qquad \text{equation 7}$$

where

M is the design ultimate moment at the centre of the span or, for a cantilever, at the support.

NOTE 2 The design service stress in the tension reinforcement in a member may be estimated from the equation:

$$f_s = \frac{2 f_y A_{s\ req}}{3 A_{s\ prov}} \times \frac{1}{\beta_b} \qquad \text{equation 8}$$

NOTE 3 For a continuous beam, if the percentage of redistribution is not known but the design ultimate moment at mid-span is obviously the same as or greater than the elastic ultimate moment, the stress f_s in this table may be taken as $2/3 f_y$.

Table 6.2: Modification factors for compression reinforcement

$100\frac{A'_{s\,prov}}{bd}$	Factor
0.00	1.00
0.15	1.05
0.25	1.08
0.35	1.10
0.50	1.14
0.75	1.20
1.0	1.25
1.5	1.33
2.0	1.40
2.5	1.45
≥ 3.0	1.50

NOTE 1 The values in this table are derived from the following equation:
Modification factor for compression reinforcement =

$$1 + \frac{100A'_{s\,prov}}{bd} \Big/ \left(3 + \frac{100A'_{s\,prov}}{bd}\right) \leq 1.5 \qquad \text{equation 9}$$

NOTE 2 The area of compression reinforcement *A* used in this table may include all bars in the compression zone, even those not effectively tied with links.

From the previous example,

M = 73.01KN.m

b = 250mm

d = 400mm

As (required) = 491.02 mm^2

As (provided = 603.00 mm^2

Therefore

$$\frac{M}{bd^2} = \frac{73.01\ x\ 1000000}{250\ x\ 400\ x\ 400} = 1.83$$

And equation 8 of Table 3. 16 of BS 8110, the service stress f_s

$$f_s = \frac{2f_yA_{s\,reuired}}{3_{As\ provided}} \text{ x } \frac{1}{\beta}\ (\beta = 1\ for \text{ simply supported beam/slab}$$

$$= \frac{2\ x\ 460\ x\ 491.02}{3\ x\ 603} \text{ x } \frac{1}{1} = 249.72 \text{ N/mm}^2.$$

From Table 3.10, the modification factor is about 1.30

This beam can be considered as a combination of simply supported plus a cantilever. The span/effective depth ratio can be taken to be the average of the two values. That is, average of 20 and 7, which is 13.5. Thus, by multiplying this value by the modification factor, the permissible span/depth ratio is obtained. That is:

$$\frac{permissible\ span}{depth} = 13.5 \text{ x } 1.3 = 17.60$$

But,

$$\frac{actual\ span}{depth}\ \text{ratio} = \frac{5000}{400} = 12.5$$

$$\frac{actual\ span}{depth}\ \text{ratio} = \frac{2500}{400} = 6.25\ \text{(for the overhang portion)}$$

Since the $\frac{permissible\ span}{depth}$ ratio (17.60) is higher than the $\frac{actual\ span}{depth}$ ratio (12. 5 or 6.25).

The deflection is OK.

Although, no moment is at the support A, it is usual to provide reinforcement equal to 40% of the initial fixed end moment calculated from maximum design load (Kong and Evans, 1987). That is

$M = 0.4 \text{ x. } \frac{wl^2}{12}$ for span BC

Also, for the compressive face of the span AB.

The sketch of the arrangement of reinforcement

The support reinforcement at B will be placed at the top, while the mid span (BC) reinforcement will be at the bottom. The arrangement, including relevant sections is shown in Figure 6.15.

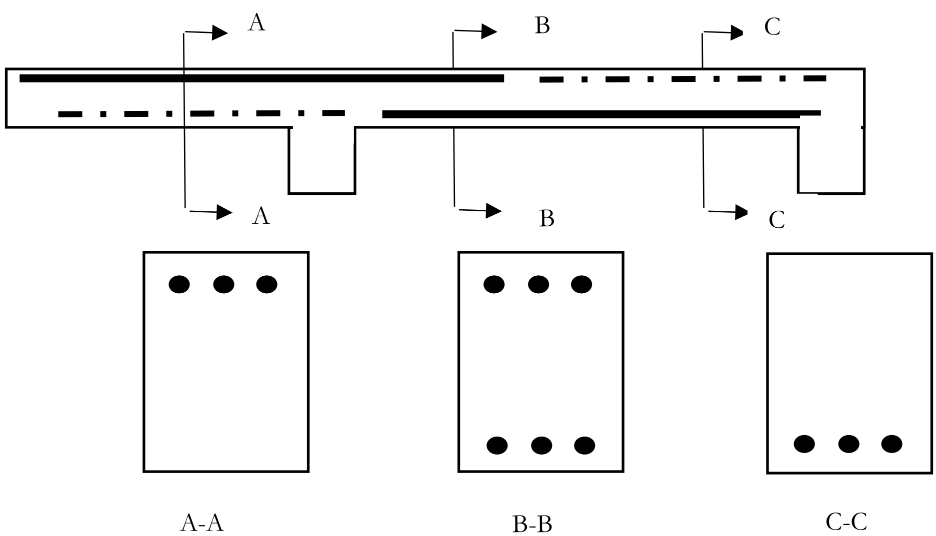


Figure 6.15: Sketch of arrangement of reinforcement

6.3 Shear in Reinforced Concrete Beam

One of the many ways that a beam can fail is if the shear capacity is exceeded. Shear failure may arise in several ways, but the two principal ways of shear failure are through diagonal compression and diagonal tension (Figure 6.16).

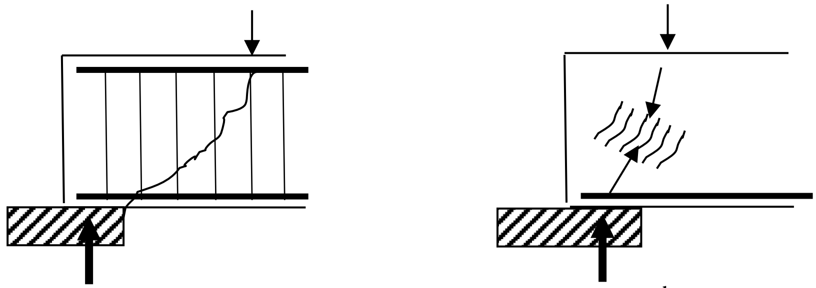


Figure 6.16: Types of shear failure: a) Diagonal Tension, b) Diagonal Compression

Diagonal tension occurs in form of inclined cracks between the edge of the support and the loading point, as the loading increases. This results in splitting of the beam into two pieces. This is prevented by proving shear reinforcement.

Diagonal compression occurs under the action of large loads acting near the support. This always result in crushing of the concrete.

This type of failure is avoided by limiting the maximum shear stress (v) to 5N/mm² or $0.8\sqrt{f_{cu}}$ whichever is lesser. The design shear stress v at any cross section is calculated from:

$$v = \frac{V}{b_v d} \qquad \textbf{6.14}$$

where:

v = the shear stress (N/mm²)

V = the design shear force due to the ultimate loading (N)

b_v = the breadth of the section

b_v = b for rectangular beams

$b_v = b_w$ for flanged beams

d = the effective depth of the section

If v exceeds **the lesser of 5N/mm² or $0.8\sqrt{f_{cu}}$,** then the product $b_v d$ must be increased to reduce v.

In order to determine the necessity of shear reinforcement, it is necessary to calculate the shear resistance of the concrete section otherwise called the "design concrete shear stress" v_c. It is found to consists in three components, which are:

i) Due to the uncracked concrete in the compression zone
ii) Aggregate interlock across the crack zone
iii) The shear force acting along the longitudinal steel (known as Dowel force/action) reinforcement

Design concrete shear stress v_c can be determine from Table 6.3 (Table 3. 8, BS 8110).

The values in Table 6.3 are given in relation to the followings observations:

i) It is given in terms of the percentage area of longitudinal reinforcement ($\frac{100A_s}{bd}$) and the effective depth of the section.

ii) The Table assumes a grade of concrete of 25 N/mm². For other grades, the design shear stress can be calculated by multiplying the values in the Table by the factor $(\frac{fcu}{25})^{0.333}$

If design shear stress v exceeds design concrete shear stress v_c, then shear reinforcement is needed. This is normally done by providing:

i) Vertical shear reinforcement, called "link"

ii) Combination of vertical and inclined (or bent-up) bars.

In order to prevent shear failure

$$V \leq V_{conc} + V_{link} \quad \textbf{6.15}$$

Where

V = design shear force due to ultimate loads

V_{conc} = shear resistance of concrete

V_{link} = shear resistance of the links

Table 6.3: Values of design concrete shear stress

$\frac{100A_s}{b_v d}$	Effective depth mm							
	125	150	175	200	225	250	300	≥ 400
	N/mm²	N/mm²	N/mm²	N/mm²	N/mm²	N/mm²	N/mm²	N/mm²
≤ 0.15	0.45	0.43	0.41	0.40	0.39	0.38	0.36	0.34
0.25	0.53	0.51	0.49	0.47	0.46	0.45	0.43	0.40
0.50	0.67	0.64	0.62	0.60	0.58	0.56	0.54	0.50
0.75	0.77	0.73	0.71	0.68	0.66	0.65	0.62	0.57
1.00	0.84	0.81	0.78	0.75	0.73	0.71	0.68	0.63
1.50	0.97	0.92	0.89	0.86	0.83	0.81	0.78	0.72
2.00	1.06	1.02	0.98	0.95	0.92	0.89	0.86	0.80
≥ 3.00	1.22	1.16	1.12	1.08	1.05	1.02	0.98	0.91

NOTE 1 Allowance has been made in these figures for a γ_m of 1.25.

NOTE 2 The values in the table are derived from the expression:

$0.79\{100A_s/(b_v d)\}^{1/3} (400/d)^{1/4}/\gamma_m$

where

$\frac{100A_s}{b_v d}$ should not be taken as greater than 3;

$\frac{400}{d}$ should not be taken as less than 1.

For characteristic concrete strengths greater than 25 N/mm², the values in this table may be multiplied by $(f_{cu}/25)^{1/3}$. The value of f_{cu} should not be taken as greater than 40.

In order to calculate the shear resistance of the links, consider a reinforced concrete beam with links uniformly spaced at distance s_v under the action of shear force V. Assuming that failure occurred along a plane approximately 45^0 to the horizontal as shown in the figure 6.17. The number of links intersecting the potential cracks is d/s_v .

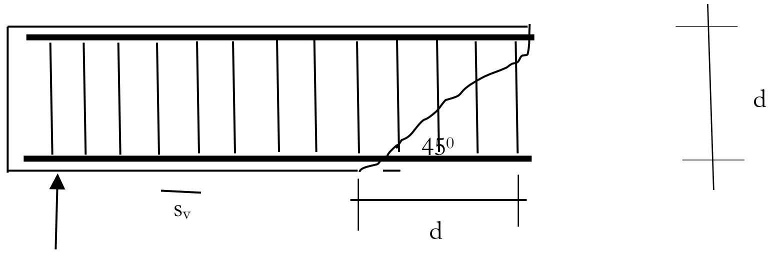


Figure 6.17: Calculation for links for beam

Thus, the V_{link} is

V_{link} = number of links x cross sectional area of link x design stress

$$= \frac{d}{S_v} \times A_{sv} \times 0.87 f_{yv} \qquad \textbf{6.16}$$

Also

$$V_{conc} = v_c bd \qquad \textbf{6.17}$$

From equation 15

$$V \leq v_c bd + \frac{d}{S_v} \times A_{sv} \times 0.87 f_{yv} \qquad \textbf{6.18}$$

By dividing equation 18 by bd, and then re-arrange gives

$$\frac{A_{SV}}{S_v} = \frac{b(v - v_C)}{0.87 f_{yv}} \qquad \textbf{6.19}$$

where A_{sv} is 2 x area of link (for 2-legged links), and 4 x area of link (for 4-legged) (Figure 6.18).

In the provision of shear reinforcement, the requirements of Table 3.8 of the Code should be observed, which are:

i) If $v < 0.5v_c$ throughout the bean, then no links is required. BUT it is the normal practice to provide nominal links in members of structural importance.

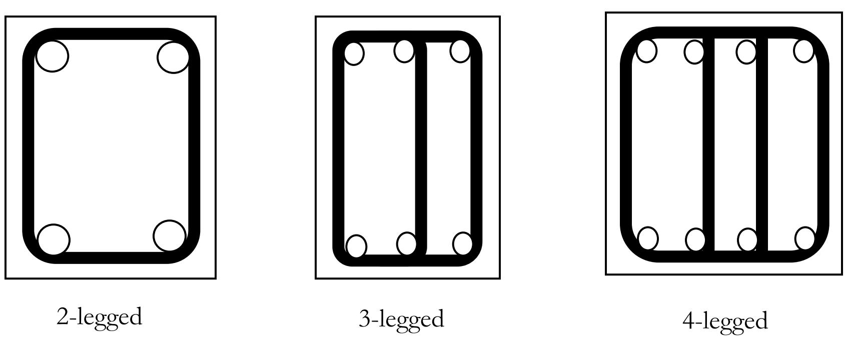


Figure 6.18: Description of links

ii) If $0.5v_c < v < (v_c + 0.4)$, then nominal reinforcement should be provided for the whole length of the beam.

$$A_{sv} \geq \frac{0.4bS_v}{0.87f_{yv}} \qquad \textbf{6.20}$$

Another way of saying it is to say that if

$v < (v_c + 0.4)$, then nominal link is to be provided where shear force $V < (v_c + 0.4)\, b_v d$

iii) If $(v_c + 0.4) < v < 0.8\sqrt{fcu}$ or $5N/mm^2$, then design the link.

$$A_{sv} = \frac{bS_v(v - v_c)}{0.87f_{yv}} \quad \text{(from equation 6.19)} \qquad \textbf{6.21}$$

$$\frac{A_{sv}}{S_v} = \frac{b(v - v_c)}{0.87f_{yv}}$$

But for practical use, it is better arranged as:

$$S_v = \frac{0.87A_{sv}f_{yv}}{b(v - v_c)} \qquad \textbf{6.22}$$

Where

A_{sv} = area of the "legs" of the link

Construction Rules for Links in Beams

1) The diameter of the link should not be less than ¼ of the diameter of the largest longitudinal bar in tension or compression zone
2) The largest size of the link is usually 10mm, and only in special large sections may larger size be used. This is because of the fact that there is a limit to the radius to which the bar may be bent at the corner. Too large radius is unsuitable at the corner of beams
   Generally, in this country, the diameter of link is usually 8 or 10 mm
3) In the singly-reinforced section, it is necessary to provide bars at the top of section in order to anchor the links. The additional bars should not be less than 12 mm

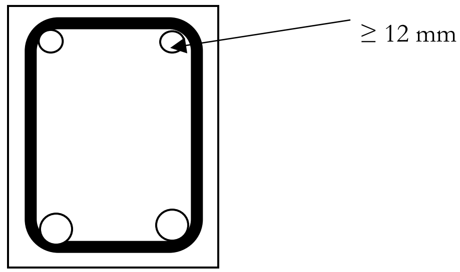


Figure 6.19: Additional reinforcement for anchor in singly-reinforced section

4) The spacing of legs S_v in longitudinal direction should not exceed 0.75 of effective depth.

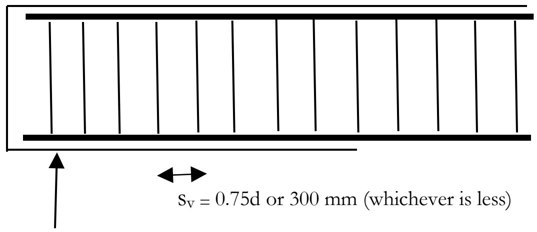


Figure 6.20: Spacing in links in singly reinforced beam section

5) In the case of doubly-reinforced section, such pitch should not exceed 12 times the size of the smallest compression bars or 0.75d or 300 mm.

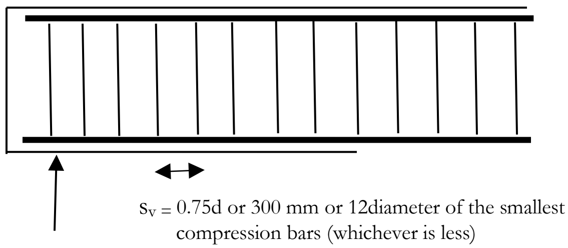


Figure 6.21: Spacing in links in doubly reinforced beam section

Example 6. 3
Design the shear reinforcement for the beam in Example 6.2

Solution

From the Shear force envelope, the ultimate Shear load at support B

For Span AB = 85 KN

For Span BC = 68 KN

The design shear stress

$$v = \frac{V}{bd} = \frac{85 \ x\ 1000}{250 \ x\ 400} = 0.85 \text{ N/mm}^2$$

The limiting value is 5N/mm² or $0.8\sqrt{f_{cu}}$ = 5 or $0.8\sqrt{30}$ = 5 or 4.38N/mm²

Since the value of v (0.85N/mm²) is less than the limiting value (2.74N/mm²). the dimensions are adequate for shear.

The area of longitudinal reinforcement provided in Example 6. 2, As is 603.00 mm²

Now,

$$\frac{100 \ x\ As}{bd} = \frac{100 \ x\ 603}{250 \ x\ 400} = 0.603$$

From Table 8 (BS 8110), the design concrete shear stress

$v_c = 0.54$ N/mm²

Because, the characteristic strength of concrete fc = 30N/mm², this value will have to be multiplied by $(\frac{f_{cu}}{25})^{0.333}$. That is:

$v_c = 0.54 \times (\frac{30}{25})^{0.333} = 0.574$ N/mm².

And $\frac{v_c}{2} = 0.287$ N/mm²

Since v > $\frac{v_c}{2}$, then shear reinforcement is needed.

But $v < v_c + 0.4$ = (0.574 + 0.4 = 0.974. That is, the shear force V = 0.974 x 250 x 400 = 97.4KN. So, provide nominal link throughout the whole length of the beam, since the maximum shear force is 85KN.

From equation 6.20

$$A_{sv} \geq \frac{0.4bS_v}{0.87f_{yv}}$$

So that:

$$S_v = \frac{0.87A_{sv}f_{yv}}{0.4b}$$

Diameter of the link = 8mm. The cross-sectional area = 50.29 mm²

Assuming a 2-legged 8 mm diameter link (50.29 x 2 legs = 100.58 mm²)

Therefore

$$S_v = \frac{0.87 \; x \; 100.58 \; x \; 460}{0.4 \; x \; 250}$$ = 402. 52 (spacing of 8 mm diameter links)

For singly reinforced beam, the maximum link spacing is 0.75d or 300mm, whichever is less. That is:

i. 0.75 x 400 = 300 mm
ii. 300 mm

Use

S_v = 300 mm (Y8@300mm)

This spacing can also be used for Span BC

The sketch of the beam with the link is shown in figure 6.22.

Although the sketch still falls of expectation required for a typical structural detailing works, it is still sufficient for now, as an expression of the picture of the beam. The rules governing detailing and curtailment of reinforcement will be dealt with later.

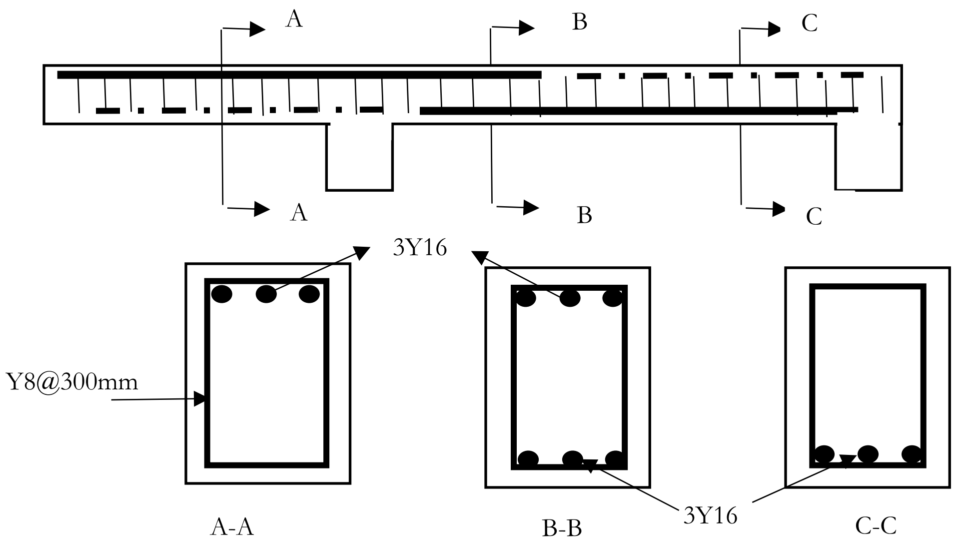


Figure 6.22: The complete sketch of the Beam

Example 6. 4
A precast slab is supported by 3 nos of 230 mm wide in-situ rectangular beams and spans in the direction shown. Design the interior beam as singly reinforced beam. Use the following data. Dead load on the slab = 1KN/m^2, Live load on the slab = 1KN/m^2, f_{cu} = 40N/mm^2, f_y = 460N/mm^2, condition of exposure = moderate, fire resistance = 3hrs, the total height of the slab, h = 150mm, maximum size of aggregates = 20mm, density of concrete = 2400kg/m^3.

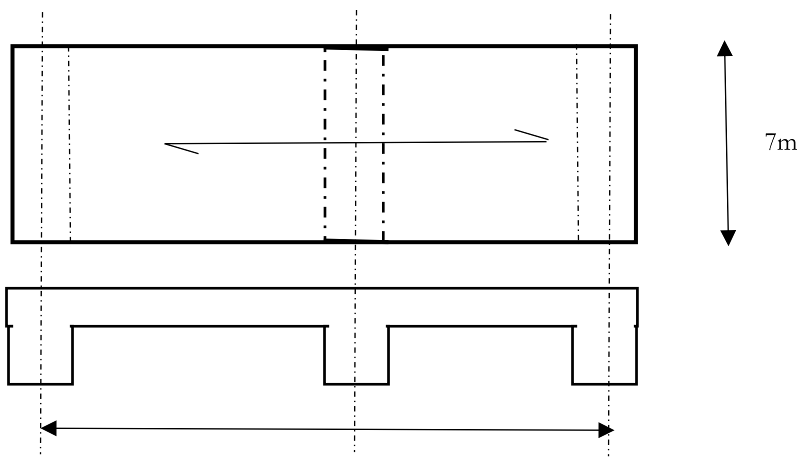


Figure 6.23: Example 6.4 diagram

Solution

Dimension of the Beam

The beam is simply supported, the effective depth can be determined from Table 3.9 (BS 8110), and taking the modification factor to be 1.0 for initial estimate.

$$\frac{l}{d} = 20 \text{ x } 1.0$$

$$= \frac{7000}{20} = 350\text{mm}$$

Take d = 400mm

Assuming 20 mm diameter main bar and 8 mm diameter link

Cover for:

i. Exposure condition = 35 mm (Table 3.3)

   = 35 – 8 = 27mm

ii. Fire resistance = 60mm (Table 3.4)

   = 60 – 8 = 52mm

iii. Concreting condition

   = 20 + 5 = 25mm

Therefore

Cover = 52mm

Thus, the total depth of the beam h

h = d + half x bar diameter + diameter of link + cover

= 400 + 10 + 8 + 52 = 470mm

Take

h = 500mm (for ease of construction). The new d = 430mm

The section is shown in figure 6.24.

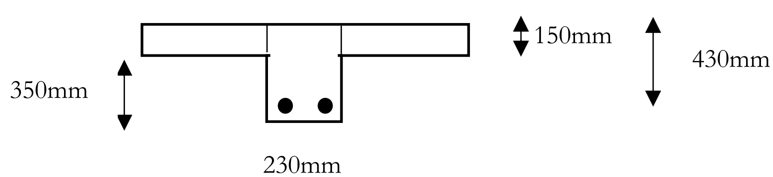


Figure 6.24: The cross section of the beam

The loadings on the Beam

The loadings on the Beam consists of three contributions

i. the loadings on the slab (gk = 1KN/m² and qk = 1KN/m²)

   gk = 1 x 3.5 = 3.5KN/m (half span from either side of the beam → 2 + 1.5 = 3.5m)

   qk = 1 x 3.5 = 3.5KN/m (half span from either side of the beam → 2 + 1.5 = 3.5m)

ii. the self-weight of the slab (half span from either side of the beam → 2 + 1.5 = 3.5m)

= 2400 x 0.15 x 3. 5 = 12.6KN/m

iii. self- weight of the beam = 2400 x 0.35 x 0.230 = 1.932KN/m

The design Load for the Beam = 1.4gk + 1.6qk

= 1.4 (3.5 + 12.6 + 1.932) + 1.6 (3.5) = 25.25 + 5.6 = 30.85KN/m

The beam is simply supported, the maximum bending moment occurs at the midspan

$M_{max} = \frac{wl^2}{8} = \frac{30.85 \ x \ 7^2}{8} = 188.96KN.m$

$Mu = 0.156f_{cu}bd^2 = 0.156 \ x \ 40 \ x \ 230 \ x \ (430)^2 = 265.37KN.m$

Since Mu > M compression reinforcement is not needed, so design the beam as singly reinforced.

$K = \frac{M}{f_{cu}bd^2} = \frac{188.96 \ x \ 1000000}{40 \ x \ 230 \ x \ 430 \ x \ 430} = 0.11$

$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$

$= d[0.5 + \sqrt{(0.25 - 0.12)}] = d[0.5 + 0.36] = 0.86d < 0.95d$ O.K

= 0.86 x 430 = 369.80 mm

Thus

As = $\frac{M}{0.87f_yz}$ **=** $\frac{188.96 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 369.80}$

= 1276. 81 mm^2

Use 5Y20 (1570mm²)

Checks

i. Spacing/arrangement of bars

The minimum spacing between vertical pairs bars = h_{agg} + 5mm = 25 mm

The width (b) required for the reinforcement arrangement

b = 2 x cover + 2 x link diameter + 3 x bar diameter + 2 x spacing

Thus,

b = 2 x cover + 2 x link diameter + 3 x bar diameter + 2 x spacing

= 2 x 52 + 2 x 8 + 3 x 20 + 2 x 25 = 230 mm = 230 mm hence OK.

ii. Deflection

From the previous example,

M = 188. 96KN.m

b = 230mm

d = 430mm

As (required) = 1276.81mm^2

As (provided = 1570.00 mm^2

Therefore

$$\frac{M}{bd^2} = \frac{188.96\ x\ 1000000}{230\ x\ 430\ x\ 430} = 4.44$$

And equation 8 of Table 3. 16 of BS 8110, the service stress f_s

$$f_s = \frac{2fyA_{s\,reuired}}{3_{As\,provided}}\ \text{x}\ \frac{1}{\beta}\ (\beta = 1\ for \text{ simply supported beam/slab}$$

$$= \frac{2\ x\ 460\ x\ \ 1276.81}{3\ x\ 1570}\ \text{x}\ \frac{1}{1} = 249.40\ \text{N/mm}^2.$$

From Table 3.10, the modification factor is about 0.90

This beam simply supported. Thus, the span/effective depth ratio is 20. Thus, by multiplying this value by the modification factor, the permissible span/depth ratio is obtained. That is:

$$\frac{permissible\ span}{depth} = 20 \text{ x } 0.90 = 18.0$$

But,

$$\frac{actual\ span}{depth} \text{ ratio} = \frac{7000}{430} = 16.28$$

Since the $\frac{permissible\ span}{depth}$ ratio (18.0) is higher than the $\frac{actual\ span}{depth}$ ratio (16.28).

The deflection is OK.

Shear Design

The beam is simply supported, the ultimate Shear load at support A and B

Shear Force at Support A = Shear Force at Support B = $\frac{wl}{2} = \frac{30.85\ x\ 7}{2} = 107.98$KN

The design shear stress

$$v = \frac{V}{bd} = \frac{107.98\ x\ 1000}{230\ x\ 430} = 1.09\ \text{N/mm}^2$$

The limiting value is 5N/mm² or $0.8\sqrt{f_{cu}}$ = 5 or $0.8\sqrt{40}$ = 5 or 5.06 N/mm²

Since the value of v (1.09N/mm²) is less than the limiting value (5.06N/mm²). the dimensions are adequate for shear.

The area of longitudinal reinforcement provided is 1570.00 mm²

Now,

$$\frac{100\ x\ As}{bd} = \frac{100\ x\ 1570}{230\ x\ 430} = 1.59$$

From Table 8 (BS 8110), the design concrete shear stress

$v_c = 0.72$ N/mm²

Because, the characteristic strength of concrete fc = 40N/mm², this value will have to be multiplied by $(\frac{fcu}{25})^{0.333}$. That is:

$$v_c = 0.72 \text{ x } (\frac{40}{25})^{0.333} = \text{ N/mm}^2.$$

$$= 0.84 \text{ N/mm}^2$$

And

$$\frac{v_c}{2} = 0.42 \text{ N/mm}^2$$

Since v > $\frac{v_c}{2}$, then shear reinforcement is needed.

But v < v_c + 0.4 (that is, 0.84 + 0.4 = 1.24). The shear force V = 1.24 x 230 x 430 = 122.64 KN.

Since the maximum ultimate shear force is less, so provide nominal link throughout the whole length.

From equation 6.20

$$A_{sv} \geq \frac{0.4bS_v}{0.87f_{yv}}$$

So that:

$$S_v = \frac{0.87A_{sv}f_{yv}}{0.4b}$$

Diameter of the link = 8mm. The cross-sectional area = 50.29 mm²

Assuming a 2-legged 8 mm diameter link (50.29 x 2 legs = 100.58 mm²)

Therefore

$$S_v = \frac{0.87 \ x \ 100.58 \ x \ 460}{0.4 \ x \ 230} = 437.\ 52 \text{ (spacing of 8 mm diameter links)}$$

For singly reinforced beam, the maximum link spacing is 0.75d or 300mm, whichever is less. That is:

i. 0.75 x 430 = 322.50 mm, OR
ii. 300 mm

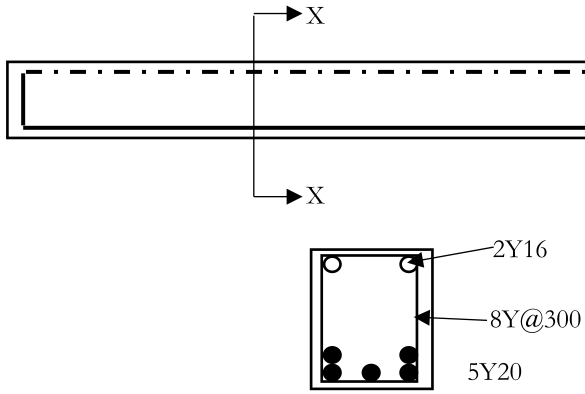
Use

S_v = 300 mm (Y8@300mm)

Figure 6.25 is the sketch of reinforcement arrangement.

Observations

1. This beam was designed as rectangular singly reinforced concrete because the beam and the slab were not cast monolithically together. The slab was a precast concrete, while the beam was cast in-situ.
2. The bars represented by dash lines are for ease of placing the links so as to form a close loop. The rules governing detailing will be discussed later



Section X-X

Figure 6.25: The sketch of reinforcement arrangement

Design Graphs for Singly Reinforced Beam

Another way of determining the area of steel required in singly reinforced rectangular beams is by using the design charts given in Part 3 of BS 8110 (Figure 6.26)

The use of the charts, based on the rectangular–parabolic stress distribution for concrete rather than the simplified rectangular distribution, results in a more economical estimate of the required area of steel reinforcement. It was issue for grade of reinforcement of 460N/mm^2.

The design procedure, using the design graphs, involves the following steps:

1. Check M ≤ Mu
2. Select appropriate chart from Part 3 of BS 8110 based on the grade of tensile reinforcement.
3. Calculate $\frac{M}{bd^2}$
4. Plot $\frac{M}{bd^2}$ ratio on chart and read off corresponding $\frac{100\,As}{bd}$ value using curve appropriate to grade of concrete selected for design.
5. Then calculate As

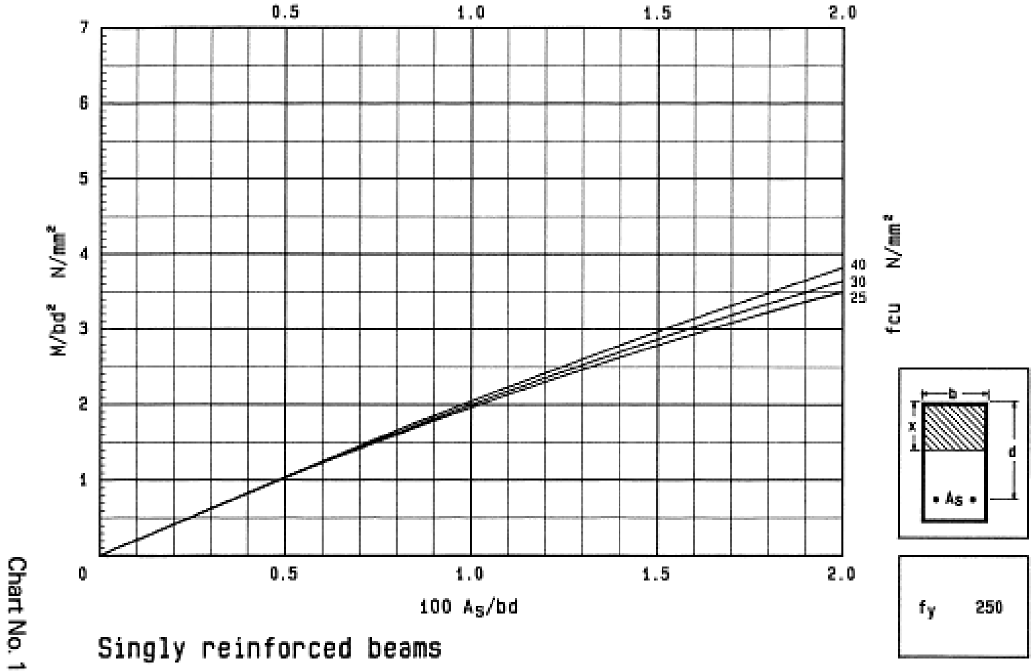


Figure 6.26: A typical design graph for singly reinforced Beam section.

6.4 Doubly Reinforced Beams

When the design Bending Moment M is greater than the ultimate moment of the section Mu, or K > K', where

$$K = \frac{M}{f_{cu}bd^2}$$

$$K' = \frac{M_u}{f_{cu}bd^2}$$

then the concrete will have insufficient strength in compression to generate this moment and maintain an under-reinforced mode of failure. The required compressive strength can be achieved by:

i. increasing the proportions of the beam, particularly its overall depth, if there is no restriction on depth.

ii. Provision of reinforcement in the compression face. The compression reinforcement will be designed to resist the moment in excess of M_u. This will ensure that the compressive stress in the concrete does not exceed the permissible value and ensure an under-reinforced failure mode.

Beams which contain tension and compression reinforcement are termed **doubly reinforced**. They are generally designed in the same way as singly reinforced beams except in respect of the calculations needed to determine the areas of tension and compression reinforcement.

The area of the compression steel is calculated in (BS 8110, Cl. 3.4.4.4) as:

$$A'_{sc} = \frac{M - M_u}{0.87 f_y (d - d'')} \quad \text{(3.18, BS 8110)} \qquad \textbf{6.23}$$

Where d" = depth of the compression steel from the compression face.

If compression steel is added to a singly reinforced concrete beam, the resulting compressive force generated produces an equivalent tensile force on the tensile face, and thus requiring additional tensile steel to be added to balance it (Figure 6.27).

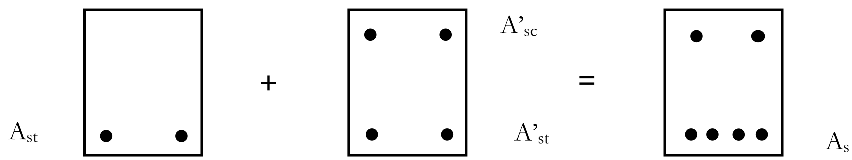


Figure 6.27: Reinforcement in doubly-reinforced beam

That is, total area of reinforcement in the tensile face is:

$$A_s = \frac{M_u}{0.87 f_y z} + A'_{sc} \quad \text{(3.19 BS 8110)} \qquad \textbf{6.24}$$
$$= A'_{st} + A'_{sc}$$

Where $z = d[0.5 + \sqrt{(0.25 - \frac{K''}{0.9})}]$, and K" = 0.156

Equations 3.18 and 3.19 were derived from the stress block used in the analysis of singly reinforced beam but for the inclusion of the compression steel. The derivations were based on the assumption that the compression steel has yielded (design stress = $0.87f_y$). And for this condition to be met:

i. $\frac{d''}{x} \leq 0.37$ or

ii. $\frac{d''}{d} \leq 0.19$

Where

$$x = \frac{d - z}{0.45} \qquad \textbf{6.25}$$

But if

$$\frac{d''}{x} > 0.37 \qquad \textbf{6.26}$$

The compression steel will not have yielded, and the compressive stress will be less than $0.87f_y$. in such situation, design stress can be obtained from Figure 2. 2 (BS 8110).

Example 6.5

A 10m simply supported reinforced concrete beam carries uniformly distributed dead (including self-weight of beam) and imposed loads of 6 and 7 kN/m respectively. The beam is rectangular in cross-section. Design the reinforcement for the beam using the following data:

b = 225mm, d = 400mm, nominal cover = 30 mm, f_{cu} = 30N/mm², f_y = 460 N/mm², bar diameter = 20mm, link diameter = 8 mm

Solution

The sketch of the beam is as shown

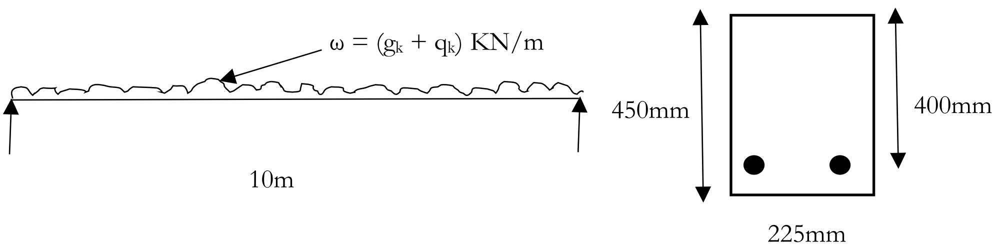


Figure 6.28: Sketch of Example 6.5

Assuming the nominal cover is to the link

h = d + half x bar diameter + link diameter + cover

= 400 + 0.5 x 20 + 8 + 30 = 448 mm

Take = 450 mm

The design load = 1.4gk + 1.6qk

= 1.4 x 6 + 1.6 x 7

= 8.4 + 11.2 = 19.6 KN/m

For a simply supported beam with uniformly distributed load

$$\text{Mmax} = \frac{wl^2}{8} = \frac{19.6 \times 10 \times 10}{8} = 245\text{KN.m}$$

The Ultimate Moment of resistance of the beam section Mu

Mu = 0.156fcubd²

= 0.156 x 30 x 225 x 400 x 400 = 168. 48KN.m

Since M > Mu, compression reinforcement is required

Compression Reinforcement

Assume the diameter of the compression bar to be 20mm, then the effective depth at the compression face d" is:

d" (from the top) = cover + link diameter + half compression bar = 30 + 8 + 10 = 48 mm

Now

$$z = d[0.5 + \sqrt{(0.25 - \frac{K"}{0.9})}]$$, where K" = 0.156

$$= 400[0.5 + \sqrt{(0.25 - \frac{0.156}{0.9})}] = 311 \text{ mm}$$

And

$$x = \frac{d-z}{0.45} = x = \frac{400 - 311}{0.45} = 197.78 \text{ mm}$$

Therefore

$$\frac{d"}{x} = \frac{48}{197.78} = 0.243$$

< 0.37 hence compression steel has yielded

Thus, the compression reinforcement

$$A'_{sc} = \frac{M - M_u}{0.87 f_y (d - d")} = \frac{(245 - 168.48)\ x\ 1000000}{0.87\ x\ 460\ (400 - 48)}$$

$$= 543.19 \text{ mm}^2$$

Provide

2Y20 (628 mm²)

Tension Reinforcement

The tension reinforcement

$$A_s = \frac{M_u}{0.87 f_y z} + A'_{sc} = \frac{168.48\ x\ 1000000}{0.87\ x\ 460\ x\ 311} + 543.19$$

$$= 1353.66 \text{mm}^2$$

Provide 5Y20 (1570 mm²) The sketch is shown in figure 6.29.

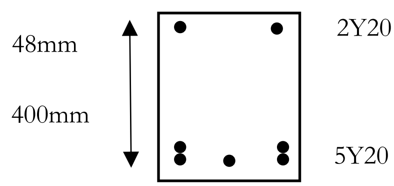


Figure 6.29: Sketch of reinforcement arrangement

Design Graphs

Just as in singly reinforced beam sections, another way of determining the area of steel required in doubly reinforced rectangular beams is by using the design charts given in Part 3 of BS 8110 (Figure 6.30) The use of the charts, based on the rectangular–parabolic stress distribution for concrete rather than the simplified rectangular distribution, results in a more economical estimate of the required area of steel reinforcement. It was issue for grade of reinforcement of 460N/mm².

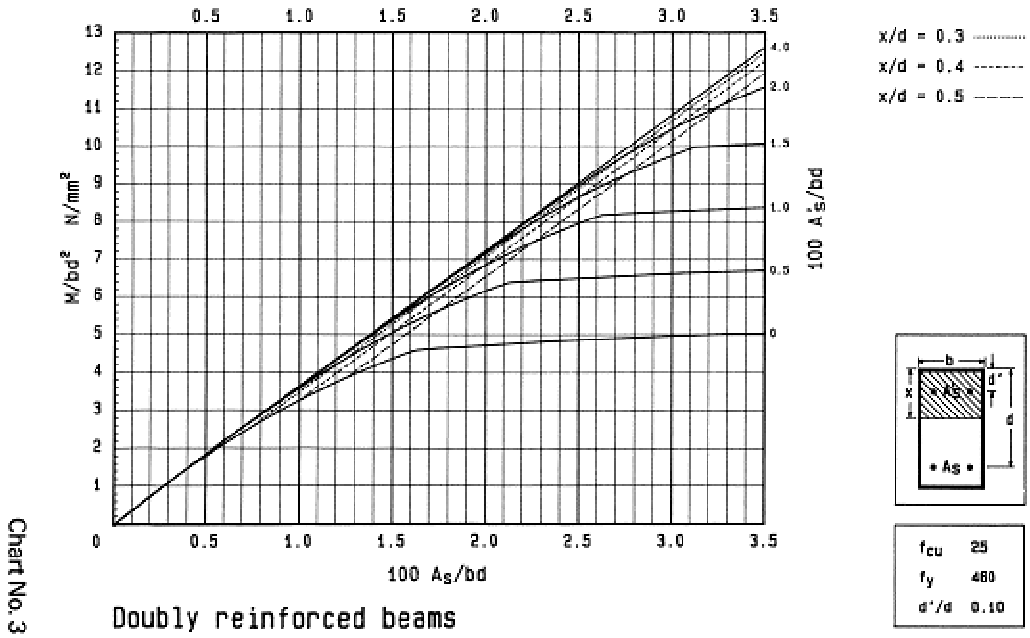


Figure 6.30: A typical design graph for doubly reinforced beam section

The design procedure, using the design graphs, involves the following steps:

1. Check M ≥ Mu
2. Select appropriate chart from Part 3 of BS 8110 based on the grade of tensile reinforcement.
3. Calculate $\frac{M}{bd^2}$
4. Plot $\frac{M}{bd^2}$ ratio on chart and read off corresponding $\frac{100\,As}{bd}$ value using curve appropriate to grade of concrete selected for design.
5. Then calculate As

Shear Reinforcement in doubly reinforced Beams

When considering doubly reinforced beam, only the tension steel should be considered in the calculation of $\frac{100\,x\,As}{b}$.

6.5 Flanged Beam Sections

In most real situations, the beams in buildings are seldom single span but continuous over the supports (Figure 6.31).

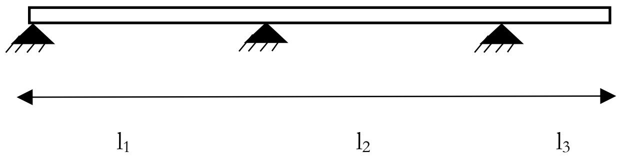


Figure 6.31: A Typical beam continuous over many supports

The design process for such beams is similar to that outlined above for single span beams. However, the main difference arises from the fact that with continuous beams the designer will need to consider the various loading arrangements and combinations in order to determine the design moments and shear forces in the beam (refer to Chapters 3 and 4).

The analysis to calculate the bending moments and shear forces can be carried out by suitable method of analysis, usually moment distribution method or method of sub-frame or computer analysis. Also, the coefficient given in Table 6.4 (Table 3.5 of BS 8110) reproduced below can be used.

Table 6.4: Shear force and Bending moment for continuous beam (Table 3.5, BS 8110)

	Outer Support	Near middle of End Span	At first interior Support	At middle of Interior Spans	Interior Support
Moment	0	0.09Fl	- 0.11Fl	0.07Fl	- 0.08Fl
Shear	0.45F	-	0.6F	-	0.55F

The usage of this table is however subject to three conditions:

i. characteristic imposed load Qk may not exceed characteristic dead load Gk;
ii. loads should be substantially uniformly distributed over three or more spans;
iii. Variations in span length should not exceed 15% of longest.

Once this has been done, the beam can be sized and the area of bending reinforcement calculated as earlier discussed. At the internal supports, the bending moment is reversed and it should be remembered that the tensile reinforcement will occur in the top half of the beam and compression reinforcement in the bottom half of the beam. Generally, beams and slabs are cast monolithically, that is, they are structurally tied. At mid-span, it is more economical in such cases to design the beam as an L or T section by including the adjacent areas of the slab as shown in figure 6.32.

The actual width of slab that acts together with the beam is normally termed **the effective width of the flange (b_{eff}).** According to clause 3.4.1.5 of BS 8110, depending on whether the beam is T or L, **the effective width of the flange** should be taken as the lesser of (a) the actual flange width and (b) the web width + l_m as follows:

For T-Beam

a. Actual flange width

b. $b_w + \frac{1}{5}kl$

For L-Beam

a. Actual flange width

b. $b_w + \frac{1}{10}kl$

where kl = distance between the points of zero moment.

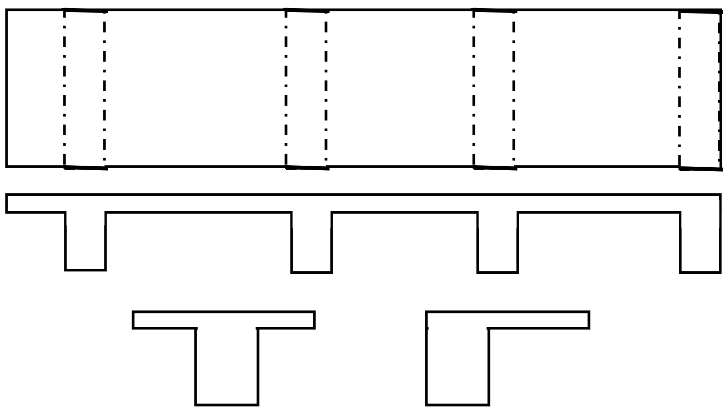


Figure 6.32: Beam-slab systems with T-section and L-section beams

The values vary according to the conditions at the support. For example:

i. simply supported beam → k = 1

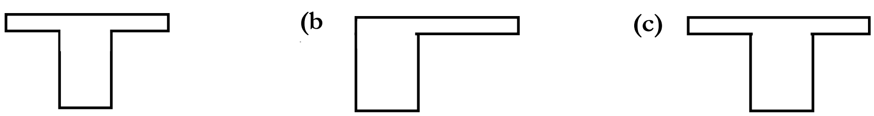
ii. for fixed ends beams → k = 0.7

iii. for propped cantilever → k = 0.85

Assignment

Determine the effective width of the beams shown in Figure 6.33

(a) **(b** **(c)**



250mm

Continuous
Span = 7m

350mm

Continuous
Span = 7m

300mm

Simply supported
Span = 7m

Figure 6.33

Answers: (a) 1230 mm (b) 840 mm (c) 1700 mm

Design of Flanged Beam

The depth of the neutral axis in relation to the depth of flange will influence the design process and must therefore be determined. The depth of the neutral axis, x, can be calculated using equation 6.25 (Section 6.4)

$$x = \frac{d-z}{0.45}$$

In the Moment of Resistance analysis of flanged beams, there are two cases to be considered

Case 1 → $x < h_f$

Where the neutral axis lies within the flange (that is Case 1), which will normally be the case in practice, the beam can be designed as being singly reinforced taking the breadth of the beam, b, equal to the effective flange width.

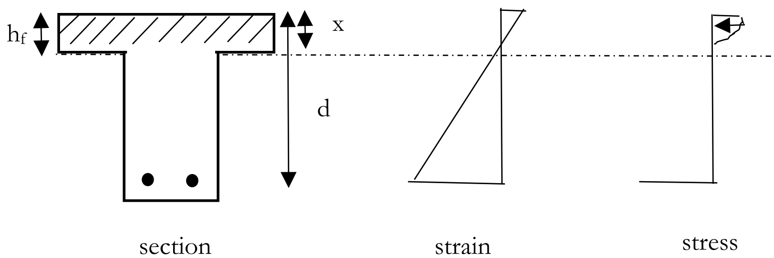


Figure 6.34: Neutral axis lies within the flange

Using the normal equation for the design of rectangular section

$$K = \frac{M}{f_{cu}bd^2}$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

$$x = \frac{d-z}{0.45}$$ → to check that it is actually Case 1

$$A_s = \frac{M}{0.87f_y z}$$

Case 1 → $x > h_f$

This is when the neutral axis lies outside the flange. Not normally occur in practice and will not be discussed here.

Example 6.6

Determine the reinforcement in Beam B in a beam-slab system shown in Figure 6.35. The slab is 200 mm and the beam are supported by 300 x 300 mm column. The design moment M = 200KN.m, f_{cu} = 30 N/mm², f_y = 460 N/mm², minimum cover to reinforcement = 25 mm, the height of the beam h = 550 mm, and bar diameter = 20 mm, and link diameter = 8 mm.

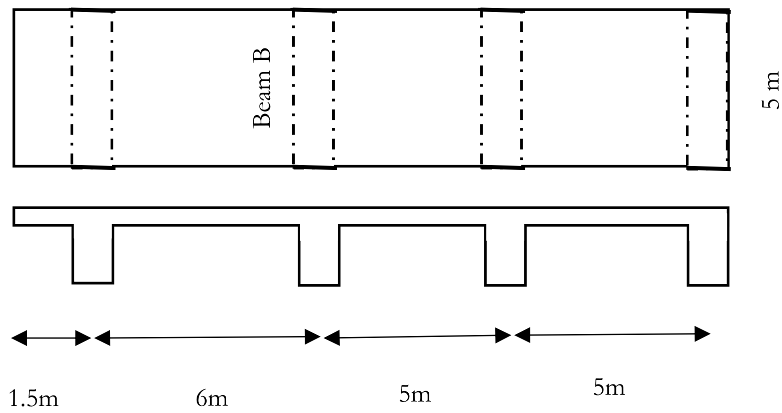


Figure 6.35

Solution.

The cover = 25 mm

The effective depth d = h – cover – link – $\frac{1}{2}$ x bar diameter

$= 550 - 25 - 8 - \frac{20}{2} = 507$ mm.

The effective width for the beam B (T-beam and simply supported on 300 x 300 mm column). Therefore, the width of the beam web (b_w) = 300 mm.

The effective width of the flange b_{ff} is:

i. actual flange width

ii. $b_w + \frac{1}{5}kl$

$b_{eff} = 3000 + 2500 = 5500$ mm **OR**

$= 300 + \frac{1}{5} 1 \, x \, 5000 = 1300$ mm

$b_{eff} = 1300$ mm.

Figure 6.36 shows the dimension of the cross section of the beam

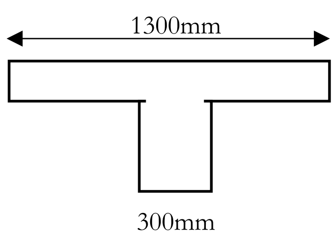


Figure 6.36: The cross-section of the beam

Now

$Mu = 0.156 f_{cu} b d^2$

$= 0.156 \times 30 \times 1300 \times 507 \times 507 = 1583.89$KN.m

Since Mu > M (200 KN.m) → design as singly reinforced beam

$$K = \frac{M}{f_{cu}bd^2} = \frac{200\ x\ 1000000}{30\ x\ 1300\ x\ 507\ x\ 507} = 0.02$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})] = d\ [0.5 + \sqrt{(0.25 - \frac{0.02}{0.9}})] = 0.98d\ \text{(Use z =0.95d)}$$

Thus,

$$z = 0.95\ x\ 507 = 481.65\ mm$$

Now

$$x = \frac{d-z}{0.45} = \frac{507 - 481.65}{0.45} = 56.33\ mm$$

This value is less than the flange depth, which is 200 mm (Case 1). Therefore,

$$A_s = \frac{M}{0.87f_yz} = \frac{200\ x\ 1000000}{0.87\ x\ 460\ x\ 481.65} = 1037.58\ mm^2$$

Provide 4Y20 (1260 mm²). The sketch of the reinforcement is shown in figure 6.37

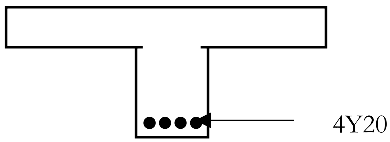


Figure 6.37: Sketch of the reinforcement arrangement

Example 6. 7

A slab is supported by 3 nos of 230 mm beams (A, B, C) and spans in the direction shown. Design the interior beam as singly reinforced beam. Use the following data. Dead load on the slab = 5.0KN/m², Live load on the slab = 4.0KN/m², f_{cu} = 25N/mm², f_y = 460N/mm², condition of exposure = moderate, fire resistance = 1.5hrs, the total height of the slab, h = 150mm, maximum size of aggregates = 20mm, density of concrete = 2400kg/m³.

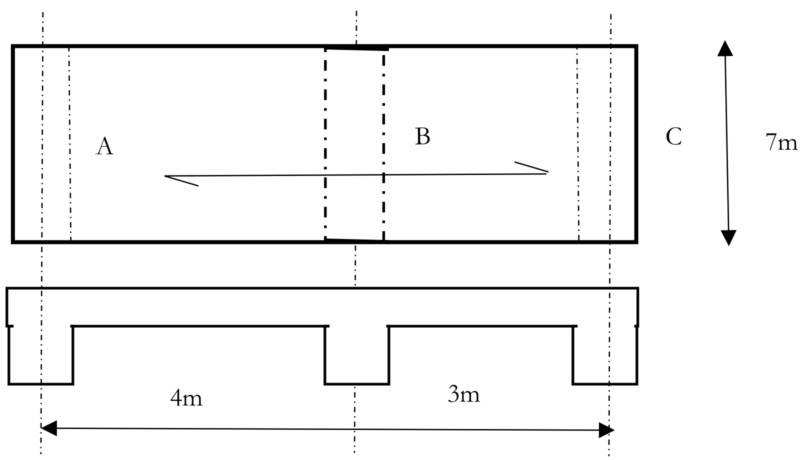


Figure 6. 38: Example 6.7

Solution

Dimension of the Beam

Assuming 20 mm diameter main bar and 8 mm diameter link

Cover for:

i. Exposure condition = 35 mm (Table 3.3)

= 35 – 8 = 27mm

ii. Fire resistance = 20mm (Table 3.4)

= 20 – 8 = 12mm

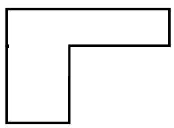
iii. Concreting condition =

= 20 + 5 = 25mm

Therefore

Cover = 27mm

The effective width of the beam (L-Beam) and simply-supported



Simply supported
Span = 7m

230mm

Figure 6.39: Dimensional and support details for the beam

b_{eff} = actual width (2000mm) or

$=b_w + \frac{1}{10}kl = 230 + \frac{7000}{10} = 930$ mm

Thus,

b_{eff} = 930mm

Now $\frac{bw}{b} = \frac{230}{930} = 0.25$

From Table 3.9 (BS 8110), the effective depth taking the modification factor to be 1.0 for initial estimate.

$\frac{l}{d} = 16 \text{ x } 1.00$

$= \frac{7000}{16} = 437.50$mm

Take d = 450mm

Thus, the total depth of the beam h

h = d + half x bar diameter + diameter of link + cover

= 437.50 + 10 + 8 + 27 = 482.50mm

Take h = 500mm and (the revised d = 455mm)

Take

h = 500mm (for ease of construction).

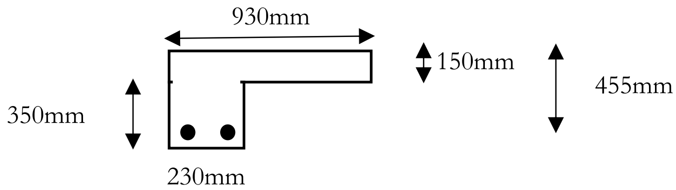


Figure 6.40: The dimensions of the cross section of the beams

The loadings on the Beam

The loadings on the Beam consists of three contributions

i. the loadings on the slab (gk = 5KN/m² and qk = 4KN/m²)

gk = 5 x 2 = 10.0KN/m (half into the span of the slab → = 2m)

qk = 4 x 2 = 8.0KN/m (half into the span of the slab → = 2m)

ii. the self-weight of the slab (half into the span of the slab → 2m)

= 2400 x 0.15 x 2.0 = 7.20KN/m

iii. self- weight of the beam = density x b x h = 2400 x 0.23 x 0.35 = 1.932KN/m

The design Load for the Beam = 1.4gk + 1.6qk

= 1.4 (10.0 + 7.20 + 1.932) + 1.6 (8.0) = 26.79 + 12.80 = 39.59KN/m

The beam is simply supported, the maximum bending moment occurs at the midspan

$M_{max} = \frac{wl^2}{8} = \frac{39.59 \ x \ 7^2}{8}$ = 242. 49KN.m

Mu = 0.156fcubd² = 0.156 x 25 x 930 x (350)² = 444. 31KN.m

Since Mu > M compression reinforcement is not needed, so design the beam as singly reinforced.

$K = \frac{M}{f_{cu}bd^2} = \frac{242.49 \ x \ 1000000}{25 \ x \ 930 \ x \ 455 \ x \ 455}$ = 0.051

$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$

= d[0.5 + √(0.25 – 0.10)] = d[0.5 + 0.44] = 0.94d < 0.95d O.K

= 0.94 x 455 = 427.27 mm

Thus

As = $\frac{M}{0.87f_yz}$ **=** $\frac{242.49 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 427.27}$

= 1418.13 mm²

Use 4Y25 (1964mm²)

Checks

i. Spacing/arrangement of bars

Bar arrangement of 3Y25 should have been more ideal, but it a quick check showed that it will not satisfy the spacing arrangement exceeding the 230mm. So, we will use two (2) vertical pairs.

The minimum spacing between vertical pairs bars = h_{agg} + 5mm = 25 mm

The width (b) requited for the reinforcement arrangement

b = 2 x cover + 2 x link diameter + 2 x bar diameter + 1 x spacing

Thus,

b = 2 x cover + 2 x link diameter + 2 x bar diameter + 1 x spacing

= 2 x 27 + 2 x 8 + 2 x (25) + 25 = 145 mm < 230 mm, hence OK.

ii. Deflection

From the previous example,

M = 242. 49KN.m

b = 930mm

d = 455mm

As (required) = 1418.13mm^2

As (provided = 1964.00 mm^2

Therefore

$$\frac{M}{bd^2} = \frac{242.49 \, x \, 1000000}{930 \, x \, 455 \, x \, 455} = 1.26$$

And equation 8 of Table 3. 16 of BS 8110, the service stress f_s

$$f_s = \frac{2fyA_{s\,reuired}}{3_{As\,provided}} \text{ x } \frac{1}{\beta} \quad (\beta = 1\, for \text{ simply supported beam/slab}$$

$$= \frac{2 \, x \, 460 \, x \, 1418.13}{3 \, x \, 1964} \text{ x } \frac{1}{1} = 221.43 \text{ N/mm}^2.$$

From Table 3.10, the modification factor is about 1.55

The beam is simply supported. Thus, the span/effective depth ratio is 20. Therefore, by multiplying this value by the modification factor, the permissible span/depth ratio is obtained. That is:

$$\frac{permissible\ span}{depth} = 20 \text{ x } 1.55 = 31.0$$

But,

$$\frac{actual\ span}{depth} \text{ ratio} = \frac{7000}{350} = 20.0$$

Since the $\frac{permissible\ span}{depth}$ ratio (31.0) is higher than the $\frac{actual\ span}{depth}$ ratio (20.0).

The deflection is OK.

Shear Design

First check the adequacy of the dimensions for shear as in Example 6.3 and 6.4

The area of longitudinal reinforcement provided is 1964.00 mm^2

Now,

$$\frac{100\ x\ As}{bd} = \frac{100\ x\ 1964}{930\ x\ 400} = 0.53$$

From Table 3.8 (BS 8110), the design concrete shear stress

v_c = 0.50 N/mm^2

Because, the characteristic strength of concrete fc = 25N/mm^2, this value need not be adjusted by multiplied by $(\frac{f_{cu}}{25})^{0.333}$. Thus,

$$\frac{v_c}{2} = 0.25 \text{ N/mm}^2$$

The beam is simply supported, the ultimate Shear load at support A and B

Shear Force at Support A = Shear Force at Support B = $\frac{wl}{2} = \frac{39.59\ x\ 7}{2} = 138.57$KN

The design shear stress

$$v = \frac{V}{bd} = \frac{138.57\ x\ 1000}{930\ x\ 455} = 0.33 \text{ N/mm}^2$$

Since v > $\frac{v_c}{2}$, then shear reinforcement is needed.

But v < v_c + 0.4 (that is, 1.17 + 0.4 = 1.57), so provide nominal link. From equation 20

$$A_{sv} \geq \frac{0.4bS_v}{0.87f_{yv}}$$

So that:

$$S_v = \frac{0.87A_{sv}f_{yv}}{0.4b}$$

Diameter of the link = 8mm. The cross-sectional area = 50.29 mm^2

Assuming a 2-legged 8 mm diameter link (50.29 x 2 legs = 100.58 mm^2)

Therefore

$S_v = \frac{0.87\ x\ 100.58\ x\ 460}{0.4\ x\ 250}$ = 402. 52 (spacing of 8 mm diameter links)

For singly reinforced beam, the maximum link spacing is 0.75d or 300mm, whichever is less. That is:

i. 0.75 x 400 = 300 mm, OR
ii. 300 mm

Use

S_v = 300 mm (Y8@300mm)

A sketch of the reinforcement arrangement is shown in Figure 6.41.

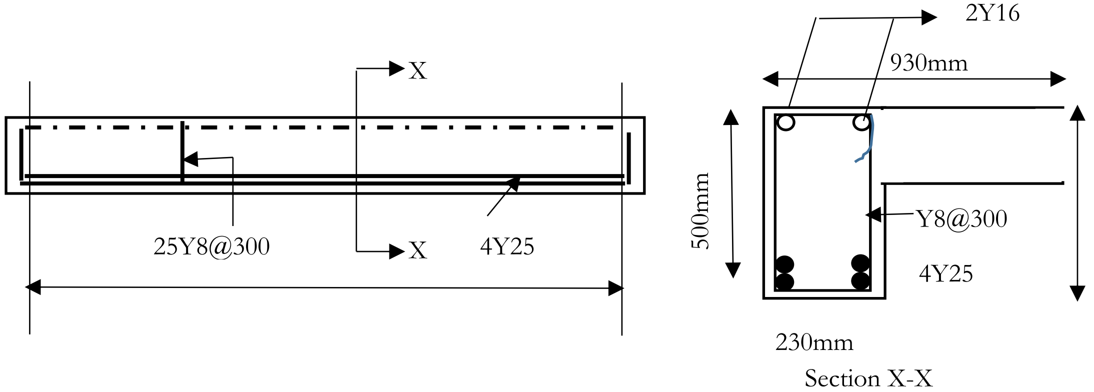


Figure 6.41: Sketch of reinforcement arrangement

Continuous Beam

In practical situations, beams in buildings are not usually single span but continuous over two or many supports. The design process for such beams is similar to that described above for single span beams. For the continuous beam however, the various loading arrangement will have to be considered in order to determine the design maximum bending moment and shear force.

Usually, analysis to calculate the bending moments and shear are carried out by using moment distribution method or, provided the conditions in clause 3.4.3 of BS 8110 are satisfied, by using the coefficients given in *Table 3.5* of BS 8110.

Once the moments and shear forces are determined, the beam can be sized and the area of bending reinforcement calculated as discussed in the proceeding section for beams. At the internal supports, the bending moment is reversed, therefore the tensile reinforcement will occur in the top half of the beam and compression reinforcement in the bottom half of the beam.

Thus, at the supports, the beam should be designed as a rectangular section, because hogging moments cause tension in slabs.

Since beams and slabs are cast monolithically in practice, that is, they are structurally tied, it is more economical in such cases to design the beam as an *L* or *T* section for internal beams, by including the adjacent areas of the slab. The actual width of slab that acts together with the beam is normally termed the effective flange, and this should be determined in accordance to clause 3.4.1.5 of BS 8110.

Example 6.8

Figure 6.42 is a part of frame. Design beam BB using the following data. Dead load on the slab = 5.0KN/m^2, Live load on the slab = 4.0KN/m^2, f_{cu} = 40N/mm^2, f_y = 460N/mm^2, condition of exposure = moderate, fire resistance = 1.5hrs, the total height of the slab, h = 150mm, maximum size of aggregates = 20mm, density of concrete = 2400kg/m^3. The beams are supported on 225 x 225mm block wall. Use bar 20mm diameter bar and 8mm diameter for link.

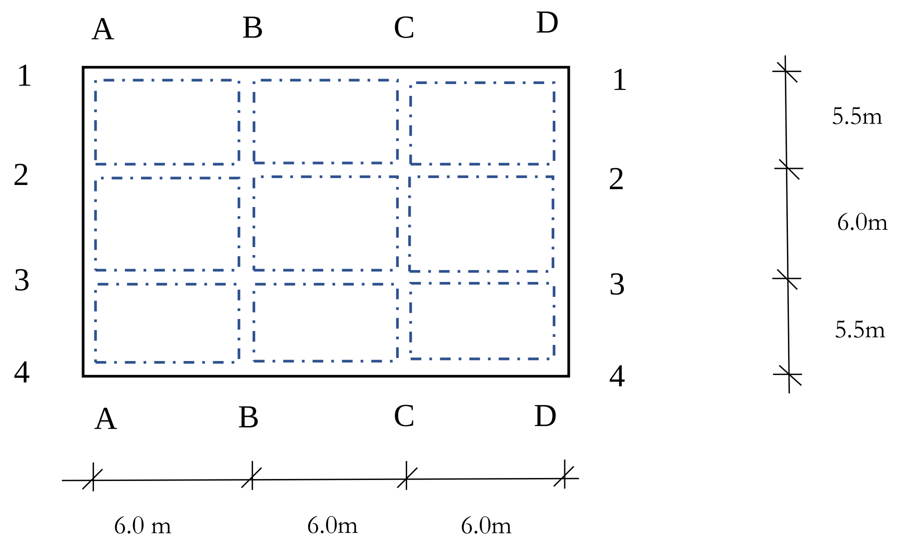


Figure 6.42: Example 6.8

Solution

Dimension of the Beam

Assuming 20 mm diameter main bar and 8 mm diameter link

Cover for:

i. Exposure condition = 35 mm (Table 3.3)

= 35 – 8 = 27mm

ii. Fire resistance = 20mm (Table 3.4)

= 20 – 8 = 12mm

iii. Concreting condition =

= 20 + 5 = 25mm

Therefore

Cover = 27mm

The effective width of the beam (L-Beam) and continuous

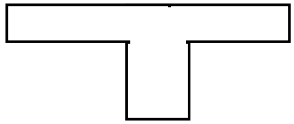


Figure 6.43: Cross section of the beam

b_{eff} = actual width (6000mm) or

$=b_w + \frac{1}{5}kl = 230 + \frac{0.7\ x\ 6000}{5}$ = 1065mm (using the longer span)

Thus,

b_{eff} = 1065mm

Now $\frac{bw}{b} = \frac{225}{1065} = 0.211$

From Table 3.9 (BS 8110), the effective depth taking the modification factor to be 1.0 for initial estimate.

$\frac{l}{d}$ = 20.8 x 1.00

$= \frac{6000}{20.8}$ = 288.46mm

Take d = 300mm

Thus, the total depth of the beam h

h = d + half x bar diameter + diameter of link + cover

= 300 + 10 + 8 + 27 = 345mm

For practical construction consideration, this height is too small. The value is then increase to 450mm.

Take h = 450mm and (the revised d = 405mm)

Loadings on the Beam

The loadings on the Beam consists of three contributions

i. the loadings on the slab (gk = 5KN/m^2 and qk = 4KN/m^2)

   gk = 5 x 6 = 30.0KN/m (half into either side of the span of the slab → = 2m)

   qk = 4 x 6 = 24.0KN/m (half into either side of the span of the slab → = 2m)

ii. the self-weight of the slab (half into either side of the span of the slab → 2m)

   = 2400 x 0.15 x 2.0 = 7.20KN/m

iii. self- weight of the beam = density x b x h = 2400 x 0.225 x 0.3 = 1.62KN/m

The design Load for the Beam = 1.4gk + 1.6qk

= 1.4 (30.0 + 7.20 + 1.62) + 1.6 (24.0) = 54.35 + 38.40 = 92.75KN/m

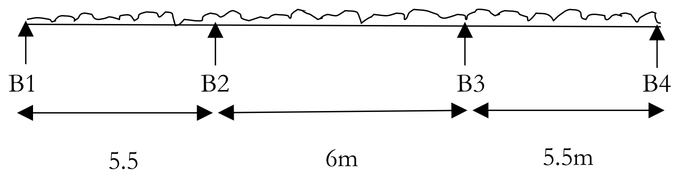


Figure 6.44: Beam BB

The beam is continuous with the following additional properties

i. The dead load gk (54.35KN/m) is more than the live load qk (38.40KN/m)
ii. The loads are uniformly distributed
iii. The variations in the span length is about 8% of the longest, and thus, does not exceed 15%.

In order to reduce the labor involved in analysis to obtain the envelopes for the bending moments and shear forces, Table 3.5 (BS 8110) can be used to determine the design ultimate bending moment and shear forces as follows.

Bending Moments

i. Supports

Outer Support Moment M_{B1} = M_{B4} = 0KN.m

Interior Support Moment M_{B2} = M_{B3} = - 0.11 FL = -0.11 x 92.75 x 6 x 6 = 367. 29KN.m

ii. Spans

Near Middle of End Spans = M_{B1-B2} = M_{B3-B3} = 0.09FL = 0.09 x 92.75 x 6 x 6 = 292. 17KN.m

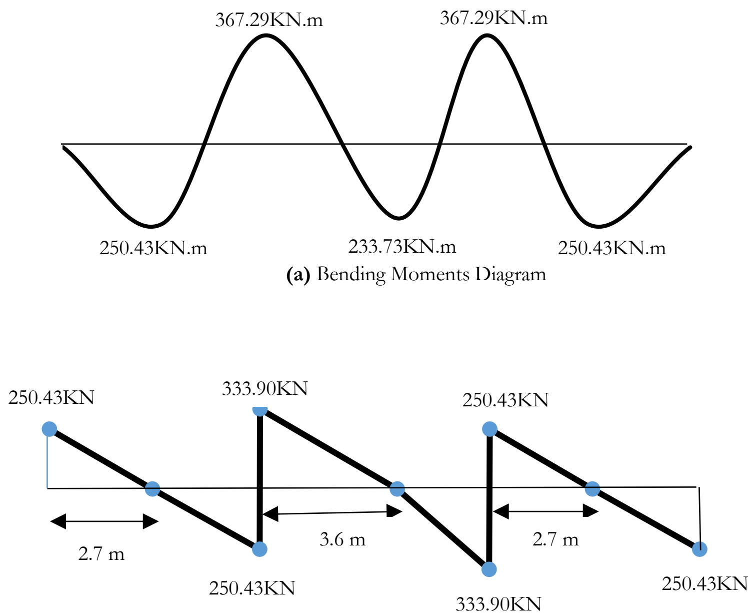
Middle Interior Span = M_{B2-B3} = 0.07FL = 0.07 x 92.75 x 6 x 6 = 233. 73KN.m

Shear Forces

i. Supports

Outer supports SF_{B1} = SF_{B4} = 0.45F = 0.45 x 92.75 x 6 = 250.43KN

First interior supports SF_{B2} = SF_{B3} = 0.6F = 0.6 x 92.75 x 6 = 333.90KN.



(b) Shear Force Diagram

Figure 6.45: The Bending Moment and Shear Force diagrams of the Beam

Calculations of areas of Reinforcement

Supports

The support will be designed as rectangular section

i. Interior Support Moments $M_{B2} = M_{B3}$

   M = 367. 29KN.m

Mu = 0.156fcubd² = 0.156 x 40 x 225 x (405)² = 147.63KN.m

Since M > Mu compression reinforcement is needed, so design the beam section as doubly reinforced.

Compression reinforcement

Assume the diameter of the compression bar to be 20mm, then the effective depth at the compression face d" is:

d" (from the top) = cover + link diameter + half compression bar = 27 + 8 + 10 = 45 mm

Now

$z = d[0.5 + \sqrt{(0.25 - \frac{K"}{0.9}})]$, where K" = 0.156

$= \mathbf{405[0.5 + \sqrt{(0.25 - \frac{0.156}{0.9}}})]$ = 405[0.5 + 0.28] = 315.90mm

And

$$x = \frac{d-z}{0.45} = x = \frac{405 - 3150.90}{0.45} = 198.00 \text{ mm}$$

Therefore

$$\frac{d"}{x} = \frac{45}{198.00} = 0.23$$

< 0.37 hence compression steel has yielded

Thus, the compression reinforcement

$$A'_{sc} = \frac{M - M_u}{0.87 f_y (d - d")} = \frac{(367.29 - 147.63)\ x\ 1000000}{0.87\ x\ 460\ (405 - 45)}$$

$= 1527.65 \text{ mm}^2$

Provide

6Y20 (1885 mm^2)

Tension Reinforcement

The tension reinforcement

$$A_s = \frac{M_u}{0.87 f_y z} + A'_{sc} = \frac{147.63\ x\ 1000000}{0.87\ x\ 460\ x\ 315.90} + 1527.65 = 1167.75 + 1527.65$$

$= 2695.40 \text{mm}^2$

Provide 6Y25 (2945 mm^2)

Span Moments

i. Near Middle of End Spans = $M_{B1\text{-}B2} = M_{B3\text{-}B3}$

M = 292. 17KN.m

It will be designed as T-section beam *(*Figure 6.46)

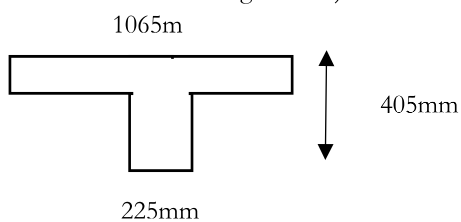


Figure 6.46

Mu = 0.156fcubd² = 0.156 x 40 x 1065 x 405 x 405 =

= 1090. 05KN.m

Since Mu > M, no compression reinforcement is needed. So, design as singly reinforced.

$$K = \frac{M}{f_{cu} b d^2} = \frac{292.17\ x\ 1000000}{40\ x\ 1065\ x\ 405\ x\ 405} = 0.042$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})]$$

= d[0.5 + √(0.25 – 0. 047)] = d[0.5 + 0.45] = 0.95d O.K

= 0.95 x 405 = 384.75 mm

Thus

As = $\frac{M}{0.87 f_y z}$ = $\frac{292.17 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 384.13}$

= 1900.56mm^2

Use 4Y25 (1964mm^2)

ii. Middle Interior Span, $M_{B2\text{-}B3}$

It will be designed as T-section

M = 233. 73KN.m

Mu = 1090. 05KN.m (from Near Middle of End Spans = $M_{B1\text{-}B2}$ = $M_{B3\text{-}B3}$**)**

Since Mu > M, no compression reinforcement is needed. So, design as singly reinforced.

K = $\frac{M}{f_{cu} b d^2}$ = $\frac{233.73 \ x \ 1000000}{40 \ x \ 1065 \ x \ 405 \ x \ 405}$ = 0.033

$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})]$

= d[0.5 + √(0.25 – 0. 037)] = d[0.5 + 0.46] = 0.96d. Use 0.95d

= 0.95 x 405 = 384.75 mm

Thus

As = $\frac{M}{0.87 f_y z}$ = $\frac{233.73 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 384.75}$

= 1517.96mm^2

Use 4Y25 (1964mm^2)

Checks

i. Spacing

Assuming the reinforcement will be arranged as vertical pairs

b = 2 x cover + 2 x link diameter + 3 x bar diameter + 2 spacing < 225 mm

= 2 x 27 + 2 x 8 + 3 x 25 + 2 x 25 = 195 mm < 225 mm

Therefore, the arrangement of bars is OK

ii. Deflection

Using largest Span Bending Moment, $M_{B1\text{-}B2}$ = $M_{B3\text{-}B3}$

M = 292. 17KN.m

b = 1065mm

d = 405mm

As (required) = 1900. 56mm^2

As (provided = 1964.00 mm^2

Therefore

$\frac{M}{bd^2}$ = $\frac{292.17 \ x \ 1000000}{1065 \ x \ 405 \ x \ 405}$ = 1.67

And equation 8 of Table 3. 16 of BS 8110, the service stress f_s

$$f_s = \frac{2fy}{3} \ (for\ continuous\ beam)$$

$$= \frac{2\ x\ 460}{3} \quad = 306.67\ \text{N/mm}^2.$$

From Table 3.10, the modification factor is about 1.13

The beam is continuous. Thus, the span/effective depth ratio is 20.8. Therefore, by multiplying this value by the modification factor, the permissible span/depth ratio is obtained.

That is:

$$\frac{permissible\ span}{depth} = 20.8 \text{ x } 1. = 23.50$$

But,

$$\frac{actual\ span}{depth} \text{ ratio} = \frac{6000}{405} \quad = 14.82$$

Since the $\frac{permissible\ span}{depth}$ ratio (23.50) is higher than the $\frac{actual\ span}{depth}$ ratio (14.82)

The deflection is OK.

The sketch of the reinforcement arrangement (without applying the rules of anchorage and curtailment) is shown in Figure 6.47.

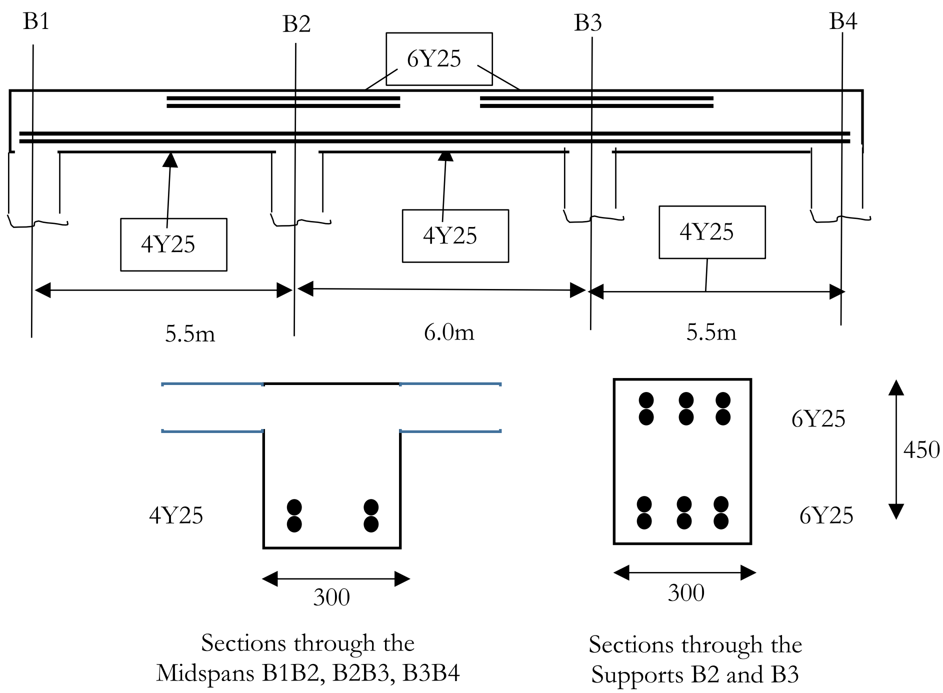


Figure 6.47: Sketch of reinforcement arrangement

Shear Design

The maximum Shear force = 333.90KN (Figure 6.45)

Thus, the design shear stress

$$v = \frac{V}{b_v d} = \frac{333.90 \ x \ 1000}{225 \ x \ 405} = 3.67 \text{ N/mm}^2$$

The limiting value is 5N/mm² or $0.8\sqrt{f_{cu}}$ = 5 or $0.8\sqrt{40}$ = 5 or 5.10N/mm²

Since the value of v (3.67N/mm²) is less than the limiting value (5.10 N/mm²). the dimensions are adequate for shear.

Now,

$$\frac{100 \ x \ As}{bd} = \frac{100 \ x \ 1964}{1065 \ x \ 405} = 0.46$$

From Table 3.8 (BS 8110), the design concrete shear stress

v_c = 0.49 N/mm²

Because, the characteristic strength of concrete fc = 40N/mm², this value will have to be multiplied by $(\frac{fcu}{25})^{0.333}$. That is:

$v_c = 0.72 \text{ x } (\frac{40}{25})^{0.333}$ = N/mm².

= 0. 84 N/mm²

Nominal links are required where the shear force V < (v_c + 0.4) x b x d < 1.24bd

V = (0.84 + 0.4) x 300 x 450 = 167.40 KN

This region is shown in Figure 6.48

The spacing for nominal links in this region is:

$$S_v = \frac{0.87 A_{sv} f_{yv}}{0.4 b}$$

Assume diameter of the link = 8mm. The cross-sectional area = 50.29 mm²

For a 2-legged 8 mm diameter link (50.29 x 2 legs = 100.58 mm²)

Therefore

$$S_v = \frac{0.87 \ x \ 100.58 \ x \ 460}{0.4 \ x \ 300}$$ = 335.43 mm (spacing of 8 mm diameter links)

For singly reinforced beam, the maximum link spacing is 0.75d or 300mm, whichever is less. That is: 0.75 x 400 = 300 mm, OR 300 mm

Provide 8Y@300

At other places (Figure 6.48), shear links are to be provided, with spacing (equation 22)

$$S_v = \frac{0.87 A_{sv} f_{yv}}{b(v - v_c)} = \frac{0.87 \ x \ 100.58 \ x \ 460}{300(2.47 - 1.17)}$$

= 103. 21mm

Provide Y8@100mm

This is shown in Figure 6.48

Shear Design

i. **for Span B2B3 - obtain the links by following the process adopted for Span B1B2.**

ii. **for Span B3B4 – obtain the links by following the process adopted for Span B1B2.**

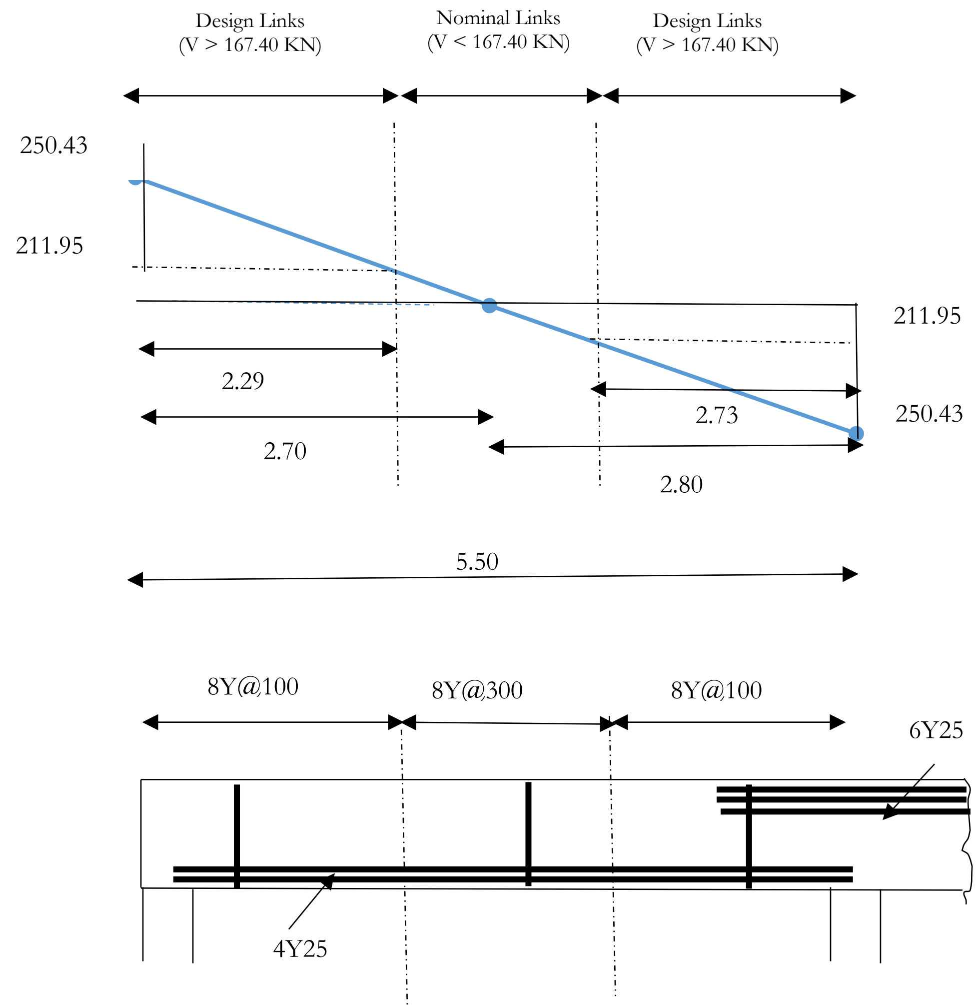


Figure 6.48: Arrangement of Links in Span B1B2

6.6 Detailing and Curtailment of Reinforcement

6.6.1 Introduction

Reinforcement is terminated in concrete due to many reasons which are:

i. to make the member fit together
ii. to make construction easier
iii. to obtain economic design through optimal usage of steel. It is usual to use the maximum moment obtained from structure analysis of the structure, but obviously, moment varies along the length of the structural member. It is thus often necessary to reduce the number of bars at convenient section.
iv. Usually reinforcement is cut into sizes, and some pieces may need to be extended to the required length or lapped with other bar. However, the lap length should be sufficiently long in order that stresses in one bar can be transferred to the other

6.6.2 Bond Stress and Anchorage length of Straight Bars

Bond stress is the shear stress acting parallel to the reinforcement but on the interface between the bar and the concrete. Effective bond exists if the relevant requirements in the code of practice are met. In BS 8110, these requirements are represented in terms of certain amount of stress. Bond is due to the combined effects of adhesion, friction and bearing (for deformed bars). Separation of bars from concrete at the ultimate limit state is prevented by shearing stress (bond stress) between the surface of steel and the surrounding concrete (Figure 6.49)

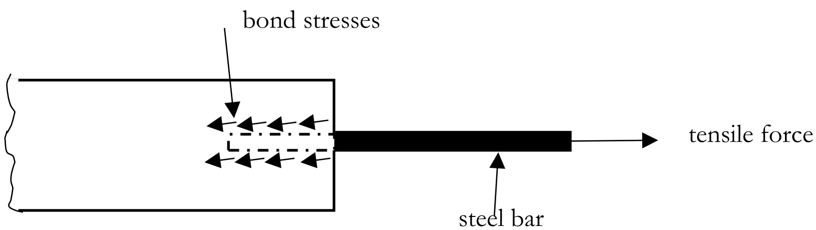


Figure 6.49: Bond stress

The code requires that the checking of the anchorage bond stress f_b, giving as:

$$f_b = \frac{F_s}{\pi \varphi l} \ngtr f_{bu} \qquad \textbf{6.27}$$

where F_s = the bar force, φ = diameter of the steel bar

since the bar force (= tensile force) Fs

$$Fs = \frac{\pi \varphi^2 f_s}{4} \qquad \textbf{6.28}$$

Then

$$f_b = \frac{\pi\varphi^2 f_s}{4\pi\varphi l} = \frac{\pi f_s}{4l} \not> f_{bu} \qquad \textbf{6.29}$$

where f_s = steel stress, fbu is given in the BS 8110 as:

$$f_{bu} = \beta\sqrt{f_{cu}} \qquad \textbf{6.30}$$

where β = bond coefficient given in clause 3.12.8.4

Table 6.5: Bond coefficient (BS 8110, Cl. 3.12.8.4)

Bar type	β	
	Bars in Tension	Bars in Compression
Plain bars	0.28	0.35
Deformed bars	0.50	0.63

Note:

1. For bars in tension in slabs, values of β are as given in Table 6. 5
2. In beams, where minimum links have been provided in accordance with BS 8110 (1997) requirements, the values of β are as given in Table 6.5.
3. In beams where minimum links have not been provided, the values used should be for those listed for plain bars, irrespective of the bars used.
4. The values in Table 6.5 already include a partial safety factor of 1.4.

The anchorage length (l) is obtained from expressions 6.27 – 6.30 as

$$l = \frac{f_s\varphi}{4\beta\sqrt{f_{cu}}} \qquad \textbf{6.31}$$

The ultimate anchorage length (or full anchorage length) is the bar length required to develop the full design strength. By putting $f_u = 0.87f_y$, equation 6.31 becomes:

$$l = \frac{0.87f_y\varphi}{4\beta\sqrt{f_{cu}}} \qquad \textbf{6.32}$$

Typically, for f_{cu} = 40N/mm² and f_y = 460N/mm² (deformed bars), the anchorage bond length is:

i. 32 φ for bars in tension
ii. 26 φ for bars in compression.

Where it is impossible to provide the necessary anchorage length for bars in tension, hooks or bends (Figure 6.50) meeting detailing requirements should be used.

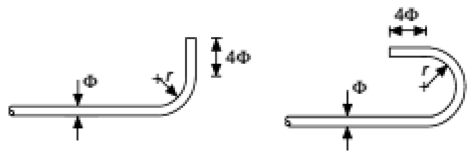


Figure 6.50

For mild steel bars, minimum $r = 2\Phi$. For high yield bars minimum, $r = 3\Phi$ or 4Φ for sizes 25 mm and above.

6.6.3 Curtailment of Bars

Economic design results if reinforcement is cut off at point when it is no longer needed to resist tensile stresses due to bending. This is called the theoretical cut of point. The theoretical curtailment point occurs where the moment of resistance of the continuing bars equals the theoretically applied bending moment. For example, in figure 6.51, assuming that 4 bars are needed to resist the maximum bending moment in the beam.

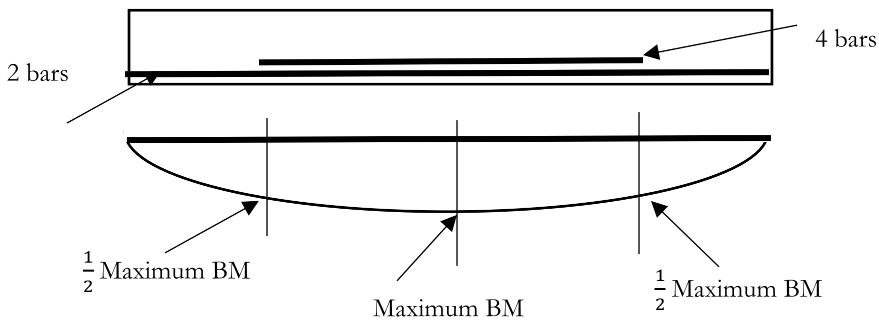


Figure 6.51: Curtailment of bars

Further along the beam when the moment has reduced to half of the maximum. Theoretically speaking, only 2 bars are needed beyond this point, so that 2 bars can be cut off.

Bars must extend beyond the theoretical cut off point for the following reasons

i. To allow for possible inaccuracies in loading and analysis
ii. To allow for inaccuracies in the placement of bars
iii. To offset the possibility of large cracks developing at the curtailment section leading to reduction in the shear and bending strength

Because of these there are Codes recommendation guarding the extension of bars beyond the theoretical cut off point. However, alternative to the recommendation is the simplify rules for reinforcement curtailment.

6.6.4 Detailing and curtailment of Reinforcement in Beams (Cl. 3. 12.10.2)

The codes recommended simplify rules for reinforcement detailing and curtailment for beams provided:

- **a.** The beams are designed for predominantly uniformly distributed loads.
- **b.** In the case of continuous beams, the spans are approximately equal

These rules are stated below.

i. Simply supported beam

At least 50% of the tension reinforcement provided at the mid span should extend to the support and having effective anchorage of 12 times the diameter past the center line of the support. The remaining 50% should extend to within 0.08l of the support (figure 6.52)

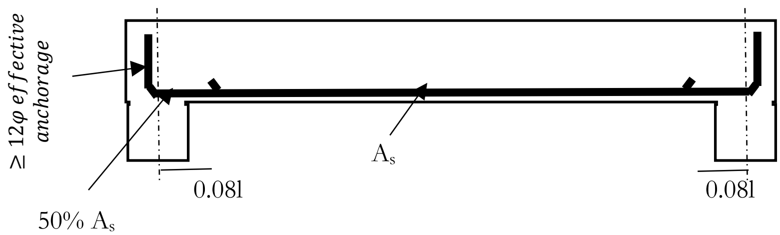


Figure 6.52: Curtailment of bars for simply supported beam

ii. Cantilever beam

At least 50% of the tension reinforcement provided at the support should extend to the end of the cantilever. The remaining 50% should extend to a distance 0.5l or 45 times bar diameter (whichever is greater) from the support (Figure 6.53)

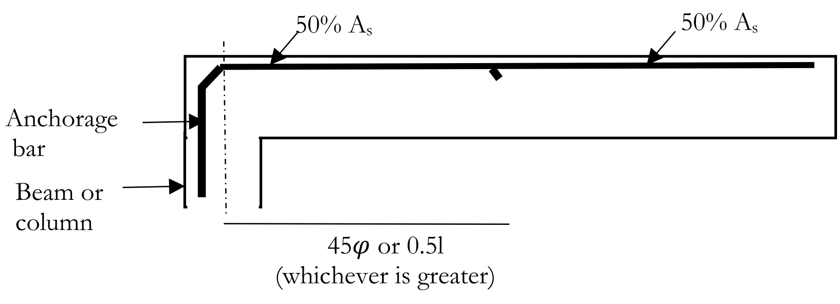


Figure 6.53 Curtailment of bars for cantilever beam

iii. **Continuous beam**

The simplify rules may be used when the **imposed load ≤ the dead load.**

a. At least 20% of the reinforcement in tension over the support should be made effectively continuous through the spans; of the remainder, half should extend to a point not less than 0.25l from the support, and the other half to a point not less than 0.15l from the support. But no bar should stop at a point less than 45 times its own size from the support

b. At least 30% of the reinforcement in tension at the mid span should extend to the support, the remainder should extend to within 0.15l of interior supports or 0.1l of exterior supports

c. At the simply supported end, the detailing should be as given for the simply supported beams

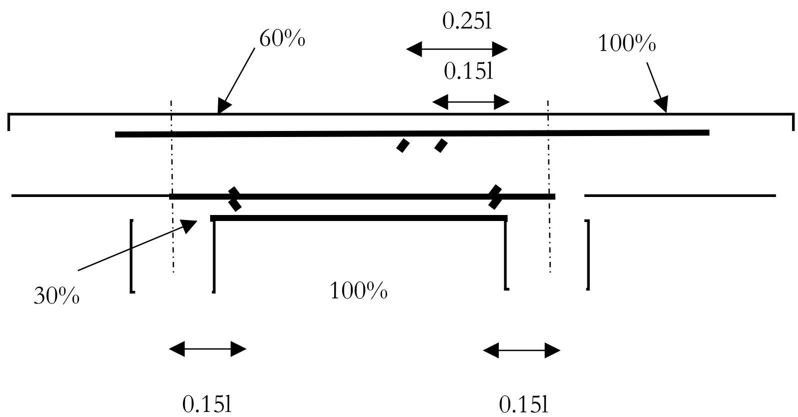


Figure 6.54: Curtailment of bars for simply supported beam

Chapter 7 - Design of Reinforced Concrete Slabs

7.1 Introduction

Slab can be defined as a "flat and plate-like element" which carries its load primarily by flexure. Reinforced concrete slabs are used to form a variety of elements in building structures such as floors, roofs, staircases, foundations and some types of walls

Slabs are usually conceived as a series of shallow and very wide rectangular beams placed side by side and connected transversely in such a way that it is possible to share loads between adjacent beams. On the basis of this model, the design of slab is similar, in principle, to that of beams. However, in slab design, the serviceability limit state of deflection is normally critical, rather than the ultimate limit states of bending and shear.

There are numbers of ways by which slabs are classified (Chapter 2). However, in the design of slab in reinforced concrete, it usual to distinguish slab on the basis of spanning action, that is either 1-way span or 2-way span. In the 2-way spanning slab, 2-way structural action results.

7.2 One-way Spanning Solid Slab

When a solid slab is supported along the two opposite edges, it is said to be a one-way spanning slab. Solid slabs are designed as if they consist of a **series of beams of 1 metre width strip.** The general procedure that are usually adopted for slab design is as follows:

i) Preliminary Sizing

   This is usually determined as it is done for the beams using $\frac{l}{d}$ ratio with appropriate modification factors (Table 3.10 and 3.11 BS 8110). But since the modification factor cannot be determined until the steel has been designed, it can be assumed initially to have the value of 1.00, so as to obtain the initial estimate). The effective span to be used is the lesser of:

   a. Clear distance between centers of the bearing
   b. Clear distance between the supports, plus the effective depth of the slab

ii) Determination of the total depth of the slab. This is useful to obtain an estimate of the self-weight.

iii) Determination of Design Loads

   Consist of the Dead Load (including the self- weight) and the Live load with the appropriate factors. For example, where only the Dead load (g_k) and live loads (q_k)are involved, the design load will be at the ultimate limit state:

   Design Load = $1.4g_k + 1.6q_k$

iv) Determination of BM and Shear Forces using suitable method of Structural Analysis.

   Analysis is carried out using the Maximum and the minimum design loads at different loading combination that will give the highest bending moment and shear force, for 1m width strip

v) Calculation of main and secondary reinforcement

The moment of resistance of the slab is calculated using

$$Mu = 0.156 f_{cu} b d^2$$

If Mu ≥ M, which is the usual condition for slabs, compression reinforcement will not be required and the area of tensile reinforcement, As, is determined using equation developed earlier

$$K = \frac{M}{f_{cu} b d^2}$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})]$$

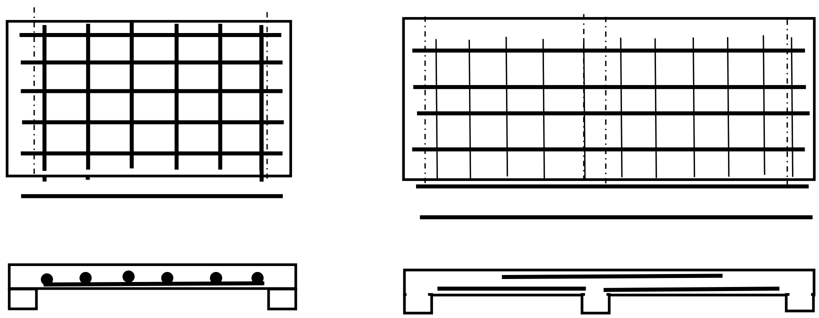
$$A_s = \frac{M}{0.87 f_y z}$$

Secondary or distribution steel is required in the transverse direction and this is usually based on the **minimum percentages of reinforcemen**t (As min) given in Table 3.25 of BS 8110.

As min = 0.24% Ac when fy = 250 N/mm2

As min = 0.13% Ac when fy = 500 N/mm2

where Ac is the total area of concrete.



(a.) Simply Supported Slab **(b)** Continuous Slab

Figure 7.1: simply-supported and continuous slabs

vi) Check compliance to appropriate Limit State requirement

a. Deflection

b. Check of critical shear stresses (Cl 3.5.5 BS 8110)

Shear resistance is generally not a problem in solid slabs subject to uniformly distributed loads. Shear reinforcement should not be provided in slabs less than 200 mm deep. In any other case shear reinforcement should be provided in form and area subject to section 3.9.1.3 and Table 3.21 of BS 8100.

c. Spacing requirements (clause 3.12.11.2.7, BS 8110).

The limiting spacing is: h_{agg} + 5mm or bar diameter < **spacing** < 3d or 750 mm

d. Crack width (clause 3.12.11.2.7, BS 8110)

To keep the crack width below 0.3 mm. No check is needed IF

- Slab depth ≤ 250 mm for 250 N/mm2
- Slab depth ≤ 200 mm for 500 N/mm2
- Percentage reinforcement < 0.3%

vii) Check of detailing requirements (clause 3.12.10.3, BS 8110).

7.2.1 Detailing and Curtailment of reinforcement in 1-way Slab (Cl. 3.12.10.3)

Simplify rules for curtailment of bars in 1-way slab are as follows.

i. Simply supported slab

At least 50% of the tension reinforcement provided at the mid span should extend to the support and having effective anchorage of 12 times the diameter past the center line of the support. The remaining 50% should extend to within 0.08l of the support. At least 50% of the tension reinforcement provided at the mid span should extend to the support and having effective anchorage of 12 times the diameter past the center line of the support. The remaining 50% should extend to within 0.08l of the support. This is approximately sketched in Figure 7.2.

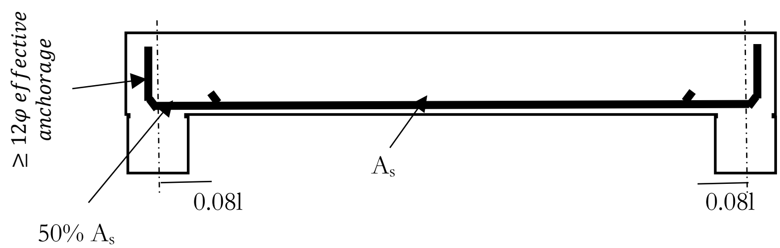


Figure 7.2: Simplified curtailment of reinforcement for simply supported slabs

ii. Cantilever slab

At least 50% of the tension reinforcement provided at the support should extend to the end of the cantilever. The remaining 50% should extend to a distance 0.5l or 45 times bar diameter (whichever is greater) from the support.

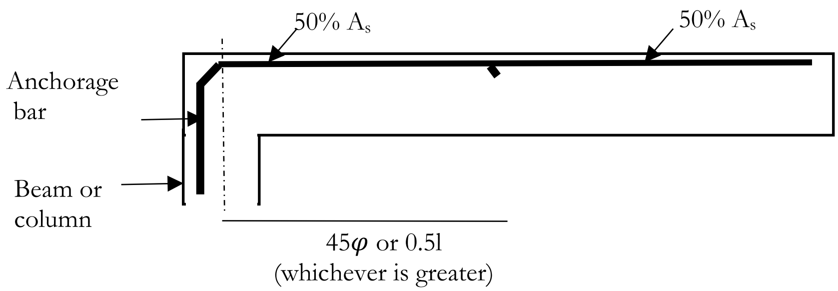


Figure 7. 3: Simplified curtailment of reinforcement for cantilever slabs

iii. **Continuous Slab**

Continuous slab with approximately equal spans or $\frac{l_y}{l_x} = 0.8 \rightarrow 1.2$, and the imposed load is ≤ the dead load

a. All tension reinforcement over support should extend a distance of 0.1l or 45 times diameter (whichever is greater) and at least 50% should extend 0.3l into the span.

b. The tension reinforcement at the midspan should extend to 0.2l of internal support and within 0.1l of external support, and at least 50% should extend into the support. It will be generally sufficient to provide the reinforcement equal to half that provided at the mid span extending 0.1l or 45 times diameter (whichever is greater) into the span from the face of the support.

This is shown in figure 7.4.

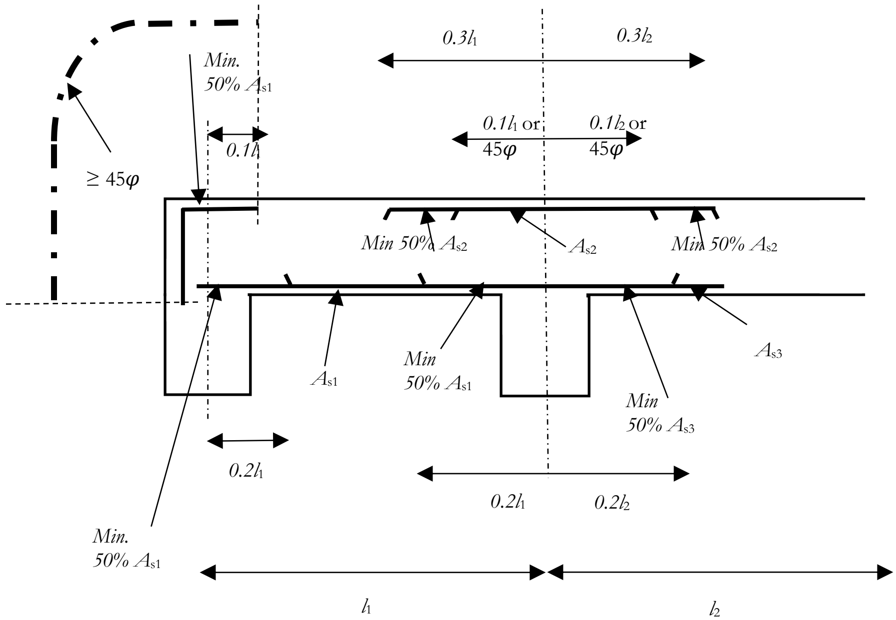


Figure 7.4: Simplified curtailment of reinforcement for continuous slabs

Example 7. 1

A reinforced concrete slab is subjected to a dead load of 3KN/m² and live load of 2.5KN/m². The slab is simply supported on load bearing wall with a span of 4m. If f_{cu} = 25N/mm², f_y = 460N/mm², exposure condition = mild, fire resistance = 2hrs, aggregate size = 20 mm, design the slab.

Solution

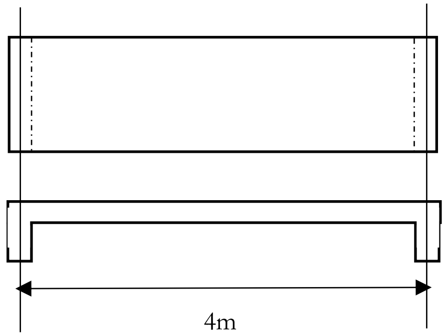


Figure 7.5: Example 7.1

The slab is simply supported, the effective depth can be determined from Table 3.9 (BS 8110), and taking the modification factor to be 1.0 for initial estimate.

$$\frac{l}{d} = 20 \times 1.0$$

$$d = \frac{4000}{20} = 200\text{mm}$$

Assuming 16 mm diameter main bar

Cover for:

i. Mild Exposure condition = 25 mm (Table 3.3)
ii. Fire resistance of 2hrs. = 35mm (Table 3.4)
iii. Concreting condition = 20 + 5 = 25mm

Therefore

Cover = 35mm

And

$$h = d + \text{cover} + \frac{diameter}{2} = 200 + 35 + 8 = 243 \text{ mm}$$

Take h = 250 mm

Loadings on the Slab

i. Dead Load = 3.0KN/m²
ii. Self-weight of the slab = density x height = 2400 x 0.25 = 6KN/m²
iii. Live load = 2.5KN/m²

Design Load = 1.4 (gk + self-weight) + 1.6 (qk)

= 1.4 (3 + 6) + 1.6 (2.5) = 16.6KN/m².

For 1m width strip of slab

The design Load = 16.6KN/m

The design moment, for simply supported structural systems, M = $\frac{wl^2}{8} = \frac{16.6 \times 4^2}{8}$ = 33.20KN.m

Mu = 0.156fcubd² = 0.156 x 25 x 1000 x 200 x 200

= 156KN.m

Since Mu > M No compression reinforcement is needed

$$K = \frac{M}{f_{cu}bd^2} = \frac{33.20 \times 1000000}{25 \times 1000 \times 200 \times 200} = 0.0332$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

$= d\ [0.5 + \sqrt{(0.25 - 0.037)}] = d\ [0.5 + 0.46] = 0.96d > 0.95d$

Therefore use 0.95d

z = 0.95d = 0.95 x 200 = 190 mm

Thus

$$\textbf{As} = \frac{M}{0.87f_yz} = \frac{33.20 \times 1000000}{0.87 \times 460 \times 190}$$

= 436.62 mm²

Use Y10@150 (523mm²)

Checks

i. **Deflection**

From the previous example,

M = 33. 20KN.m

b = 1000.00mm

d = 200mm

As (required) = 463.63mm²

As (provided = 523 mm²

Therefore

$$\frac{M}{bd^2} = \frac{33.20 \times 1000000}{1000 \times 200 \times 200} = 0.83$$

And equation 8 of Table 3. 16 of BS 8110, the service stress f_s

$f_s = \frac{2f_yA_{s\ reuired}}{3_{As\ provided}} \times \frac{1}{\beta}$ ($\beta = 1\ for$ simply supported beam/slab

$= \frac{2 \times 460 \times 463.63}{3 \times 523} \times \frac{1}{1} = 271.85$ N/mm².

From Table 3.10, the modification factor is about 1.50

The slab is simply supported. Thus, the span/effective depth ratio is 20. Therefore, by multiplying this value by the modification factor, the permissible span/depth ratio is obtained. That is:

$$\frac{permissible\ span}{depth} = 20 \text{ x } 1.5 = 30.0$$

But,

$$\frac{actual\ span}{depth} \text{ ratio} = \frac{4000}{200} = 20.0$$

Since the $\frac{permissible\ span}{depth}$ ratio (30.0) is greater than the $\frac{actual\ span}{depth}$ ratio (20.0).

The deflection is OK.

The secondary reinforcement

As min = 0.13% Ac = $\frac{0.13 \text{ Ac}}{100}$ (where Ac = total area of concrete)

$$= \frac{0.13 \ x \ 1000 \ x \ 250}{100}$$

$$= 325 \text{ mm}^2$$

Use Y8@150 (335mm^2)

Design for Shear

The slab is simply supported and symmetrically loaded

Support Reaction = $\frac{33.20 \ x \ 4}{2}$ = 66.40KN

$$v = \frac{V}{bd} = \frac{33.20 \ x \ 1000}{1000 \ x \ 200} = 0.17\text{N/mm}^2.$$

Assuming half of the tension reinforcement (50% x 523 = 261.50mm^2) is provided at the supports

Then,

$$\frac{100 \ x \ As}{bd} = \frac{100 \ x \ 261.50}{1000 \ x \ 200} = 0.131$$

From Table 3.8

$$v_c = 0.40\text{N/mm}^2$$

Since v < v_c,. No shear reinforcement is needed.

Further Checks

i. spacing between bars

Maximum spacing between bars should not exceed the lesser of 3*d* (= 2 x 200 = 600 mm) or 750 mm.

Spacing for main steel = 150 mm

Spacing for the distribution = 150 mm

Hence spacing is OK

ii. cracks

In this case, we check the percentage of reinforcement

$$\frac{100 \ x \ As}{bd} = \frac{100 \ x \ 523}{1000 \ x \ 200} = 0.262 < 0.3$$

Since reinforcement is less than 3%, the above spacing between bars will automatically ensure that the maximum permissible crack width of 0.3 mm will not be exceeded.

Reinforcement details

The sketch of the reinforcement arrangement, for a 1m strip width of the slab, is shown in figure 7.6.

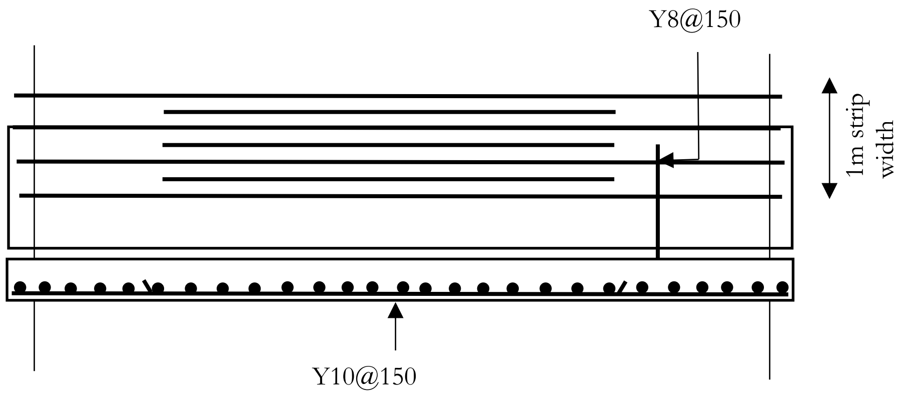


Figure 7.6: Sketch of reinforcement arrangement in the slab

Example 7. 2

A one-way continuous slab is as shown in Figure 7.2

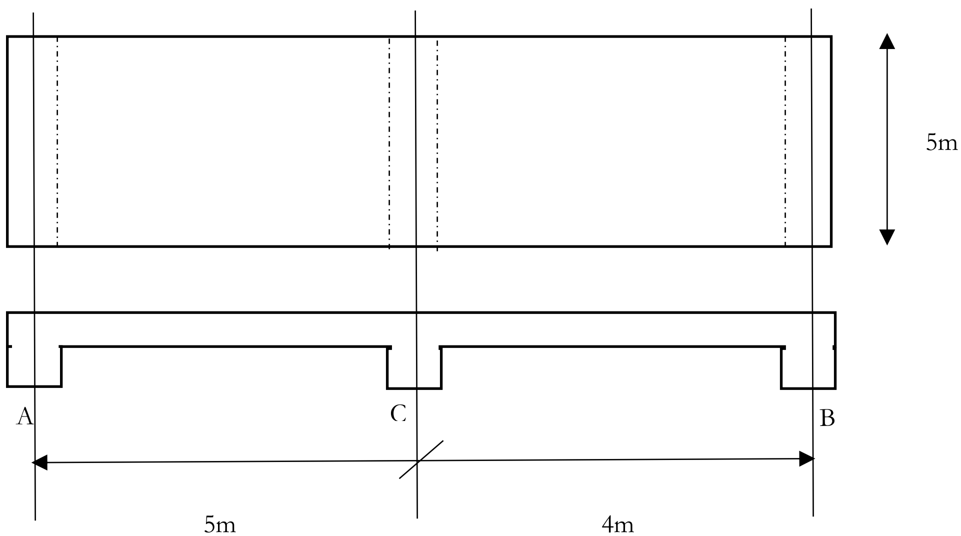


Figure 7.7: Example 7.2

Design the slab if gk = 2.5KN/ m², qk = 3.0KN/m², f_{cu} = 30 mm², f_y = 460N/mm², fire resistance = 2 hrs., exposure condition = moderate, maximum aggregate size = 20mm. Assume bar diameter = 12mm

Solution

The slab is continuous, the effective depth can be determined from Table 3.9 (BS 8110) and taking the modification factor to be 1.0 for initial estimate.

$$\frac{l}{d} = 26 \text{ x } 1.0$$

$$d = \frac{4000}{26} = 153.85\text{mm}$$

Use d = 175mm

The bar diameter is 12 mm diameter main bar

Cover for:

i. Mild moderate condition = 35mm (Table 3.3)
ii. Fire resistance of 2hrs. = 35mm (Table 3.4)
iii. Concreting condition = 20 + 5 = 25mm

Therefore

Cover = 35mm

And

$$h = d + \text{cover} + \frac{diameter}{2} = 175 + 35 + 6 = 216\text{mm}$$

Take h = 225 mm (for ease of construction)

Loadings on the Slab

i. Dead Load = 2.50KN/m^2
ii. Self-weight of the slab = density x height = 24 x 0.225 = 5.4KN/m^2
iii. Live load = 3.0KN/m^2

Design Load

Maximum design load = 1.4 (gk + self-weight) + 1.6 (qk)

= 1.4 (2.5 + 5.4) + 1.6 (3.0) = 15.86KN/m^2.

For 1m width strip of slab

The maximum design Load = 15.86KN/m

Minimum design load = 1.0gk = 1.0 (2.5 + 5.4) = 7.90KN/m^2

For 1m width strip of slab

The minimum design Load = 7.90KN/m

The structure, with the loadings, is shown in Figure 7.3. The analysis was done using moment distribution method and considering 4 loading cases. The shear forces and bending moments envelopes, showing the maximum values are presented in Figure 7.4.

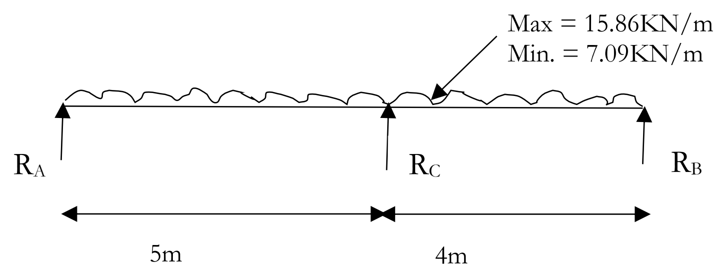


Figure 7. 3: The Slab with loadings

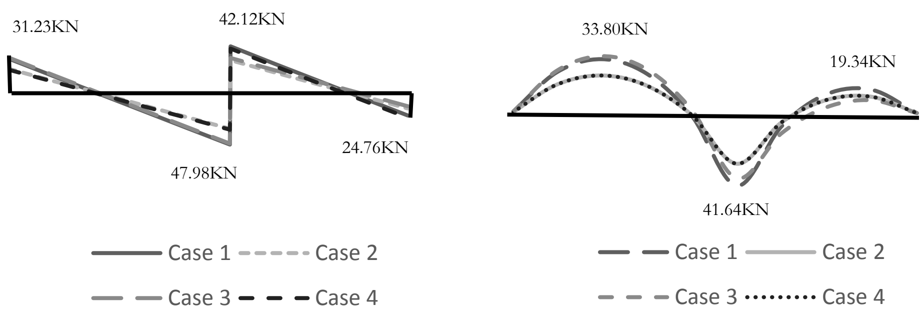


Figure 7.4: The Shear Forces and Bending Moments Envelopes Diagrams

Design of Reinforcement

Mu = 0.156fcubd² = 0.156 x 30 x 1000 x 175 x 175

= 143. 33KN.m

Since Mu > M No compression reinforcement is needed

Span AC (Maximum BM = 33. 80KN.m)

$$K = \frac{M}{f_{cu}bd^2} = \frac{33.80\ x\ 1000000}{30\ x\ 1000\ x\ 175\ x\ 175} = 0.04$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

= d [0.5 + √(0.25 – 0.0444)] = d [0.5 + 0.453] = 0.953 > 0.95d

Therefore use 0.95d

z = 0.95d = 0.95 x 175 = 166.25 mm

Thus

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{33.80\ x\ 1000000}{0.87\ x\ 460\ x\ 166.25}$$

= 508.02 mm²

Use Y12@200 (566 mm²)

Support C (Maximum BM = 41.64KN.m)

$$K = \frac{M}{f_{cu}bd^2} = \frac{41.64 \ x \ 1000000}{30 \ x \ 1000 \ x \ 175 \ x \ 175} = 0.05$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{0.05}{0.9})}]$$

$= d \ [0.5 + \sqrt{}(0.25 – 0.06)] = d \ [0.5 + 0.44] = 0.94d$

Therefore,

z = 0.94d = 0.94 x 175 = 164.50 mm

Thus

$$\textbf{As} = \frac{M}{0.87f_y z} = \frac{41.64 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 164.50}$$

= 632.51 mm^2

Use Y12@175 (646 mm^2)

Span CB (Maximum BM = 19. 34KN.m)

$$K = \frac{M}{f_{cu}bd^2} = \frac{19.34 \ x \ 1000000}{30 \ x \ 1000 \ x \ 175 \ x \ 175} = 0.02$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{0.02}{0.9})}]$$

$= d \ [0.5 + \sqrt{}(0.25 – 0.022)] = d \ [0.5 + 0.48] = 0.98d > 0.95d$

Therefore,

z = 0.95d = 0.95 x 175 = 166.25 mm

Thus

$$\textbf{As} = \frac{M}{0.87f_y z} = \frac{19.34 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 164.50}$$

= 293.77 mm^2

Use Y12@ 300 (377 mm^2)

Checks

i. **Deflection for Span AC**

From the previous example,

M = 33. 80KN.m

b = 1000.00mm

d = 175mm

As (required) = 508.02 mm^2

As (provided = 566 mm^2

Therefore

$$\frac{M}{bd^2} = \frac{33.80 \ x \ 1000000}{1000 \ x \ 175 \ x \ 175} = 1.10$$

And equation 8 of Table 3. 16 of BS 8110, the service stress f_s

$f_s = \frac{2fy}{3}$ (for continuous slab/beam)

$= \frac{2 \ x \ 460}{3} = 306.67 N/mm^2$

From Table 3.10, the modification factor is about 1.26

Therefore

$$\frac{permissible \ span}{depth} = 26 \ x \ 1.26 = 32.76$$

But,

$$\frac{actual \ span}{depth} \text{ ratio} = \frac{5000}{175} = 28.57$$

Since the $\frac{permissible \ span}{depth}$ ratio (32.66) is greater than the $\frac{actual \ span}{depth}$ ratio (28.57).

The deflection is OK.

The secondary reinforcement

As min = 0.13% Ac = $\frac{0.13 \ Ac}{100}$ (where Ac = total area of concrete)

$$= \frac{0.13 \ x \ 1000 \ x \ 225}{100}$$

$$= 292.50 \ mm^2$$

Use Y10@250 (314 mm^2)

Design for Shear

Support C Shear Force V = 47.98KN

$$v = \frac{V}{bd} = \frac{47.98 \ x \ 1000}{1000 \ x \ 175} = 0.274 N/mm^2.$$

Then,

$$\frac{100 \ x \ As}{bd} = \frac{100 \ x \ 566}{1000 \ x \ 175} = 0.32$$

From Table 3.8

$v_c = 0.53 N/mm^2$

Since v < v_c,. No shear reinforcement is needed.

Further Checks

i. spacing between bars

Maximum spacing between bars should not exceed the lesser of 3*d* (= 2 x 175 = 525 mm) or 750 mm.

Spacing for main steel = 300 mm

Spacing for the distribution = 300 mm

Hence spacing is OK

ii. cracks

In this case, we check the percentage of reinforcement

$$\frac{100 \ x \ As}{bd} = \frac{100 \ x \ 566}{1000 \ x \ 175} = 0.\ 32 < 0.30$$

Since reinforcement is less than 3%, the above spacing between bars will automatically ensure that the maximum permissible crack width of 0.3 mm will not be exceeded.

The sketch of the arrangement of reinforcement is shown in Figure 7.5. Because of the possibility of development of negative moments at the supports which could cause cracking, an amount of reinforcement equal to half of area of bottom steel at midspan – but not less than minimum reinforcement – should be provided. In this case, half of 566 = 283 mm², or 0.013 x 1000 x 225 = 292.5 mm². Thus, use Y10@250 mm (314 mm²) at the supports

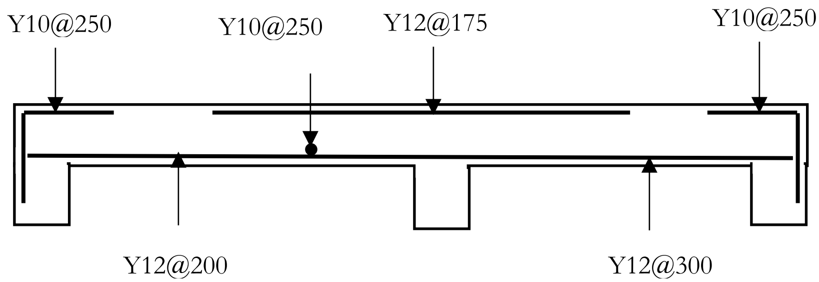


Figure 7.5 – A sketch of the arrangement of reinforcement

7.3 Two-way Spanning Solid Slab

A slab is said to be a two-way spanning when it is proportioned and supported in such a way that two-way structural action results. The simple one-way slab deforms under load into a cylindrical surface.

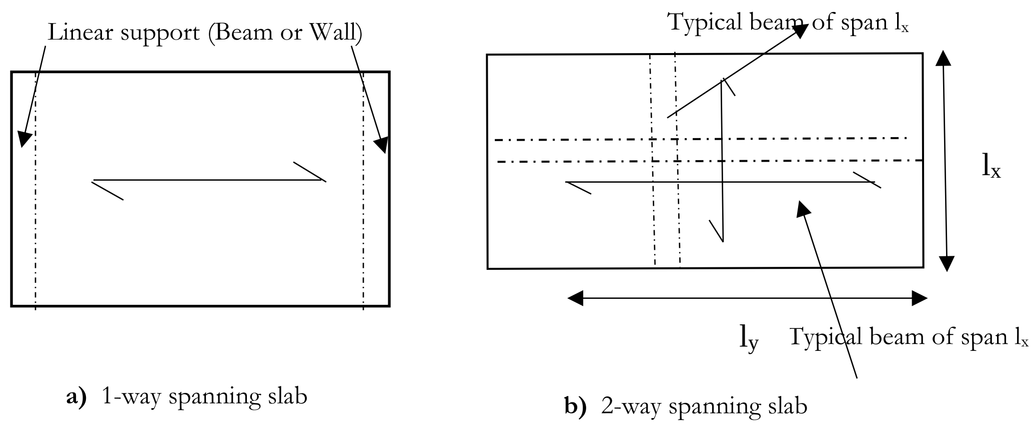


Figure 7.6: Two-way solid slabs

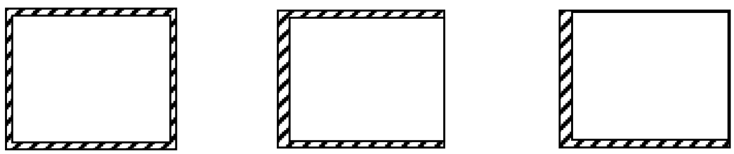
The main structural action is one-way, in the direction normal to the support on two opposite edges of the rectangular panel. In many cases however, rectangular slabs are of such proportion and supports that two-way structural action results. When loaded, such slabs bend into dished-shape surface consisting of series interlocking beams (Figure 7.6)

What this means is that, at any point, the slab is curved in both principal directions, and since bending moments are proportional to curvature, moments also exist in both directions. Therefore, to resist the BM, the slab must be reinforced in both directions by mutually perpendicular layers of bars which should be also be perpendicular respectively to 2 pairs of edges. Usually, two-way results, when:

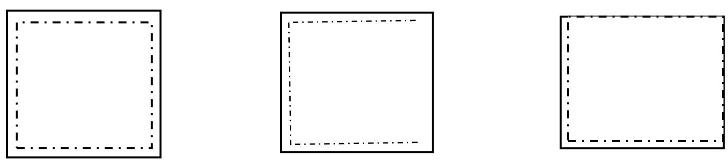
i. it is supported on two, or three, or four sides, and
ii. the ratio of the length of the longer side to the shorter side is equal to 2 or less

Types of RC slabs which are characterized by 2-way structural actions are:

1) Slabs supported by wall or beams along all the sides or along 3 sides or along two adjacent sides or solid slab on continuous supports



(a) Slabs supported by beams on 4, 3, and 2 sides



(b) Slabs supported by walls on 4, 3, and 2 sides

Figure 7. 7: Two-way slabs supported on sides by beams and walls

2) Flat Plates (beamless slab-column structure)

Loads from the slabs are transmitted directly to the columns. It is directly supported by columns (Figure 7. 8). The use of flat plate construction offers a number of other advantages, absent from other flooring systems, including reduced storey heights, no restrictions on the positioning of partitions, windows can extend up to the underside of the slab and ease of installation of horizontal services. The main drawbacks with flat plate are that they may deflect excessively and are vulnerable to punching failure.

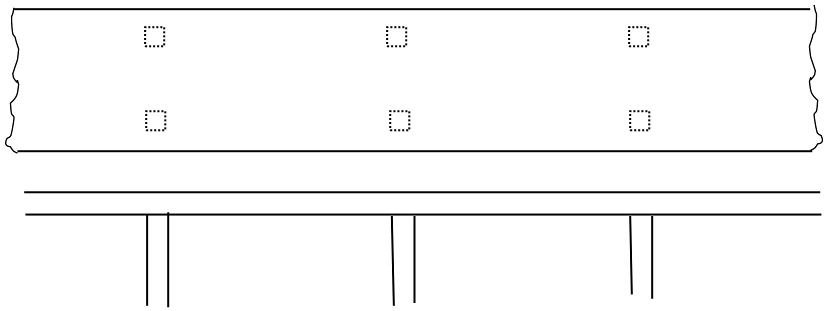


Figure 7. 8: A typical Flat plates

Punching failure arises from the fact that high live loads results in high shear stresses at the supports which may allow the columns to punch through the slab unless appropriate steps are taken.

3) Flat Slab

This is an attempt to solve the problem of punching failure in flat plates. Though still without beams, but intermediate column heads or drop panels (Figure 7.9) are introduced to arrest the punching failure.

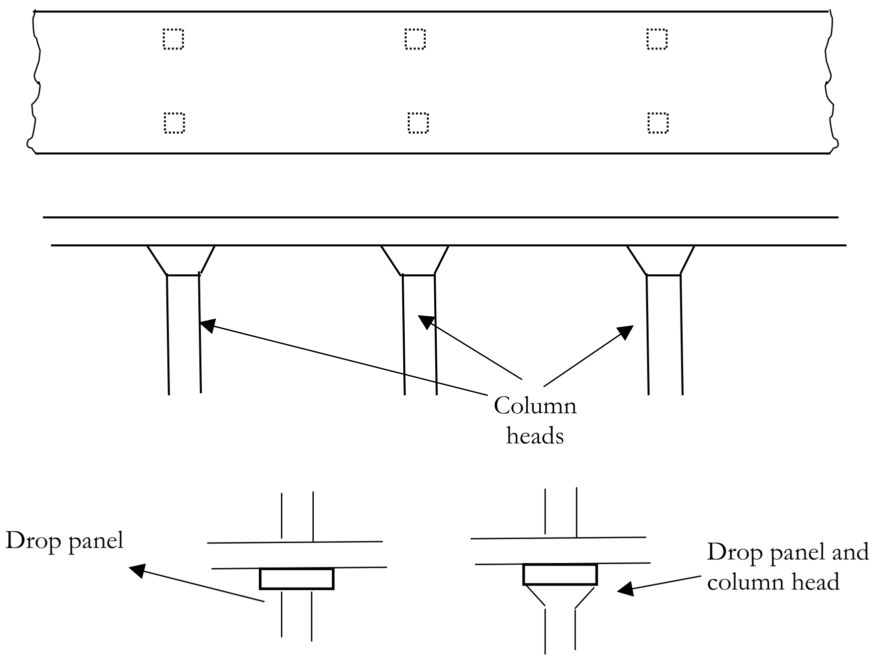


Figure 7. 9: Two-way flat slabs

4) Two-way Ribbed (or Waffle) Slabs

A two-way ribs slab (or waffle slab) is a reinforced concrete slab that is characterized by ribs in two directions giving it the appearance of a waffle, that is, slab is flat on top and joists create a grid like surface on the bottom (Figure 7: 10). This slab is preferred for spans longer than 10 m, over systems like flat plates, flat slabs, and two-way slabs. It is used for laboratories, theatres and buildings requiring big open spaces.

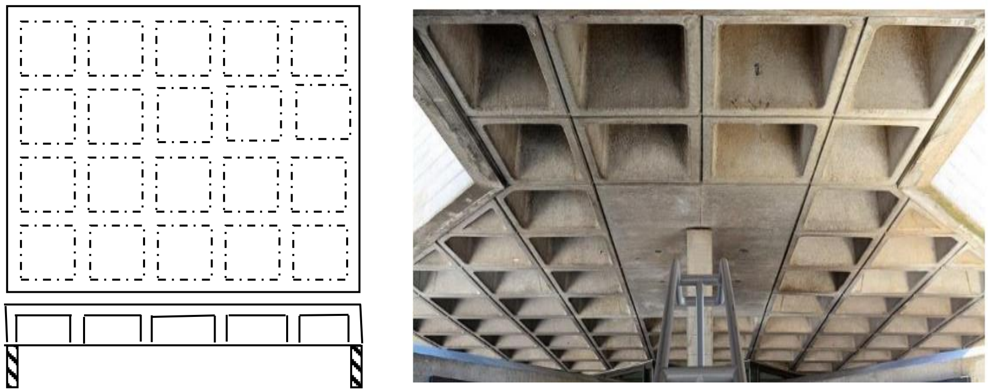


Figure 7.10: Two-way ribbed slab

5) Grid slabs

The slab which is resting on the beams running in two directions is known as grid slab. It is considered as a system of beams. In these types of slab, a mesh or grid of beams run in both directions in the main structure, and the slab is of nominal thickness.

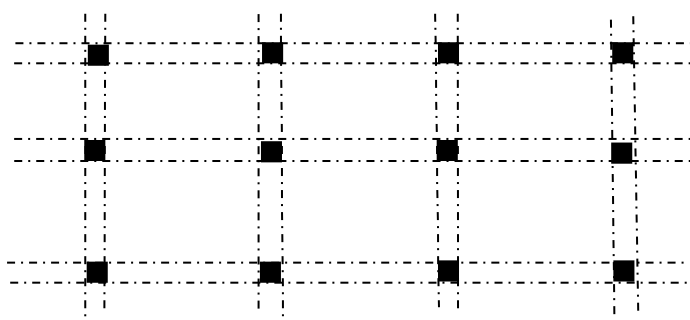


Figure 7. 11: A typical two-way grid slab

It is an assembly of intersecting beams placed at regular interval and interconnected to a slab of nominal thickness. These slabs are used to cover a large column free area and therefore are good choice for public assembly halls. They are generally employed for architectural reasons for large rooms such as auditoriums, vestibules, theatre halls, show rooms of shops where column free space is often the main requirement. The structure is monolithic in nature and has more stiffness. It gives pleasing appearance. For these kind of structures, 2-way action should be considered if the proportions of the spans are in the range of $0.5 \leq \frac{l_1}{2} \leq 2.0$.

In practice, the choice of slab for a particular structure will largely depend upon economy, buildability, the loading conditions and the length of the span. For slabs with short spans, of say ≤ 5 m, the most economical solution is to provide a solid slab of constant thickness over the complete span. With medium size spans from 5 to 9 m it is more economical to provide flat slabs since they are generally easier to construct. The ease of construction chiefly arises from the fact that the floor has a flat soffit. This avoids having to erect complicated shuttering, thereby making possible speedier and cheaper construction. Using deep slabs with large diameter columns, providing drop panels and/or flaring column heads, can avoid this problem. However, all these methods have drawbacks, and research effort has therefore been directed at finding alternative solutions. For design purpose however, two-way slabs consist in two cases, namely:

i. Simply supported 2-way slab
ii. Restrained 2-way slab

7.3.1 Design of Simply Supported 2-Way Slab

When a slab is supported on all the four sides, it can be said to effectively spans in both directions. And it will be economical to design the slab on the basis of this. The amount of bending in each direction will depend:

i. on the ratio of the two spans and
ii. on the conditions of restraint at the support.

If the slab is square and restraints are similar along the four sides, then the load will span equally in both direction.

But if the slab is rectangular, then more than one half of the load will be carried on the stiffer and shorter direction than the longer direction. The assumption is that both spans (l_x, l_y) approximately acts like simple beam uniformly loaded with q_x and q_y respectively. And for equilibrium

$$q = q_x + q_y$$

That is, the sum of these imaginary loads on the spans is "q". But because the imaginary strips are part of the same monolithic surface, the deflection at intersection points must be the same. That is:

$$\frac{5\,q_x {l_x}^4}{384EI} = \frac{5\,q_y {l_y}^4}{384EI}$$

Consequently, the shorter span carries the larger share of the load

$$\frac{q_x}{q_y} = \frac{{l_y}^4}{{l_x}^4}$$

This result is however approximate, because the actual behavior of a slab is complex than that of two intersecting strips. Because the strips are not only bent but also twisted as a result of transversal interaction between parallel strips. This twisting results in torsional stresses and torsional moments. Twisting moments are not pronounced at the corners.

For simply supported 2-way slabs, moments in each direction of the spans are calculated using the equations 10 and 11 of BS 8110.

$$M_{sx} = \alpha_{sx} n {l_x}^2 \qquad 10$$

$$M_{sy} = \alpha_{sy} n {l_x}^2 \qquad 11$$

Where

M_{sx} and M_{sy} = the maximum moments at mid-span on strips of unit width spans l_x and l_y respectively

n = total ultimate load per unit area (1.4gk + 1.6qk)

ly = length of the longer side

lx = length of the shorter side

α_{sx} *and* α_{sy} = *the moment coefficients as shown Table* 3.13

Table 7.1: Bending Moments Coefficient for Slabs spanning in two directions at right angles and simply supported on four sides (BS 8110, Table 3.13)

l_y/l_x	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
α_{sx}	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118
α_{sy}	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029

Once the forces and moments have been calculated, the design of the section follows that for one way spanning slabs, irrespective of whether the slab is solid, ribbed or voided.

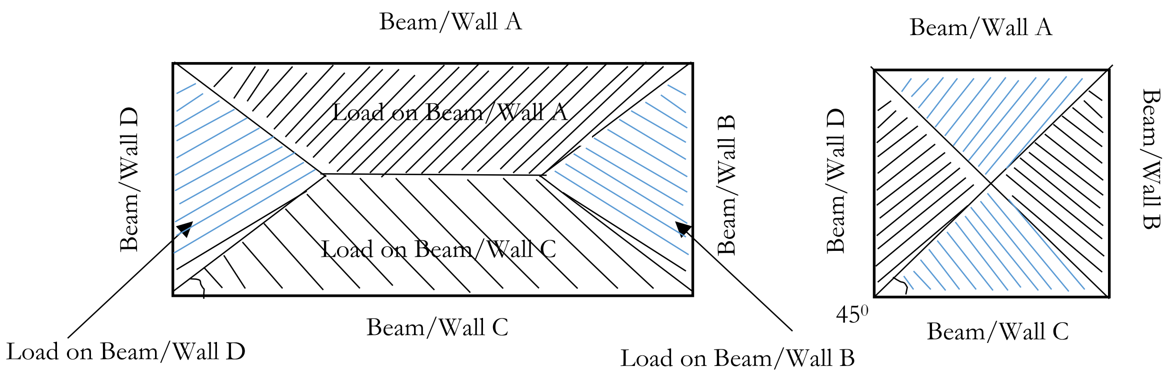
Areas of reinforcement to resist the moments are determined independently for each direction of the span. The slab is reinforced with bars in both directions parallel to the span.

NOTE

1) the design moments are tied to the shorter span
2) But the reinforcement for the shorter span will be placed at a distance farther from the neutral axis to give a greater effective depth.
3) The span-effective depth ratios are based on the shorter span.

Loads on 2-way on Beams/Walls supports

For two-way slabs, the loadings on the edge beams or walls can be estimated as indicated by considering failure yield lines forming at 45^0 as shown in Figure 7.12.



(a) 2-way rectangle slab **(a) 2-way square slab**

Figure 7.12: Estimation of loads on adjacent beams or walls

Example 7.3

A simply supported slab spans in two directions. The effective span in each direction 4.0 m and 5.0 m. if the dead load g_k is 3.0KN/m^2 and live load q_k is 3.5KN/m^2. Design the reinforcement for the slab. Take f_{cu} = 30N/mm^2, f_y = 460N/mm^2, condition of exposure = moderate., fire resistance = 2hrs, bar diameter = 16mm.

Solution.

The plan of the slab is as shown

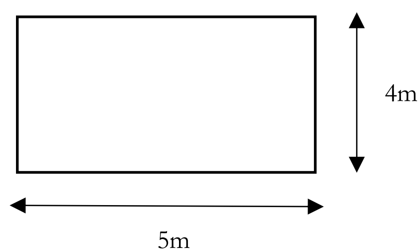


Figure 7.13: Example 7.3

The minimum span-depth ratio will be based on the shorter span

$$\frac{l}{d} = 20$$

$$d = \frac{4000}{20} = 200\text{mm}$$

Minimum Cover from the conditions:

i. Condition of exposure (Table 3.3)

$c = 35$ mm

ii. Fire resistance (Table 3 .4)

$c = 35$

Thus,

c = 35 mm

Overall depth of the slab

h = d +half bar diameter + cover

= 200 +8 + 35 = 243

= 250mm

Loading on the Slab

i. Self-weight = 2400 x h = 2400 x 0.25 = 6KN/m^2

ii. Dead Load = 3KN/m^2

iii. Live load – 3.5KN/m^2

Design Load (n) = 1.4gk + 1.6qk

= 1.4 (3 + 6) + 1.6 (3.5) = 12.6 + 5.6

= 18.2KN/m^2

= 18.2KN/m (per 1meter width)

Now

$\frac{l_y}{l_x} = \frac{5000}{4000} = 1.25$

From Table 3.13

$\alpha_{sx} = 0.089$

$\alpha_{sy} = 0.057$

Therefore

i. Bending for Short Span

$M_{sx} = \alpha_{sx} n l_x^2 \quad = 0.089 \times 18.2 \times 4^2 = 25.92$KNm

And

ii. Bending for Longer Span

$M_{sy} = \alpha_{sy} n l_x^2 \quad = 0.057 \times 18.2 \times 4^2 = 16.60$KNm

Reinforcement for Short Span

Mu = 0.156fcubd^2 = 0.156 x 30 x 1000 x (200)^2 = 187. 20KN.m

Since

Mu > M, no compression reinforcement is needed

Thus, K = $\frac{M}{f_{cu}bd^2}$ = $\frac{25.92\ x\ 1000000}{30\ x\ 1000\ x\ 200\ x\ 200}$ = 0.022

$$z = d[0.5 + \sqrt{(0.}25 - \frac{K}{0.9})]$$

= d[0.5 + √(0.25 – 0.024] = d[0.5 + 0.48] = 0.98d > 0.95d

Therefore use 0.95d

z = 0.95d = 0.95 x 200 = 190 mm

Thus

As = $\frac{M}{0.87f_y z}$ **=** $\frac{25.92\ x\ 1000000}{0.87\ x\ 460\ x\ 190}$

= 340.88 mm^2

Use Y10@200 (393mm^2)

Checks

i. **Deflection**

For the Short span,

M = 25.92KN.m

b = 1000.00mm

d = 200mm

As (required) = 340.88mm^2

As (provided = 393 mm^2

Therefore

$\frac{M}{bd^2}$ = $\frac{25.92\ x\ 1000000}{1000\ x\ 200\ x\ 200}$ = 0.65

And equation 8 of Table 3.16 of BS 8110, the service stress f_s

f_s = $\frac{2f_yA_{s\ reuired}}{3_{As\ provided}}$ x $\frac{1}{\beta}$ ($\beta = 1\ for$ simply supported beam/slab

= $\frac{2\ x\ 460\ x\ 340.88}{3\ x\ 393}$ x $\frac{1}{1}$ = 266.00 N/mm^2.

From Table 3.10, the modification factor is about 1.81 The slab is simply supported. Thus, the span/effective depth ratio is 20. Therefore, by multiplying this value by the modification factor, the permissible span/depth ratio is obtained.

That is:

$\frac{permissible\ span}{depth}$ = 20 x 1.81 = 36.20

But,

$\frac{actual\ span}{depth}$ ratio = $\frac{4000}{200}$ = 20.0

Since the $\frac{permissible\ span}{depth}$ ratio (30.0) is greater than the $\frac{actual\ span}{depth}$ ratio (20.0).

The deflection is OK.

The new d = 250 – 35 – 5 = 210 mm (this will result in even greater Mu)

Reinforcement for Longer Span

Design Moment = 16.60KNm

The longer span will have a reduced effective depth.

d = h – cover – diameter of short span bar diameter – half of long span bar diameter

= 250 – 35 – 10 – 5 =

= 200mm

Mu = 0.156fcubd^2 = 0.156 x 30 x 1000 x 200 x 200 = 187.20KNm

Since

Mu > M, no compression reinforcement is needed

Thus

$$K = \frac{M}{f_{cu}bd^2} = \frac{16.60\ x\ 1000000}{30\ x\ 1000\ x\ 200\ x\ 200} = 0.014$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

= d[0.5 + √(0.25 – 0.016] = d[0.5 + 0.48] = 0.98d > 0.95d

Therefore use 0.95d

z = 0.95d = 0.95 x 200 = 190 mm

Thus

$$\textbf{As} = \frac{M}{0.87f_yz} = \frac{16.60\ x\ 1000000}{0.87\ x\ 460\ x\ 190}$$

= 218mm^2

` Use Y10@300 (262mm^2)

The sketch of reinforcement arrangement is shown in figure 7.14.

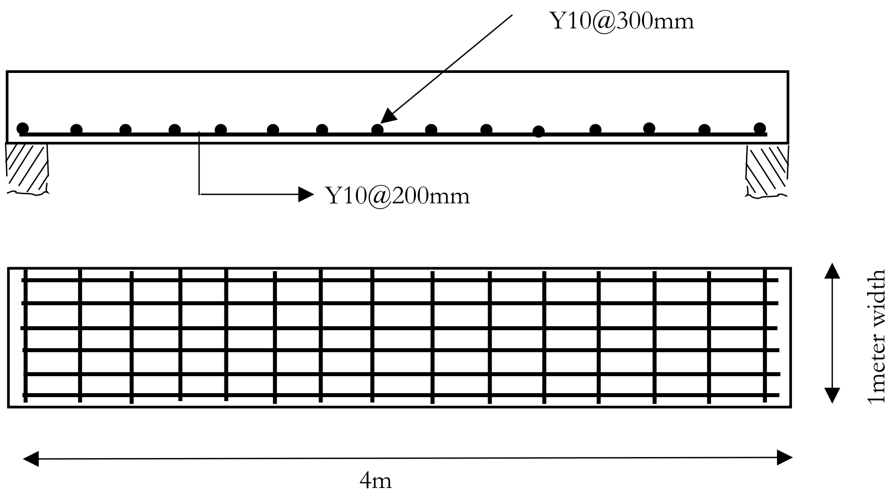


Figure 7.14: Sketch of reinforcement arrangement

7.3.2 Restrained Two Way Slab

A slab may have its edges restrained to a greater or lesser degree depending on whether it is continuous over it support or cast monolithically with the supporting slabs. This will result in hogging or negative moments leading to tension at the top face of the slab at the supported sides in the same way as the beams. Although, the design of two-way spanning restrained slabs supporting uniformly distributed loads is generally similar, in principle to the ones outlined for one-way spanning slabs, it is however difficult to determine their design bending moments and shear forces. The code provided alternative way – in form of expression – for calculating the bending moment and shear force in two-way restrained slabs. The moments in each direction of the spans are calculated using the equations 14 and 15 of BS 8110.

$$M_{sx} = \beta_{sx} n l_x^2 \qquad 12$$

$$M_{sy} = \beta_{sy} n l_x^2 \qquad 13$$

Where

M_{sx} and M_{sy} = the maximum moments at mid-span on strips of unit width and spans l_x and l_y respectively

n = total ultimate load per unit area (1.4gk + 1.6qk)

l_y = length of the longer side

l_x = length of the shorter side

β_{sx} *and* β = *the moment coefficients as shown Table* 3.14

Also, the BS 8110 provides expression for calculating the design shear forces at supports in the long span direction, V_{sy}, and short span direction, V_{sx}, according to the following equations

$$V_{sx} = \beta_{vx} n l_x \qquad 14$$

$$V_{sy} = \beta_{vy} n l_x \qquad 15$$

The values of $\beta_{sx}, \beta_{sy}, \beta_{vx}$ and β_{vy} depend on case type of slab panel according to Tables 3.14 and 3.15 of BS 8100. Code recognizes 9 cases of slab types, as identified in the figure 7.14. The cases in Figure 7.14 can be identified as follows:

Case 1 = interior panel

Case 2 = one short edge discontinuous

Case 3 = one long edge discontinuous

Case 4 = two adjacent edges discontinuous

Case 5 = two short edges discontinuous

Case 6 = two long edges discontinuous

Case 7 = three edges discontinuous (one long edge continuous)

Case 8 = three edges discontinuous (one short edge continuous)

Case 9 = four edges discontinuous

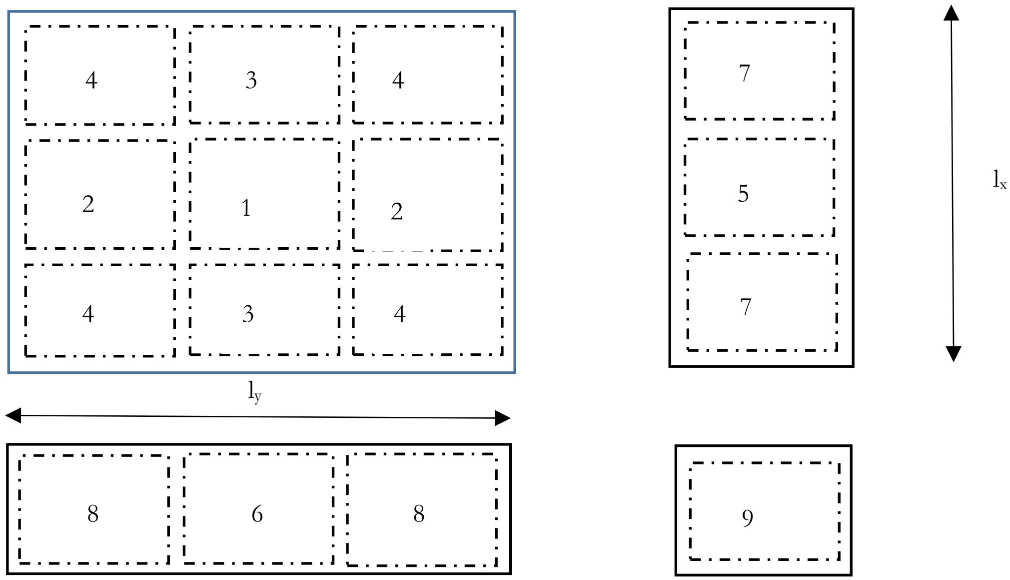


Figure 7.14: Identification of cases in restrained 2-way slab ($l_y > l_x$)

The bending moments and shear force coefficient for each case are shown in Table 7.2 and 7.3 (Tables 3.14 and 3.15 of BS 8100)

Table. 7.2: Bending moment coefficients for rectangular panels supported on four sides with provision for torsion at corners (Table 3.14, BS 8110)

Type of panel and moments considered	Short span coefficients, β_{sx}								Long span coefficients, β_{sy} for all values of l_y/l_x
	Values of l_y/l_x								
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Interior panels									
Negative moment at continuous edge	0.031	0.037	0.042	0.046	0.050	0.053	0.059	0.063	0.032
Positive moment at mid-span	0.024	0.028	0.032	0.035	0.037	0.040	0.044	0.048	0.024
One short edge discontinuous									
Negative moment at continuous edge	0.039	0.044	0.048	0.052	0.055	0.058	0.063	0.067	0.037
Positive moment at mid-span	0.029	0.033	0.036	0.039	0.041	0.043	0.047	0.050	0.028
One long edge discontinuous									
Negative moment at continuous edge	0.039	0.049	0.056	0.062	0.068	0.073	0.082	0.089	0.037
Positive moment at mid-span	0.030	0.036	0.042	0.047	0.051	0.055	0.062	0.067	0.028
Two adjacent edges discontinuous									
Negative moment at continuous edge	0.047	0.056	0.063	0.069	0.074	0.078	0.087	0.093	0.045
Positive moment at mid-span	0.036	0.042	0.047	0.051	0.055	0.059	0.065	0.070	0.034
Two short edges discontinuous									
Negative moment at continuous edge	0.046	0.050	0.054	0.057	0.060	0.062	0.067	0.070	—
Positive moment at mid-span	0.034	0.038	0.040	0.043	0.045	0.047	0.050	0.053	0.034
Two long edges discontinuous									
Negative moment at continuous edge	—	—	—	—	—	—	—	—	0.045
Positive moment at mid-span	0.034	0.046	0.056	0.065	0.072	0.078	0.091	0.100	0.034
Three edges discontinuous (one long edge continuous)									
Negative moment at continuous edge	0.057	0.065	0.071	0.076	0.081	0.084	0.092	0.098	—
Positive moment at mid-span	0.043	0.048	0.053	0.057	0.060	0.063	0.069	0.074	0.044
Three edges discontinuous (one short edge continuous)									
Negative moment at continuous edge	—	—	—	—	—	—	—	—	0.058
Positive moment at mid-span	0.042	0.054	0.063	0.071	0.078	0.084	0.096	0.105	0.044
Four edges discontinuous									
Positive moment at mid-span	0.055	0.065	0.074	0.081	0.087	0.092	0.103	0.111	0.056

Table. 7.3: Shear Force coefficients for rectangular panels supported on four sides with provision for torsion at corners (Table 3.15, BS 8110)

Type of panel and location	β_{vx} for values of l_y/l_x								β_{vy}
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Four edges continuous									
Continuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
One short edge discontinuous									
Continuous edge	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36
Discontinuous edge	—	—	—	—	—	—	—	—	0.24
One long edge discontinuous									
Continuous edge	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36
Discontinuous edge	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	—
Two adjacent edges discontinuous									
Continuous edge	0.40	0.44	0.47	0.50	0.52	0.54	0.57	0.60	0.40
Discontinuous edge	0.26	0.29	0.31	0.33	0.34	0.35	0.38	0.40	0.26
Two short edges discontinuous									
Continuous edge	0.40	0.43	0.45	0.47	0.48	0.49	0.52	0.54	—
Discontinuous edge	—	—	—	—	—	—	—	—	0.26
Two long edges discontinuous									
Continuous edge	—	—	—	—	—	—	—	—	0.40
Discontinuous edge	0.26	0.30	0.33	0.36	0.38	0.40	0.44	0.47	—
Three edges discontinuous (one long edge discontinuous)									
Continuous edge	0.45	0.48	0.51	0.53	0.55	0.57	0.60	0.63	—
Discontinuous edge	0.30	0.32	0.34	0.35	0.36	0.37	0.39	0.41	0.29
Three edges discontinuous (one short edge discontinuous)									
Continuous edge	—	—	—	—	—	—	—	—	0.45
Discontinuous edge	0.29	0.33	0.36	0.38	0.40	0.42	0.45	0.48	0.30
Four edges discontinuous									
Discontinuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33

Once the moments and shear forces have been calculated, based on the case type for the slab panel, the design of the section follows that for one way spanning slabs, irrespective of whether the slab is solid, ribbed or voided. Areas of reinforcement to resist the moments are determined independently for each direction of the span. **The reinforcement in the direction parallel to the shorter span should be placed in the lower layer to carry greater BM with greater lever arm**

Rules for Detailing

The rules to be observed when the equations are applied to restrained slabs (continuous or discontinuous) are as follows.

1) Slabs are considered as divided into two in each direction:
   a. The middle strips
   b. The edge strips

The middle strip being three-quarters of the width and each edge strip one-eighth of the width (Figure 7.15).

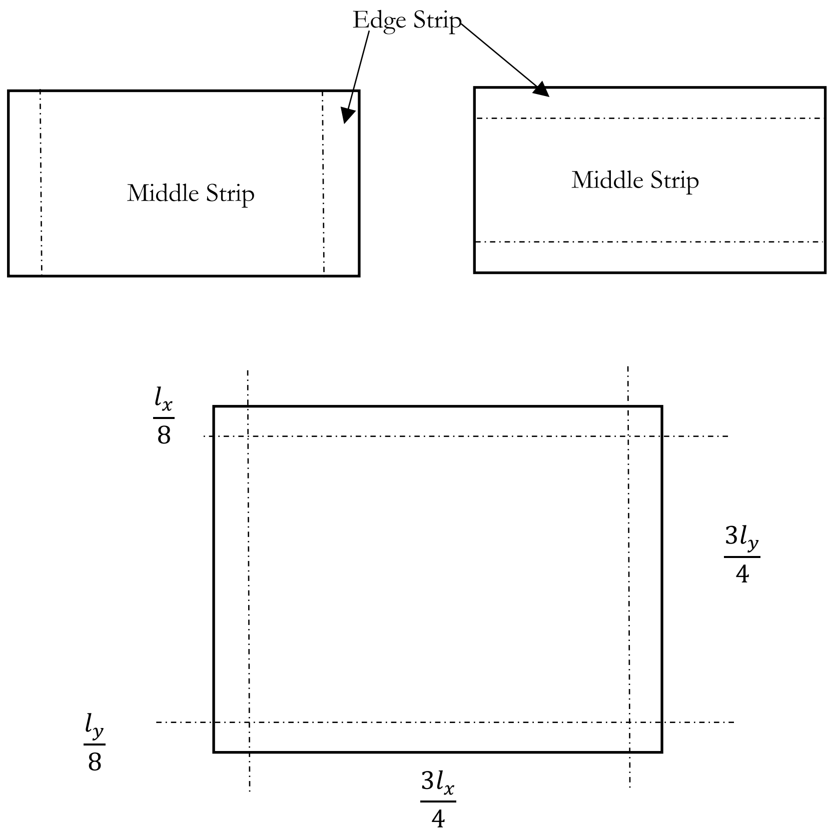


Figure 7.15: Division of a 2-way slab

2) The maximum design moments calculated as above apply only to the middle strips and no redistribution should be made.
3) The middle span reinforcement placed at the lower part of slab sections should extend to with 0.25l of a continuous edge AND 0.15l of a discontinuous edge

   In the first case, at least half of the reinforcement should extend to within 0.15l of a continuous edge and to within 50 mm of a discontinuous edge

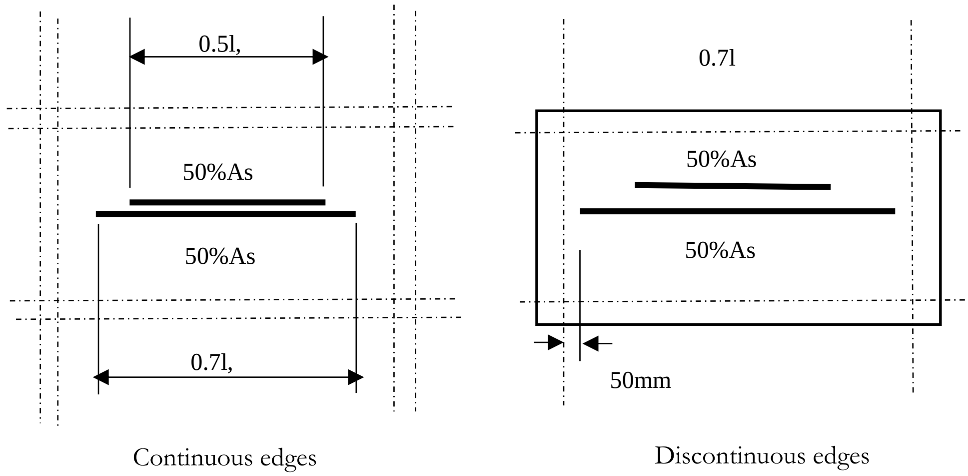


Figure 7.16: Reinforcement arrangement at continuous and discontinuous edges

4) Reinforcement calculated for negative moments over the continuous edge of middle strip should extend in the upper part of the slab section a distance 0.15l from the support AND at least 50% of this reinforcement a distance of 0.30l (Figure 7.17)

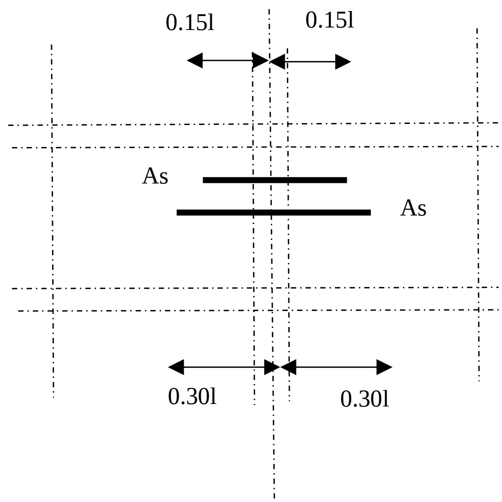


Figure 7.17

5) Because the negative reinforcement at the discontinuous edge depends on the degree of fixity at the support, therefore, upper reinforcement equal to 50% of the mid span reinforcement, extending 0.1l into the span (Figure 7.18)

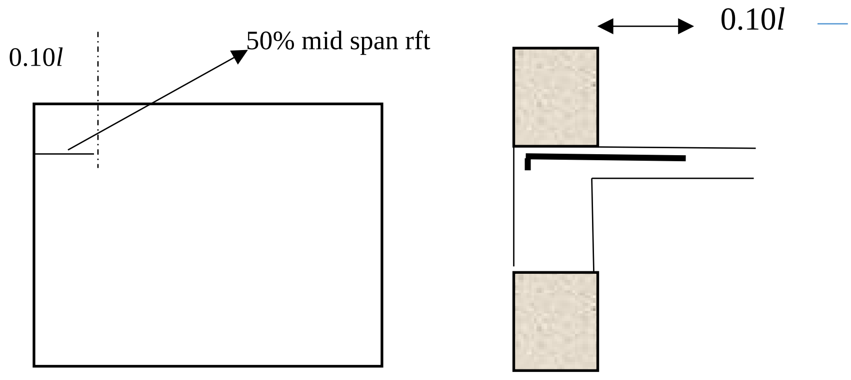


Figure 7.18

6) In the edge strip, reinforcement parallel to those edges should be at least equal to the minimum reinforcement. That is:

0.24% bh for mild steel (fy = 250 N/mm2)

0.13% bh for high yield steel (fy = 500 N/mm2)

7) At any corner when the slab is simply supported on both adjacent edges, the torsion reinforcement should be provided. It should consist of top and bottom RFT in both directions parallel to sides and extending 0.2 of the shorter span. The area of the RFT in each of the four-layers should be 0.75 of the mid span maximum reinforcement (figure 7.19).

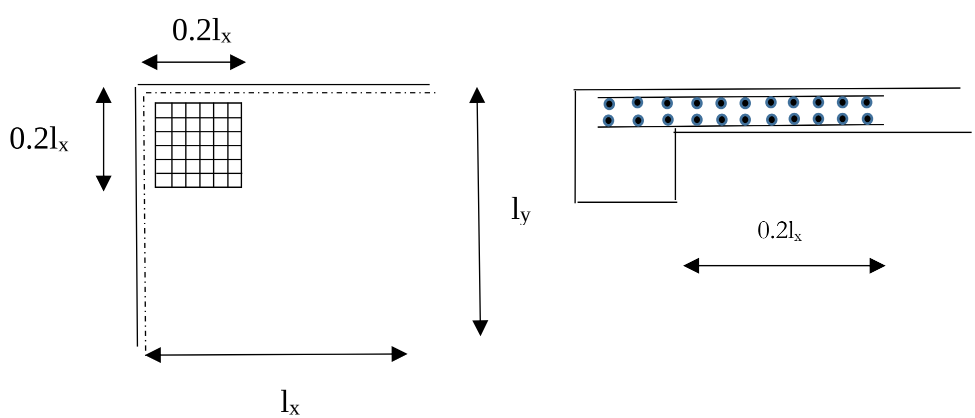


Figure 7.19: Reinforcement at corners

8) If only one edge is simply supported, half of the above discussed torsional reinforcement should be provided.

9) Torsion reinforcement need not be provided at any corner contained by edges over both of which the slab is continuous.

Example 7.4

Design the panel slab shown in the figure

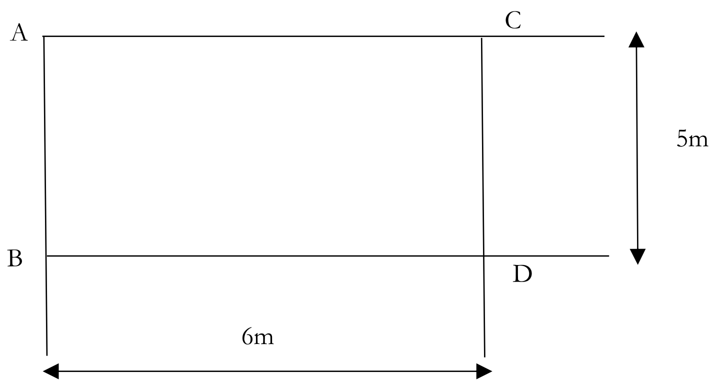


Figure 7.20: Example 7.4

The effective span in each direction 6.0 m and 5.0 m. if the dead load g_k is 3.0KN/m^2 and live load q_k is 3.5KN/m^2. Design the reinforcement for the slab. Take f_{cu} = 30N/mm^2, f_y = 250N/mm^2, condition of exposure = very severe., fire resistance = 2hrs, bar diameter = 16mm.

Solution

From Table 3.14, the slab is **CASE 4**, that is, two adjacent edges discontinuous.

The minimum span-depth ratio will be based on the shorter span; and treating it as continuous. That is,

$$\frac{l}{d} = 26$$

$$d = \frac{5000}{26} = 192.31\text{mm}$$

Minimum Cover from the conditions:

i. Condition of exposure (Table 3.3)

c = 50 mm

ii. Fire resistance (Table 3 .4)

c = 35

Thus,

c = 35 mm

Overall depth of the slab h is:

h = d +half bar diameter + cover

= 192.31 +8 + 50 = 250.31 mm

Use h = 300mm

Loading on the Slab

i. Self-weight = 2400 x h = 2400 x 0.30 = 7.2KN/m^2
ii. Dead Load = 3KN/m^2
iii. Live load – 3.5KN/m^2

Design Load (n) = 1.4gk + 1.6qk

= 1.4 (3 + 7.2) + 1.6 (3.5) = 14.28 + 5.6

= 18.2KN/m^2

= 19.88KN/m (per 1meter width)

Now

$\frac{l_y}{l_x} = \frac{6000}{5000}$ = 1.20

Calculation of Design Moments

From Table 3.14, moments coefficients are:

At the Short Span

i. Mid span moments = M_{sx} = $\beta_{sx} n l_x{}^2 =$ 0.047 x 19.88 x 5^2 =

= 23. 36KN.m

ii. Moments over the longer continuous edge = M_{sx} = $\beta_{sx} n l_x{}^2$ = 0.063 x 19.88 x 5^2

= 31. 311KN.m

At the long span

i. Mid span moment = M_{sy} = $\beta_{sy} n l_x{}^2$ = 0.034 x 19.88 x 5^2

= 16. 90KN.m

ii. Moment over the shorter continuous edge = M_{sy} = $\beta_{sy} n l_x{}^2 =$ 0.045 x 19.88 x 5^2

= 22. 37KN.m

Calculations of Reinforcement

Short Span

i. Mid Span (M = 23. 36KN.m)

Mu = 0.156fcubd^2 = 0.156 x 30 x 1000 x 250 x 250 = 292.50KNm

Since

Mu > M, no compression reinforcement is needed

Thus

$$K = \frac{M}{f_{cu}bd^2} = \frac{23.36 \ x \ 1000000}{30 \ x \ 1000 \ x \ 250 \ x \ 250} = 0.012$$

$$z = d[0.5 + \sqrt{(0}.25 - \frac{K}{0.9})]$$

= d[0.5 + √(0.25 – 0.013] = d[0.5 + 0.49] = 0.99d > 0.95d

Therefore use 0.95d

z = 0.95d = 0.95 x 250 = 237.50 mm

Thus

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{23.36 \ x \ 1000000}{0.87 \ x \ 250 \ x \ 237.5}$$

= 452.22mm²

Use R10@150 (524mm²)

→ **Labelled as R1-10**

Checks

Deflection

For the Short span,

M = 23.36KN.m

b = 1000.00mm

d = 250mm

As (required) = 452.22mm²

As (provided = 524.00 mm²

Therefore

$$\frac{M}{bd^2} = \frac{23.36 \ x \ 1000000}{1000 \ x \ 250 \ x \ 250} = 0.37$$

And equation 8 of Table 3.16 of BS 8110, the service stress f_s

$$f_s = \frac{2 f_y A_{s\ reuired}}{3_{As\ provided}} \text{ x } \frac{1}{\beta} \ (using \ \beta = 1)$$

$$= \frac{2 \ x \ 250 \ x \ 452.22}{3 \ x \ 524} \text{ x } \frac{1}{1} = 143.84 \text{ N/mm}^2.$$

From Table 3.10, the modification factor is about 1.99

The slab is simply supported. Thus, the span/effective depth ratio is 20. Therefore, by multiplying this value by the modification factor, the permissible span/depth ratio is obtained. That is:

$$\frac{permissible\ span}{depth} = 20 \text{ x } 1.99 = 39.80$$

But,

$$\frac{actual\ span}{depth} \text{ ratio} = \frac{5000}{200} = 25.0$$

Since the $\frac{permissible\ span}{depth}$ ratio (39.80) is greater than the $\frac{actual\ span}{depth}$ ratio (25.0).

The deflection is OK.

The new d = 300 – 35 – 5 = 260mm (this will result in even greater Mu)

ii. **Longer Continuous edge (M =** 31. 311KN.m)

Mu > M, no compression reinforcement is needed

Thus

K = $\frac{M}{f_{cu}bd^2}$ = $\frac{31.311\ x\ 1000000}{30\ x\ 1000\ x\ 260\ x\ 260}$ = 0.015

$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})]$

= d[0.5 + √(0.25 – 0.017] = d[0.5 + 0.48] = 0.98d > 0.95d

Therefore use 0.95d

z = 0.95d = 0.95 x 260 = 247.00 mm

Thus

As = $\frac{M}{0.87f_yz}$ = $\frac{31.311\ x\ 1000000}{0.87\ x\ 250\ x\ 247}$

= 582.83mm^2

Use R10@120 (654mm^2)

→ Labelled as **R2-10**

Longer Span

i. **Mid Span (M =** = 16. 90KN.m)

The effective depth = 300 – 35 – 10 – 5 = 250mm

Mu > M, no compression reinforcement is needed

Thus

K = $\frac{M}{f_{cu}bd^2}$ = $\frac{16.9\ x\ 1000000}{30\ x\ 1000\ x\ 250\ x\ 250}$ = 0.009

$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})]$

= d[0.5 + √(0.25 – 0.01] = d[0.5 + 0.49] = 0.99d > 0.95d

Therefore use 0.95d

z = 0.95d = 0.95 x 250 = 237.50 mm

Therefore,

As = $\frac{M}{0.87f_yz}$ = $\frac{16.9\ x\ 1000000}{0.87\ x\ 250\ x\ 237.5}$

= 327.16 mm^2

Use R10@200 (393mm^2)

→ Labelled as **R3-10**

ii. **Shorter Continuous edge (M** = 22. 37KN.m)

Mu > M, no compression reinforcement is needed

Thus

K = $\frac{M}{f_{cu}bd^2}$ = $\frac{22.37 \ x\ 1000000}{30\ x\ 1000\ x\ 250\ x\ 250}$ = 0.012

$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})]$

= d[0.5 + √(0.25 – 0.013] = d[0.5 + 0.49] = 0.99d > 0.95d

Therefore use 0.95d

z = 0.95d = 0.95 x 250 = 237.50 mm

Thus

As = $\frac{M}{0.87f_yz}$ **=** $\frac{22.37\ x\ 1000000}{0.87\ x\ 250\ x\ 237.5}$

= 433.06mm^2

Use R10@160 (491mm^2)

→ Labelled as **R4-10**

Other Design Considerations

i. **At the discontinuous edge**

As = 50% x maximum mid span reinforcement

= 0.5 x 524

= 262 mm^2

Use R10@250 (314mm^2)

→ Label as **R5-10**

Distribution, use miminum reinforcement

As = 0.24% bh for mild steel

= $\frac{0.24}{100}$ x 1000 x 300 = 720mm^2

= R10@100 (785mm^2)

→ Labelled as **R6-10**

ii. **The reinforcement (As) for the edge strip**

As = 0.24% bh for mild steel

= $\frac{0.24}{100}$ x 1000 x 300 = 720mm^2

= R10@100 (785mm^2)

→ Labelled as **R7-10**

The widest edge strip is $\frac{6000}{8}$ = 750mm wide

iii. **Torsion reinforcement for corners**

For corner A, with two (2) discontinuous sides, 4 layers (Top and bottom) are needed. For each layer, reinforcement As= $\frac{3}{4}$ x mid span maximum reinforcement. That is

As = 0.75 x 524

As = 393mm^2

Use R10@180mm (436mm^2) for each layer)

→ Labelled as **R8-10**

For corner B and C with one discontinuous edge, half of the above is needed. That is:

As = $\frac{3}{8}$ x 524

As = 196.5mm^2

Use R10@300mm (262mm^2)

→ Labelled as **R9-10**

Note

The length of each bar is 0.2 of the shorter length. That is 0.2 x 5 = 1m in each direction

Shear Design

From Table 3.25, the highest coefficient is the support at the continuous edge = 0.47

Design shear $V_{sx} = \beta_{vx} n l_x$ = 0.47 x 19.88 x 5 = 46.72KN

Thus,

Shear Stress, v = $\frac{V}{bd} = \frac{46.72 \ x \ 1000}{1000 \ x \ 250}$ = 0.19N/mm^2

From Table 3.8 (BS 8110)

$\frac{100As}{bd} = \frac{100 \ x \ 524}{1000 \ x \ 250}$ = 0.21

v_c = 0.40

Adjustment for f_{cu} = 30N/mm^2

v_c =0.40 x $(\frac{30}{25})^{0.333}$

= 0.43N/mm^2

Since v_c > v, no shear reinforcement is required

Other Checks

i. Spacing requirements **(clause 3.12.11.2.7, BS 8110).**

The limiting spacing is: h_{agg} + 5mm or bar diameter < **spacing** < 3d or 750 mm

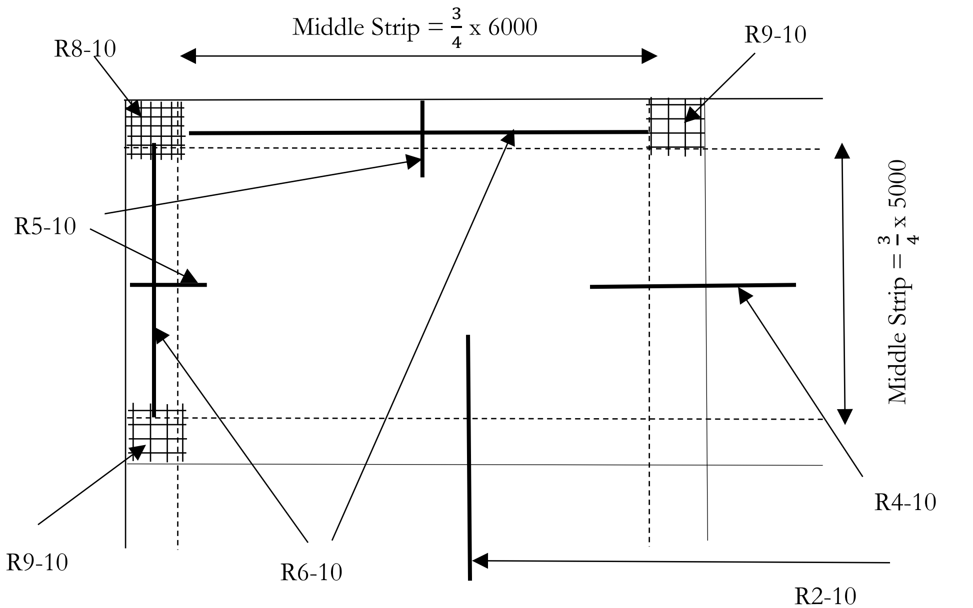
The highest spacing is 300 mm < 3 x 250 = 750

Spacing is OK

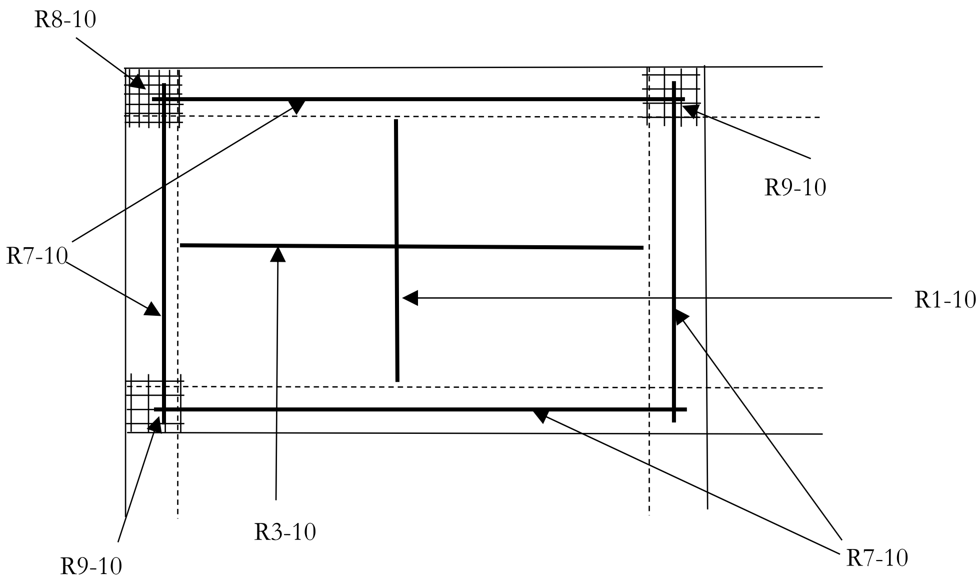
e. Crack width **(clause 3.12.11.2.7, BS 8110)**

The percentage of reinforcement provided ($\frac{100As}{bd}$) is < 0.3%. No check is needed.

The sketch of the reinforcement arrangement (without the rules of curtailment for clarity) is shown in Figure 7.21.



(a) Reinforcement arrangement in the Top of Slab



(b) Reinforcement arrangement in the Bottom of Slab

Figure 7.21: Sketch of reinforcement arrangement in both Top and Bottom of the slab

Chapter 8 - Design of Reinforced Concrete Stairs

8.1 Introduction

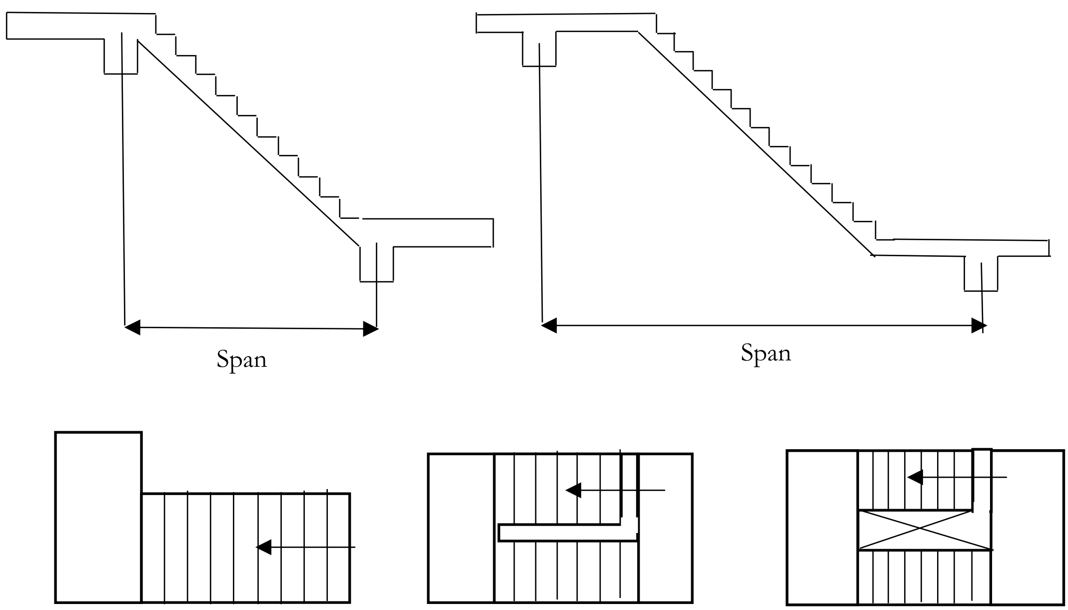
Stairs can be defined as structural member that enable vertical movement or translation between floors in multi-storey houses. It can also be used between places of different topographical elevations, or as part of pedestrian bridges, etc. Stairs can be divided into two:

1. Common stairs
2. Special Type of Stairs

8.2. Common Stairs

i. Supported on Beams

This is the simplest of RC stairway. It consists of an inclined slab supported at ends on beams (Figure 8.1). Such a slab with stairs is usually designed as a simple slab with the span equal to the horizontal distance between support.



a **b** **c**

Figure 8.1: Stairs supported on Beams

The slab may be simply supported, or also be considered as interior panel of a continuous beam. This will depend on the stiffness of the adjacent parts of the structure between floors and landings. Whether considered as simply supported or otherwise, main reinforcement should be placed only in the direction of the length of the flight. Transverse reinforcement, usually one bar in each step is used as secondary reinforcement and to assist in the distribution of loads, to protect against temperature shrinkage. The effective depth of the slab section is calculated according to the thickness of the waist. If the stair is between two floors, it is called straight flight stairs (a). If there is an intermediate landing (known as half landing) between the two floors, then stair is called half-turn (180^0) stair (b). If there are three flights between two floors, with two intervening landing, such stairs are called quarter-turn stair (c).

ii. Beamless Stairs

This means stairs without longitudinal or transverse beams (Figure 8.2). The slab in beamless stairs is usually considered as simply supported on walls or elastically restrained when supported on beams in skeletal structure

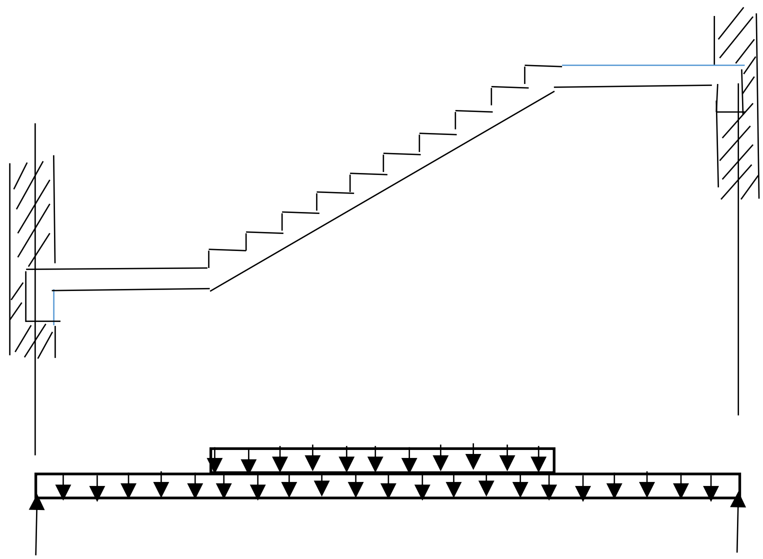


Figure 8.2: Beamless Stair

iii. Stairs Supported on both sides

This type of concrete stairs is used when steps are supported on both sides. It may be supported by wall on one side and by beam on the other side (Figure 8.3).

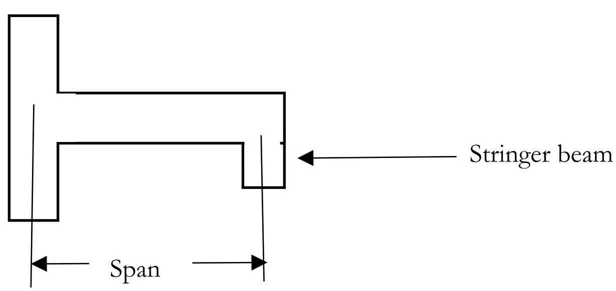


Figure 8.3: Stair supported on both sides

iv. Cantilever Stairs

It may also be cantilevered from the supporting wall. If cantilevered from the wall, each step is working as cantilever jutting out of the wall perpendicularly. This is shown in figure 8.4. Cantilever stairs are sometimes designed as precast consisting of separate steps.

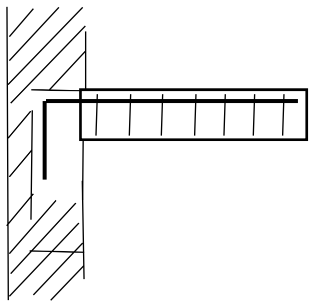


Figure 8.4: Cantilever Stair

v. Stairs on String Boards

In this type of stairs, slab is supported on two inclined beams or wall ties at both sides. The slab then acts transversely as a simple beam. The longitudinal beams now transmit the loads to upper and lower supports.

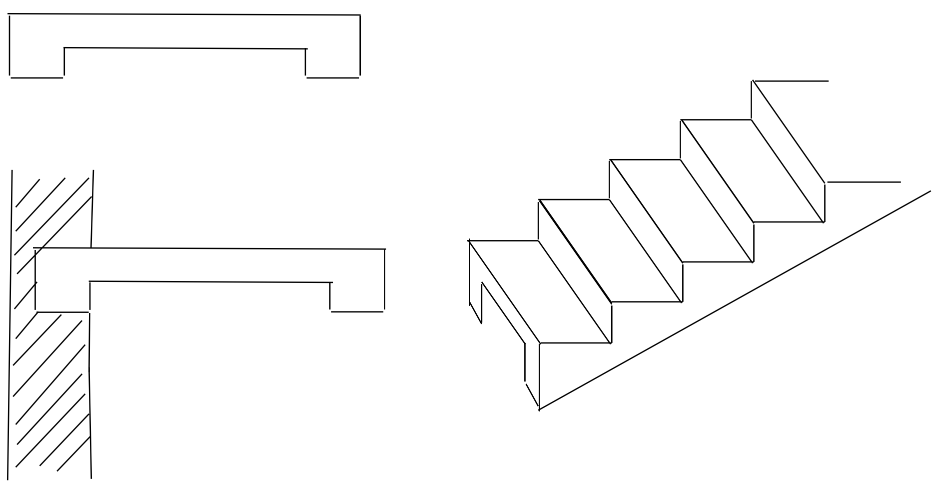


Figure 8.5: Stair on String Boards

8.3 Special Types of Stairs

i. **Open Skeleton Staircase**

Open Skeleton Staircase – simple staircase without linear support. The very clear structure is difficult for exact analysis. Usually, hidden beams are provided for the stairs and landings.

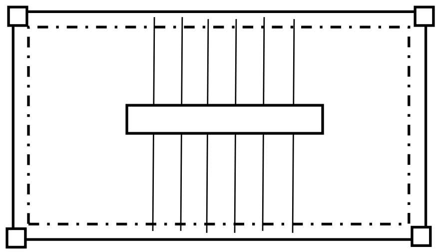


Figure 8.6

ii. **One Wall Supported Staircase**

The Flight and landing acts as cantilever

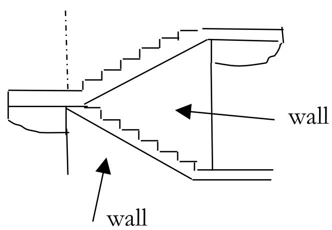


Figure 8.8

iii. **External Cantilever Staircase**

Load from one acts as point load on the other. There is torsion to be considered.

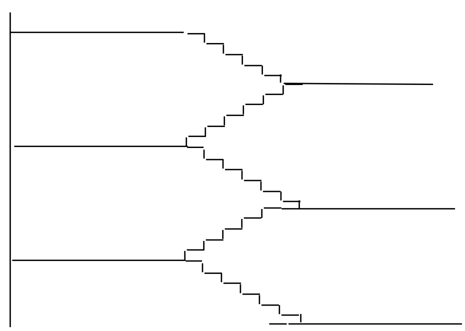


Figure 8.7: External Cantilever Staircase

iv. Spiral

In spiral stairs, the stairs are steps are arranged to from a spiral around a central column. Each of the steps is designed as simple cantilever, and the steps are usually tapered, the free end being wider. The steps are usually precast

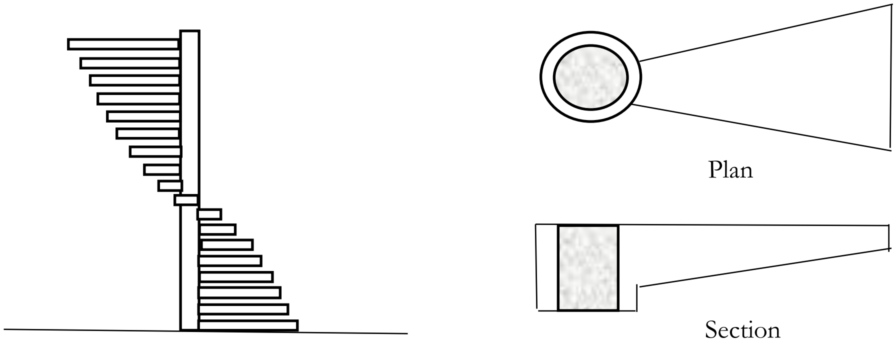


Figure 8.9: Spiral Stair

v. Helical Staircase

Helical stairs look like a helix from floor to floor. It usually occupies less space than straight flight stairs. It usually a straight flight stair but turns as it rises. In this type of stair, torsion at the end of the ends of the stair must be checked to ensure its adequacy.

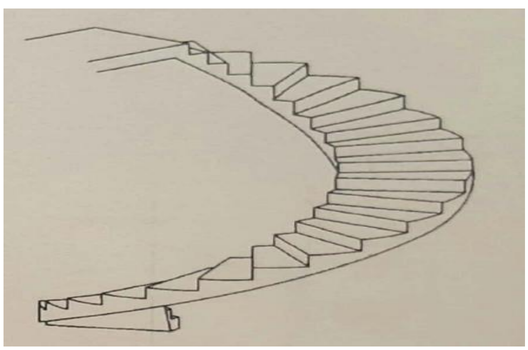


Figure 8.10: Helical Stair

8.4 General Functional Requirements for Stairs

In order to obtain a good stair design, combining economy with comfort of usage, the followings are guides. Some of the main technical terms used in stairs are shown in Figure 8.11.

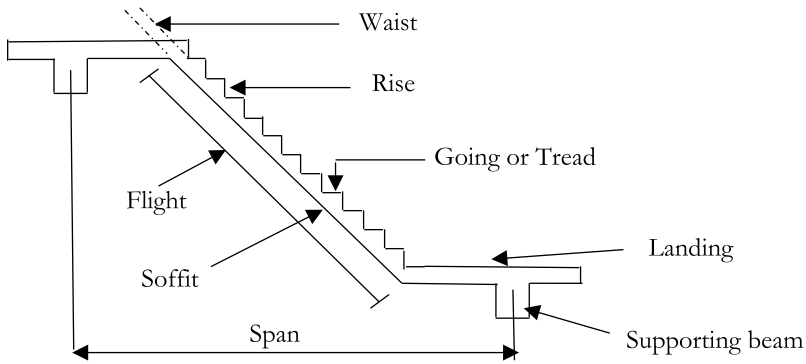


Figure 8.11: Functional Terms Used in Stairs

Definitions

i. Going or Tread = horizontal distance between the faces of two consecutive risers
ii. Rise = the vertical distance between the tops of two consecutive going
iii. Riser = vertical portion of a step
iv. Nosing = the intersection of the tread and the riser
v. Landing = a horizontal slab provided between two flights.
vi. Waist = the least thickness of a stair slab.
vii. Flight = a series of steps provided between two landings
viii. Supporting beam =
ix. Soffit = the bottom surface of a stair slab.
x. Headroom: the vertical distance from a line connecting the nosings of all treads and the soffit above.

The followings are recommendations for safe and user-friendly stairs.

i. No single stair should have varied goings or varied rises, because this may lead to accident during usage.
ii. Goings and rises should have their dimensions constant throughout each flight or the entire stair.
iii. The minimum dimension of a rise should be 150 mm. rise should not be more than 195 mm.
iv. The dimension of goings should be between 225 mm and 250 mm maximum.
v. For comfort, it has been found out that the best relationship (κ) between goings and rises should be:

κ = Goings + 2 x Risers = between 580 and 600 mm

For example, a step of 300mm goings and 150 mm rise, then

$\kappa = 300 + 2 \times 150 = 600$mm

vi. The minimum vertical headroom above any step should be 2 m.

vii. The imposed loading is assumed to act vertically on the plan area of the staircase and should be at least equal to the imposed load on the floor to which it gives access.

8.5 Design of Stairs

The design of stairs is dealt with in section 3.10 of BS 8110. Although the general conditions that are applicable to beams and slabs, the followings should however be noted.

i. The design loads should be obtained from the ultimate combinations as specified in Clause 3.2.1.2.2.

ii. The design ultimate load should be assumed to be uniformly distributed over the plan area of a staircase. But when staircases surrounding open wells include two spans that intersect at right angles, the load on the areas common to both spans may be assumed to be divided equally between the two spans

iii. The effective span of simply-supported staircases without stringer beams should be taken as the horizontal distance between the centre-lines of the supports or the clear distance between the faces of supports plus the effective depth, whichever is the lesser

iv. Strength, deflection and crack control recommendations for beams and slabs given in **3.4** and **3.5, also apply** except for the span/depth ratio of a staircase without stringer beams where **3.10.2.2** applies.

v. For staircases without stringer beams, permissible span/effective depth ratio can be increased by 15%.

For the purpose of design, stairs are classified as (i) Stairs spanning Longitudinally, and (ii) Stairs spanning horizontally

a. Stairs spanning Longitudinally (in the direction of movement)

These stairs span between supports at the top and bottom of a flight and unsupported at the sides. Stairs

spanning longitudinally supported stairs may be supported in any of the following manners:

i. Beams or walls at the outside edges of the landings.

ii. Internal beams at the ends of the flight in addition to beams or walls at the outside edges of the landings.

iii. Landings which are supported by beams or walls running in the longitudinal direction.

iv. A combination of (i) or (ii), and (iii).

v. Stairs with quarter landings associated with open-well stairs.

Longitudinal stairs are designed like slab with the waist as the depth (h). Since the horizontal distance will be used, the loads must be converted by multiplying all inclined loads (self-weight of waist, finishes, etc) by a factor α, given as:

$$\alpha = \frac{\sqrt{(R^2 + T^2)}}{T}$$

Where R = the rise of the step, T = goings of the step

The simplest type of stairs is the straight flight. Its effective span however depends on the supporting members as shown in Figure 8. 12.

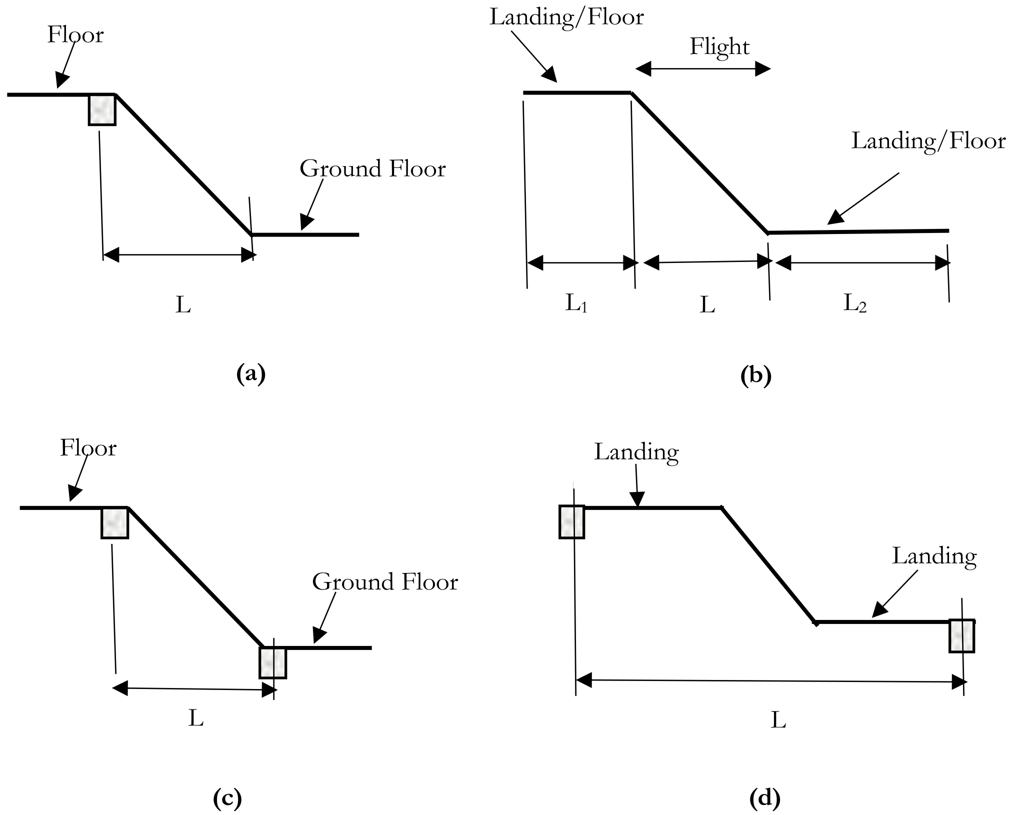


Figure 8: 12: Typical Effective depth for some types of Stairs

NOTE:

For cases (a), (c), and (d), the Span = L; and the bending moment = 0.125wl²

For case (b), Span = L + 0.5(L_1 + L_2); and the bending moment = 0.10wl²

b. Stairs spanning horizontally

This involves the design of each individual steps. This is common in stairs in the following instances.

i. Simply supported steps supported by two walls or beams or a combination of both.

ii. Steps cantilevering from a wall or a beam.

iii. Stairs cantilevering from a central spine beam.

For stairs designed to act horizontally, each tread can be treated as a separate rectangular beam of width (b), as shown in Figure 8.13.

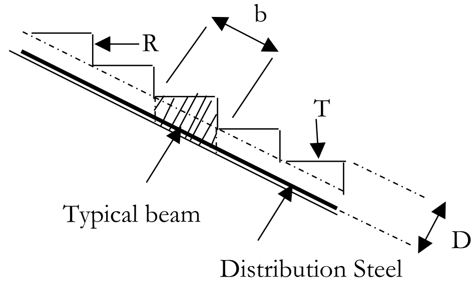


Figure 8.13: Rectangular beam for horizontal stairs design

Where

$$b = \sqrt{(R^2 + T^2)}$$

and the effective depth $d = \frac{D}{2}$

where

$$D = \text{waist} + \frac{RT}{b}$$

Example 8. 1

A straight flight stair, spanning between two beams (500 x 300 mm in cross section), is to be designed for a residence. The stair is to have 15 treads. If the going is 250 mm and the rise is 150 mm for each step, design the stairs. Use the followings data: f_{cu} = 25N/mm², fy = 250N/mm², qk = 1.5KN/m², finishes = 1.0KN/m², Concrete density = 24KN/m³, exposure = mild.

Solution

The stair is shown in Figure 8.14

Total length of goings = 15 x 250 = 3.75m

Span L = 3.75 + 0.5 (0.5) + 0.5 (0.3) = 4.05 m

Using the modification factor of 1.25, then the effective depth of waist can be obtained from

$$d = \frac{l}{20\ x\ mf} = \frac{4050}{20\ x\ 1.25} = 168.65 \text{ mm}$$

Thus, making allowance for cover, take the depth of the waist to be:

h = 200 mm

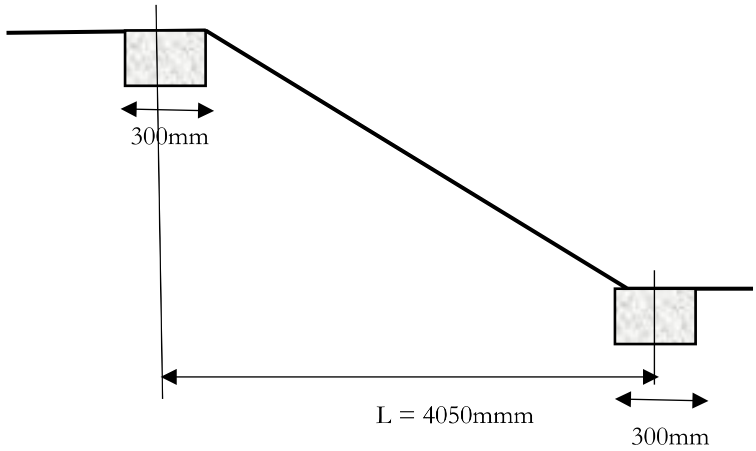


Figure 8.14: Example 8.1

Loadings

Self-weight of waist = 0.2 x 24 = 4.8KN/m^2

Weight of stairs = (0.5 x 0.15) x 24 = 1.8KN/m^2

Finishes = 1.00 KN/m^2

Live load = 1.5 KN/m^2

The Slope factor

$$\alpha = \frac{\sqrt{(R^2 + T^2)}}{T} = \frac{\sqrt{(150^2 + 250^2)}}{250^2} = 1.17$$

The design load = 1.4gk + 1.6qk

= 1.4 ([4.8 + 1.0] x 1.17 + 1.8) + 1.6 (1.5) = 12.02 + 2.4 = 14.42KN/m^2

= 14.42KN/m (per m width of slab)

Note

i. The self-weight of the stair is calculated by treating the step to be equivalent slab of thickness equal to half the rise (that is $\frac{R}{2}$)
ii. The shape factor applies only to the self-weight of the waist and the finishes)

The Bending Moment M = 0.125wl^2

= 0.125 x 14.42 x 4.05 x 4.05 = 29. 57KN.m

Now,

b = 1000mm (considering 1-meter width)

d = 200 – 20 – 6 = 174 mm (cover = 20mm for mild exposure, and assuming bar diameter of 12mm)

Moment of the section, Mu

Mu = 0.156fcubd^2

= 0.156 x 25 x 1000 x 174 x 174 = 118.08 KN.m

Since Mu > M, no compression reinforcement is required

$K = \frac{M}{f_{cu}bd^2} = \frac{29.57 \ x\ 1000000}{25\ x\ 1000\ x\ 174 x\ 174} = 0.04$

$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})] = 0.96d$

= d [0.5 + √(0.25 – 0.04] = d [0.5 + 0.46] = >0.96d

= 0.95d

z = 0.95d = 0.95 x 174 = 165.30 mm

Thus

$\mathbf{As} = \frac{M}{0.87 f_y z} = \frac{29.57\ x\ 1000000}{0.87\ x\ 250\ x\ 165.30}$

= 822.47mm2

Use R12@125 (905mm^2)

Check deflection

$\frac{M}{bd^2} = \frac{29.57\ x\ 1000000}{1000\ x\ 174\ x\ 174} = 0.98$

The service stresses

$fc = \frac{2\ x\ 250\ x\ 822.47}{3\ x\ 905} = 151.\ 47 N/mm^2$

From Table 3.10, modification factor = 1.98

Thus

$\frac{permissible\ span}{effective\ depth} = 20\ x\ 1.98 = 39.60$

But

$\frac{actual\ span}{effective\ depth} = \frac{4050}{174} = 23.28$

Since $\frac{permissible\ span}{effective\ depth} > \frac{actual\ span}{effective\ depth}$, the deflection is OK

Secondary reinforcement

$As = \frac{0.24}{100}\ x\ 1000\ x\ 200 = 480\ mm^2$

Use R10@150 (523mm^2)

Continuity bar at the top and bottom should be provided to prevent formation of cracks. This is usually taken to be 50% of main reinforcement, but subject to the maximum spacing requirement of clause 3.12.11.2.7 (BS 8110). 50% of main reinforcement = 0.5 x 905 = 402.5mm^2). Therefore, provide R10@175 (449mm^2). The sketch of reinforcement arrangement is shown in figure 8.15.

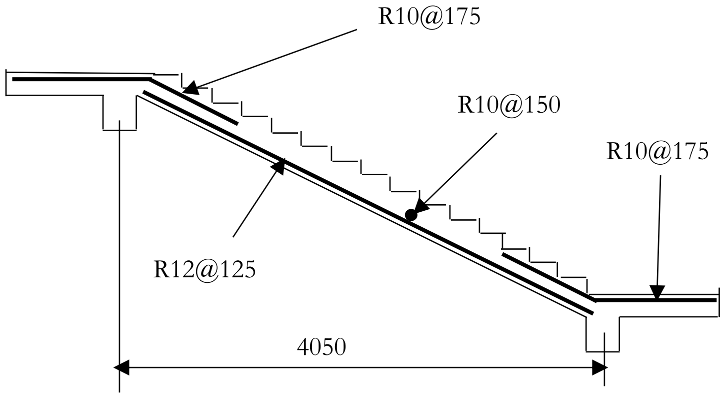


Figure 8.15: Sketch of arrangement of reinforcement

Now when straight flights are long, and there is problem satisfying the deflection criterium of the Code, it is advisable to design such flight in either of the following ways:

i. Each step of the stair is designed as spanning horizontally between two supporting walls or beams or a combination of both. The step is assumed to be simply supported, where span is the width of the stair, and bending moment is $0.125wl^2$

ii. Each Step is treated as cantilevering rectangular beam from a wall or a beam, spanning between the beams or walls

Example 8.2

A stair, is supported by a wall on one side, and by a stringer beam on the other side. The stair is to have 15 treads. If the going is 250 mm and the rise is 150 mm for each step, and the width of the stair is 1.5 m, design the stairs. Use the followings data: f_{cu} = 25N/mm², fy = 250N/mm², qk = 1.5KN/m², finishes = 1.0KN/m², Concrete density = 24KN/m³, exposure = mild.

Solution

Each step of the stair will be designed as a beam spanning horizontally between a wall and stringer beam.

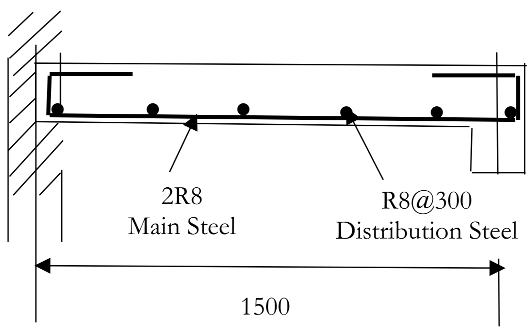


Figure 8.16: Example 8.2

b = $\sqrt{(R^2 + T^2)}$ = $\sqrt{(150^2 + 250^2)}$ = 291.55mm

take b = 300 mm

taking the waist = 80mm

D = waist + $\frac{RT}{b}$ = 80 + $\frac{150 \times 250}{300}$ = 205 mm

Effective depth d = $\frac{D}{2}$ = 102.5mm

Loadings

Waist = 0.08 x 0.292 x 24 = 0.56KN/m

Steps = 0.5 x (0.15 x 0.25) x 24 = 0.45KN/m

Finishes = 1.0 x 0.25 = 0.25KN/m

Live load = 1.5 x 0.25 = 0.38KN/m

The design Load = 1.4gk + 1.6qk

= 1.4 (0.56 + 0.45 + 0.25) + 1.6 (0.38) = 1.764 + 0.608 = 2.372KN/m

Moment = $0.125wl^2$ = 0.125 x 2.372 x 1.5 x 1.5 = 0. 67KN.m

Moment of the section, Mu

Mu = $0.156f_{cu}bd^2$

= 0.156 x 25 x 300 x 102.5 x 102.5 = 12. 292KN.m

Since Mu > M, no compression reinforcement is required

K = $\frac{M}{f_{cu}bd^2}$ = $\frac{0.67 \times 1000000}{25 \times 300 \times 102.5 \times 102.5}$ = 0.009

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

= d[0.5 + √(0.25 – 0.01] = d[0.5 + 0.47] = 0.99d

= 0.95d

z = 0.95d = 0.95 x 102.5 = 97.38 mm

Thus

As = $\frac{M}{0.87f_y z}$ **=** $\frac{0.67 \times 1000000}{0.87 \times 250 \times 97.38}$

= 31.63mm²

Use 2R8 (101mm²) per step

Provide distribution reinforcement = $\frac{0.24}{100}$bh = 0.24 x 300 x 80 = 57.6mm²

Use R8@300 (168mm²)

The reinforcement arrangement is as shown Figure 8.16

Example 8. 3

Design the stairs shown in Example 8.1 as half-turn stairs, consisting of 7 steps in the first flight and 8 steps in the second flight. The going is 250 mm and the rise is 150 mm for each step. The data is as in Example 8.1: f_{cu} = 25N/mm², fy = 250N/mm², qk = 1.5KN/m², finishes = 1.0KN/m², Concrete density = 24KN/m³, exposure = mild. Assume the walls to be 300mm thick.

Solution

The diagram of the stairs is as shown in figure 8.17.

Span $= \frac{L1}{2} + L_2 = \frac{2}{2} + 0.25 \text{ x } 7 = 2.75\text{m}$

Using the modification factor of 1.25, then the minimum effective depth of waist can be obtained from

$$d = \frac{l}{20 \, x \, mf} = \frac{2750}{20 \, x \, 1.25} = 110.00 \text{ mm}$$

For mild condition of exposures, cover = 20 mm, and using 12mm bar diameter.

Then

h = 110 + 20 + 6 = 136 mm

Take

h = 150mm (so that d = 124mm)

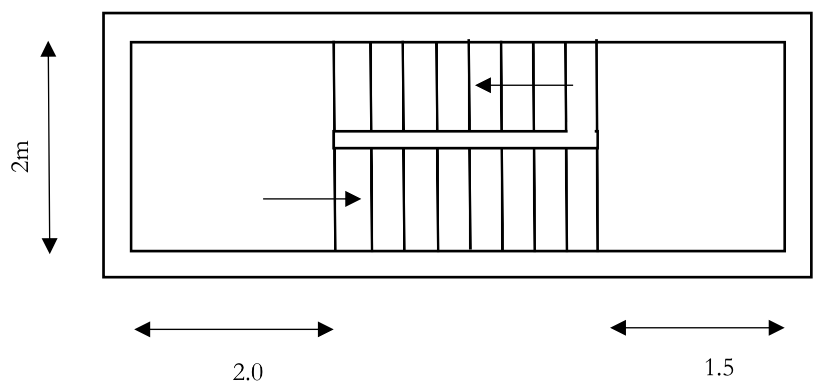


Figure 8.17: Example 8. 3

Loading (as for Example 8.1) = 14.42KN/m

For the flight, the ultimate design moment M

$M = 0.125wl^2 = 0.125 \text{ x } 14.42 \text{ x } 2.75 \text{ x } 2.75 = 13.63\text{KN.m}$

$Mu = 0.156f_{cu}bd^2 = 0.156 \text{ x } 25 \text{ x } 1000 \text{ x } 124 \text{ x } 124 = 59.97\text{KN.m}$

The First flight (Figure 8.18)

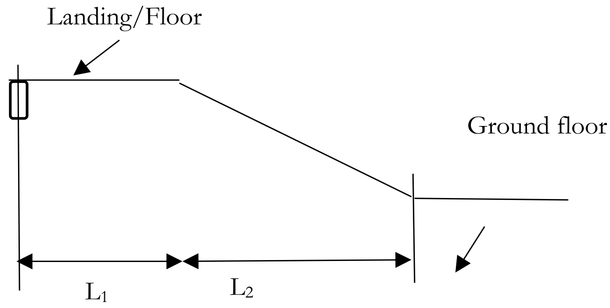


Figure 8.18: First flight

Mu > M, no compression reinforcement is needed.

Therefore

$$K = \frac{M}{f_{cu}bd^2} = \frac{13.63 \ x \ 1000000}{25 \ x \ 1000 \ x \ 124 \ x \ 124} = 0.04$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

$= d[0.5 + \sqrt{(0.25 – 0.04]} = d[0.5 + 0.46] = 0.96d$

$= 0.95d$

$z = 0.95d = 0.95 \ x \ 124 = 117.80$ mm

Thus

$$\mathbf{As} = \frac{M}{0.87f_yz} = \frac{13.63 \ x \ 1000000}{0.87 \ x \ 250 \ x \ 117.80}$$

$= 531.98mm^2$

Use R12@200 (566mm²) [1-R12@200]

Check deflection

$$\frac{M}{bd^2} = \frac{13.63 \ x \ 1000000}{1000 \ x \ 124 \ x \ 124} = 0.89$$

The service stresses

$$fc = \frac{2 \ x \ 250 \ x \ 531.98}{3 \ x \ 566} = 156.\ 65N/mm^2$$

From Table 3.10, modification factor = 1.94

Thus

$$\frac{permissible\ span}{effective\ depth} = 20 \ x \ 1.94 = 38.80$$

But

$$\frac{actual\ span}{effective\ depth} = \frac{2175}{124} = 17.54$$

Since $\frac{permissible\ span}{effective\ depth} > \frac{actual\ span}{effective\ depth}$, the deflection is OK

Secondary reinforcement

$$As = \frac{0.24}{100} \text{ x } 1000 \text{ x } 150 = 360 \text{ mm}^2$$

Use R10@200 (393mm^2) [2-R10@200]

The Second flight (Figure 8.19)

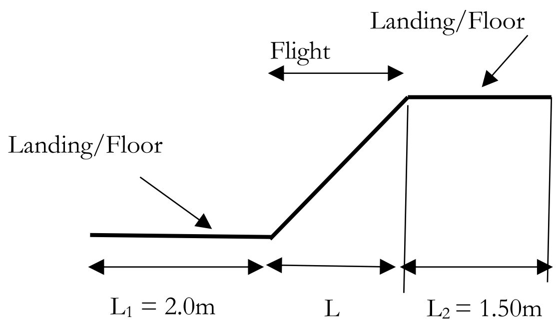


Figure 8.19: Second flight

Span = L + 0.5(L_1 + L_2) = 8 x 0.25 + 0.5 (2.0 + 1.5) = 3.75m

The Loadings

The loading as in First Flight = 14.42KN/m

The ultimate design moment M

M = 0.10wl^2 = 0.1 x 14.42 x 3.75 x 3.75 = 20. 28KN.m

M < Mu, thus no need for compressive reinforcement

Therefore

$$K = \frac{M}{f_{cu}bd^2} = \frac{20.28 \ x\ 1000000}{25\ x\ 1000\ x\ 124\ x\ 124} = 0.053$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

$$= d[0.5 + \sqrt{(0.25 - 0.06]} = d[0.5 + 0.44] = 0.94d$$

z = 0.94d = 0.94 x 124 = 116.56 mm

Thus

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{20.28\ x\ 1000000}{0.87\ x\ 250\ x\ 116.56}$$

= 799.94mm^2

Use R12@140 (808mm^2) [3-R12@140]

Check deflection

$$\frac{M}{bd^2} = \frac{20.28\ x\ 1000000}{1000\ x\ 124\ x\ 124} = 1.32$$

The service stress

$$fc = \frac{2\ x\ 250\ x\ 799.94}{3\ x\ 808} = 165.00\text{N/mm}^2$$

From Table 3.10, modification factor = 1.73

Thus $\frac{permissible\ span}{effective\ depth}$ = 20 x 1.73 = 34.60

But

$$\frac{actual\ span}{effective\ depth} = \frac{3750}{124} = 30.25$$

Since $\frac{permissible\ span}{effective\ depth} > \frac{actual\ span}{effective\ depth}$, the deflection is OK

Secondary reinforcement

$$As = \frac{0.24}{100}\ x\ 1000\ x\ 150 = 360\ \text{mm}^2$$

Use R10@200 (393mm^2) [2-R10@200]

Half Landing

Span = 2 + $\frac{0.300}{2}$ + $\frac{0.300}{2}$ = 2.3m

Retaining h = 150mm, and d = 124mm as for the flights

Loadings

Self-weight = 0.15 x 2.0 (width) x 24 = 7.2KN/m

Finishes = 1 x 2 = 2KN/m

Live Load =1.5 x 2 = 3KN/m

Design Load = contributions from landing + contribution from the flight

= 1.4 (7.2 + 2) + 1.6 (3) + 14.42 x $\frac{7\ x\ 0.25}{2}$ = 12.88 + 4.8 + 12.62

(Note that flight load is already factored, and half of it is assumed to be taken by the landing)

= 30.30KN/m

Design Moment M

= 0.125wl^2 = 0.125 x 30.30 x 2.3 x 2.3 = 20. 04KN.m

Mu = 59. 97KN.m (as calculated for flight 1) > M, therefore no compression reinforcement is required.

$$K = \frac{M}{f_{cu}bd^2} = \frac{20.04\ x\ 1000000}{25\ x\ 1000\ x\ 124\ x\ 124} = 0.052$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})]$$

$= d[0.5 + \sqrt{}(0.25 – 0.06] \quad = d[0.5 + 0.44] = 0.94d$

z = 0.94d = 0.94 x 124 = 116.56 mm

Thus

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{20.28 \ x \ 1000000}{0.87 \ x \ 250 \ x \ 116.56}$$

= 799.94mm^2

Use R12@140 (808mm^2) [4-R12@140]

Check deflection

$$\frac{M}{bd^2} = \frac{20.04 \ x \ 1000000}{1000 \ x \ 124 \ x \ 124} = 1.30$$

The service stresses

$$fc = \frac{2 \ x \ 250 \ x \ 799.94}{3 \ x \ 808} = 165.00 N/mm^2$$

From Table 3.10, modification factor = 1.73

Thus

$$\frac{permissible\ span}{effective\ depth} = 20 \ x \ 1.72 = 34.40$$

But

$$\frac{actual\ span}{effective\ depth} = \frac{2300}{124} = 18.55$$

Since $\frac{permissible\ span}{effective\ depth} > \frac{actual\ span}{effective\ depth}$, the deflection is OK

Secondary reinforcement

$$As = \frac{0.24}{100} \ x \ 1000 \ x \ 150 = 360 \ mm^2$$

Use R10@200 (393mm^2)

Final landing

Span = $1.5 + \frac{0.300}{2} + \frac{0.300}{2} = 1.8m$

Retaining the value of h = 150mm, and d = 124mm as for the flights

Loadings

Self-weight = 0.15 x 2.0 (width) x 24 = 7.2KN/m

Finishes = 1 x 2 = 2KN/m

Live Load =1.5 x 2 = 3KN/m

Design Load = contributions from landing + contribution from the flight

$$= 1.4 (7.2 + 2) + 1.6 (3) + 14.42 \ x \ \frac{8 \ x \ 0.25}{2} = 12.88 + 4.8 + 14.42$$

(Note that flight load is already factored, and half of it is assumed to be taken by the landing)

= 32.10KN/m

Design Moment M

= 0.125wl² = 0.125 x 32.10 x 1.8 x 1.8 = 13. 00KN.m

Mu = 59. 97KN.m (as calculated for flight 1) > M, therefore no compression reinforcement is required.

$$K = \frac{M}{f_{cu}bd^2} = \frac{13.00 \ x \ 1000000}{25 \ x \ 1000 \ x \ 124 \ x \ 124} = 0.034$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

= d[0.5 + √(0.25 – 0.04] = d[0.5 + 0.47] = 0.97d

z = 0.95d = 0.94 x 124 = 117.80 mm

Thus

$$\mathbf{As} = \frac{M}{0.87f_yz} = \frac{13.00 \ x \ 1000000}{0.87 \ x \ 250 \ x \ 117.80}$$

= 507.34mm²

Use R12@200 (565mm²) [5-R12@200]

Check deflection

$$\frac{M}{bd^2} = \frac{13 \ x \ 1000000}{1000 \ x \ 124 \ x \ 124} = 0.85$$

The service stresses

$$fc = \frac{2 \ x \ 250 \ x \ 507.34}{3 \ x \ 565} = 149.65 N/mm^2$$

From Table 3.10, modification factor = 1.99

Thus

$$\frac{permissible \ span}{effective \ depth} = 20 \ x \ 1.99 = 39.80$$

But

$$\frac{actual \ span}{effective \ depth} = \frac{1800}{124} = 14.52$$

Since $\frac{permissible \ span}{effective \ depth} > \frac{actual \ span}{effective \ depth}$, the deflection is OK

Secondary reinforcement

$$As = \frac{0.24}{100} \ x \ 1000 \ x \ 150 = 360 \ mm^2$$

Use R10@200 (393mm²) [2-R10@200]

The of reinforcement arrangement is shown in Figure 8.20.

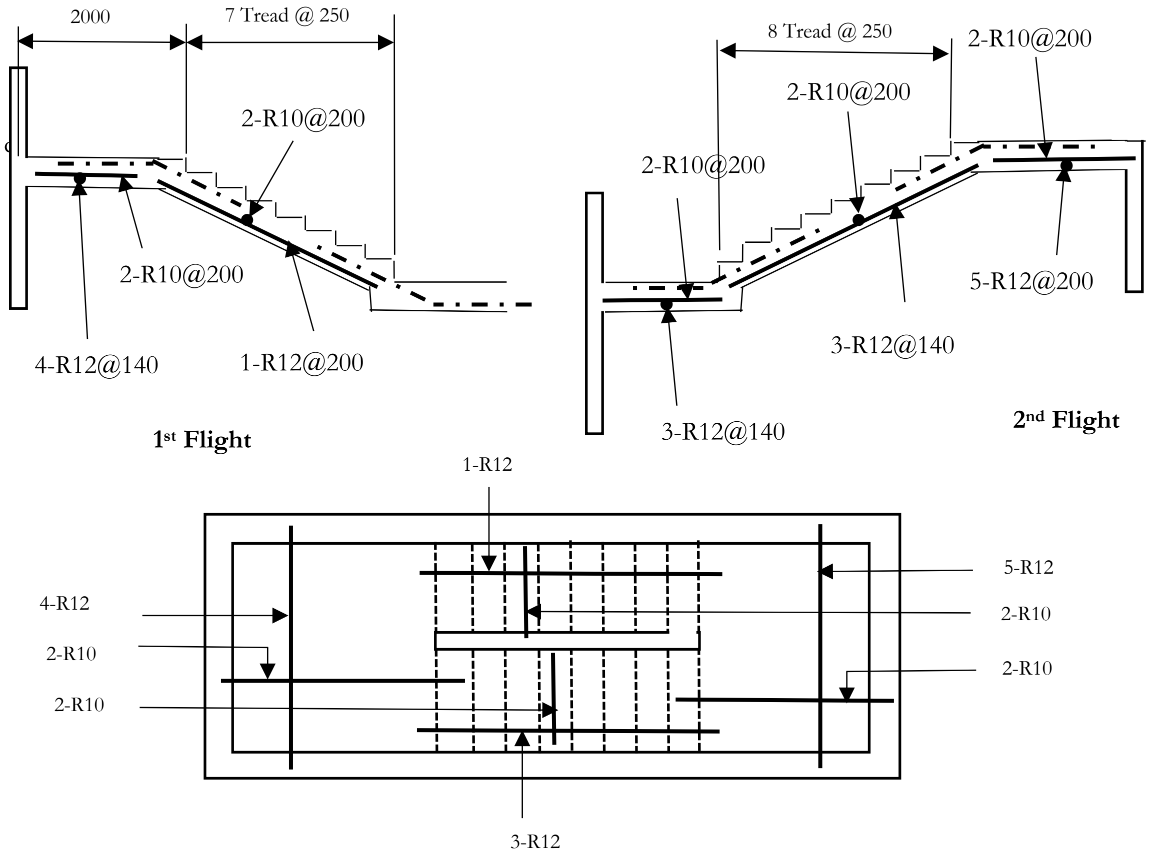


Figure 8.20: The sketch of reinforcement arrangement

Chapter 9- Design of Reinforced Concrete Columns

9.1 Definitions

Columns can be defined as structural elements subjected to compression and with dimensions such that its length is relatively larger than its lateral dimensions. The strength of all columns depends on:

i. The strength of the materials
ii. The elasticity of the materials
iii. The shape of and size of the cross section
iv. The length of the column
v. The degree of positional and directional restraint of the ends of the columns

Column can fail in three ways, namely

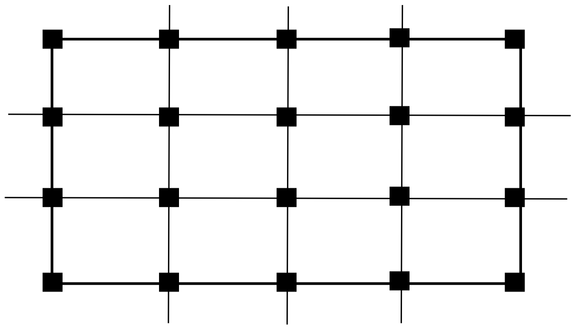
1. Compression failure
   This happens in columns that short relative to its cross-section dimension (short column). For concrete, it takes the form of longitudinal splitting and spalling of concrete
2. Buckling failure
   If a compression member is long relative to its lateral dimensions, it can fail by buckling at a load much lower than that indicated by the strength of the materials.
3. Combination of compression and Buckling
   This when failure of compression member is a combination of crushing of the material and buckling.

9.2 Classifications of Columns

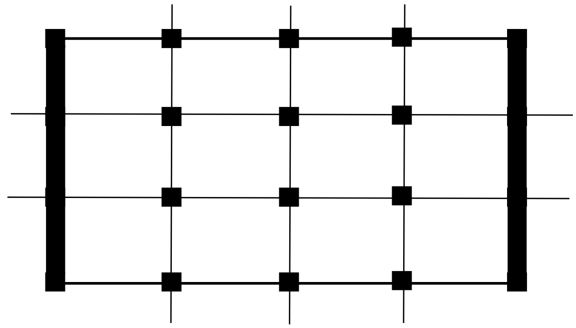
There are many methods used to classify columns, and are discussed below

i. On the basis of response to lateral restraint (Braced or Unbraced Columns)

On the basis of response to lateral restraint, columns are classified as either braced or unbraced. Columns are considered braced if the lateral loads, due to wind for example, are resisted by shear walls or cross bracing or brick or concrete panel or adjacent structures, or some other form of bracing. Simply put, in braced column, side sway is prevented. Unbraced column is usually part of an unbraced frame which is free to deflect horizontally by flexure of the columns when horizontal forces are applied. For example, Figure 9.1a is example of unbraced columns in both x and y axis. On the other hand, Figure 9.1b represents columns that are braced in one direction only, that is, along the y axis.



a. Columns unbraced in both directions



b. Columns unbraced in one (x) directions only

Figure 9.1: Unbraced and Braced Columns

ii. Classification on the basis of slenderness ratio (Short or Slender/Long Columns)

Columns can also be classified on the basis of slenderness ratio. Slenderness ratio is defined as the ratio effective height to the radius of gyration. The radius of gyration is defined as:

$$\lambda = \frac{l_o}{i} \qquad \textbf{9.1}$$

Where

l_o = effective length of the column

$$i = \sqrt{\left(\frac{second\ moment\ of\ area\ of\ the\ section\ about\ the\ axis\ (I)}{cross\ sectional\ area\ of\ the\ column\ (A)}\right)} \qquad \textbf{9.2}$$

But for the purpose of the design of rectangular reinforced concrete columns, "i" is roughly proportional to breadth of the cross section. The slenderness ratio is thus defined as effective height divided by the breadth.

$$\lambda = \frac{l_o}{breadth} \qquad \textbf{9.3}$$

In laboratory testing, the effective height is usually connected with the least radius of gyration. But in practical conditions, the effective height is relating to all possible values of slenderness ratio must be considered. For columns, there will two values of slenderness ratio to consider, which will be those corresponding to the two principal axes of the cross section.

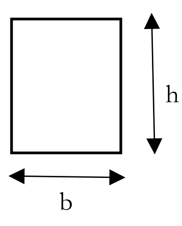


Figure 9.2 – Column lateral dimensions

That is, $\frac{l_{ex}}{h}$ and $\frac{l_{ey}}{b}$.

Thus, on the basis of slenderness ratio, columns are classified by BS8110 (Cl. 3.8.1.3) as either short or long.

Short columns **9.4**

$$\frac{l_{ex}}{h} < 15 \quad \text{and} \quad \frac{l_{ey}}{b} < 15 \text{ (for braced columns)}$$

$$\frac{l_{ex}}{h} < 10 \quad \text{and} \quad \frac{l_{ey}}{b} < 10 \text{ (for unbraced columns)}$$

Slender columns **9.5**

$$\frac{l_{ex}}{h} > 15 \quad \text{and} \quad \frac{l_{ey}}{b} > 15 \text{ (for braced columns)}$$

$$\frac{l_{ex}}{h} > 10 \quad \text{and} \quad \frac{l_{ey}}{b} > 10 \text{ (for unbraced columns)}$$

Where:

l_{ex} = effective height of the column in respect of the major axis

l_{ey} = effective height of the column in respect of the minor axis

b = width of the column cross section

h = depth of the cross section

However, in order to determine the effective length of the column, careful evaluations of the support conditions of the column is essential. The effective length of column depends on the degree of fixity. The effective height of the column is defined as either:

a. the height which corresponds to the height of a pin-ended column which carry the same axial load, or
b. as the height between points of contraflexure of the buckled column.

Understanding of the bucked shape of for a typical Euler's column (that is, columns to which Euler's equation may be applied), is useful in determining the effective length as in Table 9.1.
Table 9. 1 may be difficult to use for designers with limited experience. For design purposes, the codes also gave practical guidance on the choice of effective height in practical situations.

According to BS 8110, the effective height (l_e) of the column can be obtained by multiplying the clear height (l_o)between lateral restraint by a coefficient (β). This coefficient is a function of the fixity at the column ends, and it is obtained in Table 3.19 (BS 8110). So that

$$l_e = l_o\beta \qquad \textbf{9.6}$$

The value of the coefficient β, depend on the condition at the end, as stated in Tables 3.19 and 3.20 (BS 8110) for braced and unbraced columns, and reproduced below as Tables 9.2 and 9.3.

Table 9. 1: Effective length of Euler's Slender Column

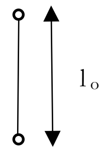
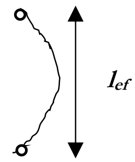
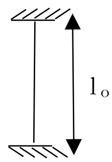
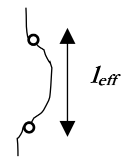
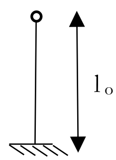
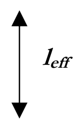
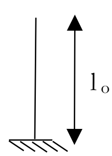
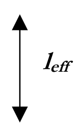
Column	Shape	Bucked Shape	Effective length (BS 8110)
Both ends Pinned (braced column but not restrained in direction)			l_{eff} = **0.75l_o - 1**
Both ends Fixed (braced, but the ends are rigidly restrained in direction			l_{eff} = **0**.75l
One end Fixed and one end Pinned			l_{eff} = **0**.75l – l
One end Fixed and one end Free			l_{eff} = **1** - 2l

Table 9.2: Values for Coefficient for Braced Columns (Table 3: 19 BS 8110)

End condition of top	End condition at bottom		
	1	2	3
1	0.75	0.80	0.90
2	0.80	0.85	0.95
3	0.90	0.95	1.00

Table 9.3: Values for Coefficient for Unbraced Columns (Table 3: 20 BS 8110)

End condition of top	End condition at bottom		
	1	2	3
1	1.2	1.3	1.6
2	1.3	1.5	1.8
3	1.6	1.8	-
4	2.2	-	-

The descriptions of the four end conditions are as follows:

Condition 1. The end of the column is connected monolithically to beams on either side which are at least as deep as the overall dimension of the column in the plane considered. Where the column is connected to a foundation structure, this should be of a form specifically designed to carry moment.

Condition 2. The end of the column is connected monolithically to beams or slabs on either side which are shallower than the overall dimension of the column in the plane considered.

Condition 3. The end of the column is connected to members which, while not specifically designed to provide restraint to rotation of the column will, nevertheless, provide some nominal restraint.

Condition 4. The end of the column is unrestrained against both lateral movement and rotation (e.g. the free end of a cantilever column in an unbraced structure)

The degree of restraint at the column ends can be inferred from the diagrams shown in Figure 9.3

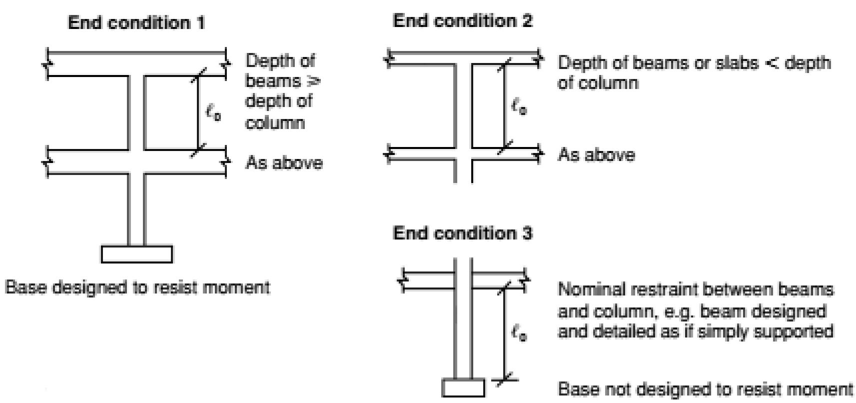


Figure 9.3: Diagrammatic representation of restraint at column ends

Example 9.1

Using Tables 9.2 and 9.3, classify the column shown in Figure 9.4, if the clear distance of the column up to the soffit of the beam is 4m, assuming the column is (a) braced and (b) unbraced.

Solution

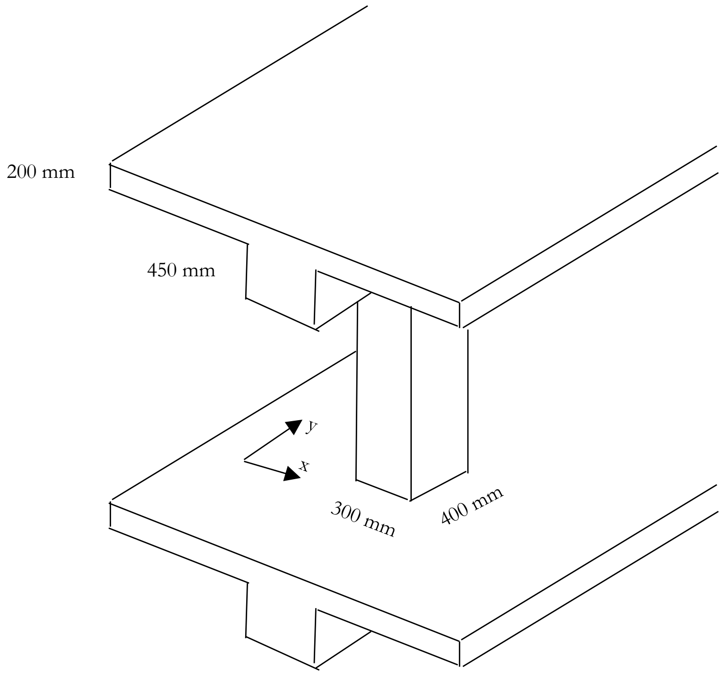


Figure 9.4: Example 9.1

Assuming that the column is braced

Bending in the y direction,

Depth of the beam = 450 mm

Depth of the column = 400 mm

Since depth of the beam > depth of the column, end condition at the top = 1, and end condition at the bottom is 1. Therefore β = 0.75 (Table 9.2)

Thus,

$$\frac{l_{ey}}{h} = \frac{\beta l_{ey}}{h} = \frac{0.75 \ x \ 4000}{400} = 7.5$$

Bending in the x direction

Depth of the slab = 200 mm

Depth of the column = 300 mm

Since depth of the slab < the depth of the column, the end condition at the top = 2, and end condition at the bottom is = 2. Therefore β = 0.85

Thus

$$\frac{l_{ex}}{h} = \frac{\beta l_{ey}}{h} = \frac{0.75\ x\ (4000+450)}{300} = 12.61$$

Since both $\frac{l_{ey}}{h}$ and $\frac{l_{ex}}{h}$ < 15, the column is Short

Assuming the column is unbraced

Bending in the y direction, for end condition at the top = 1, and end condition at the bottom is 1, β = 1.2 (Table 9.3)

Thus,

$$\frac{l_{ey}}{h} = \frac{\beta l_{ey}}{h} = \frac{1.2\ x\ 4000}{400} = 12.0$$

Bending in x direction, for end condition at the top = 2, and end condition at the bottom is 2, β = 1.5 (Table 9.3)

Thus,

$$\frac{l_{ex}}{h} = \frac{\beta l_{ex}}{h} = \frac{1.5\ x\ 4450}{300} = 22.25$$

On the basis of the slenderness ratio in the x direction (> 15), the column should be treated as slender

iii. Classification of Columns based on induced axial bending

On the basis of the structural configuration of the beams framing into columns, columns can be classified as:

a. Axially loaded column
b. Uni-axially bending column
c. Bi-axially bending column.

This concept can be explained through an illustration, using figures 9.5 and 9.6.

For example, in the Figure 9.5, Columns B2, B3, B4, C2, C3, and C4 are axially loaded column because of the fact that the beams framing into them are structurally symmetrical. This is because the beams framing into the columns are equal in dimensions.

On the other hand, columns A2, A3, A4, D2, D3 and D4 are **uni-axial bending** columns (in y-direction). This is because the moments in x-directions cancel out because of symmetry and equal beam dimensions, leaving bending in the y-direction. But the corner columns A1, A5, D1, and D5 are in bi-axial bending. Because there is bending of adjacent beams in both x-x and y-y directions. However, for unsymmetrically arranged beams framing into columns, identification will depend on which of the bending in the axis predominates.

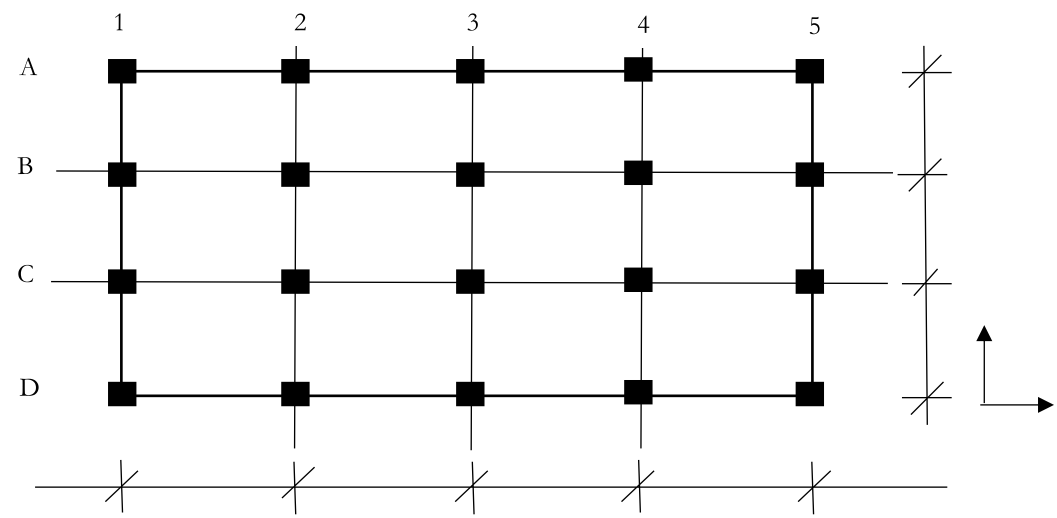


Figure 9.5: Demonstration of types of column from symmetrical beams

For example, in Figure 9.6, the corner columns A1, A5, D1, and D5 are still in biaxial bending. But A2 and A4 are in biaxial bending.

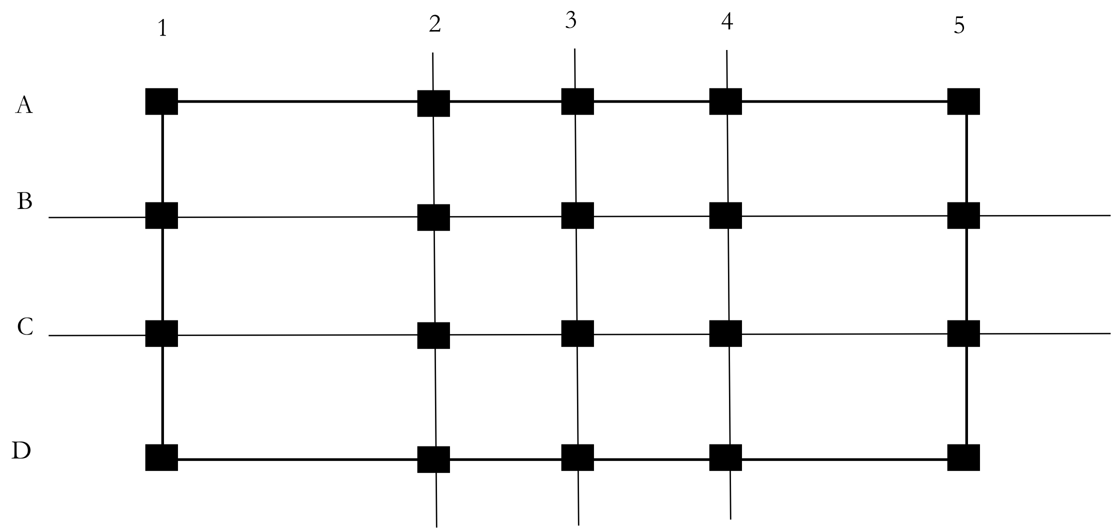


Figure 9.6: Demonstration of types of column from non-symmetrical beams

In column A2, it is obvious that bending in beams 1-2 will predominate over bending in beam 2-3 resulting in net bending moment in the x direction. This in addition to bending along y-axis. Similar reason can be giving for column A4. Column B3 and B4 will be axially loaded columns.

Thus, it can be seen that the structural arrangement of beams (loadings, spans and dimensions) is very important in identifying whether a column is axially-loaded, or in uni-axial bending, or in bi-axial bending in a frame structure.

9.3 Reinforcement detailing in Columns

In order to ensure structural stability, durability and practicability of construction BS 8110 lays down various rules governing the minimum size, amount and spacing of (i) longitudinal reinforcement and (ii) links.

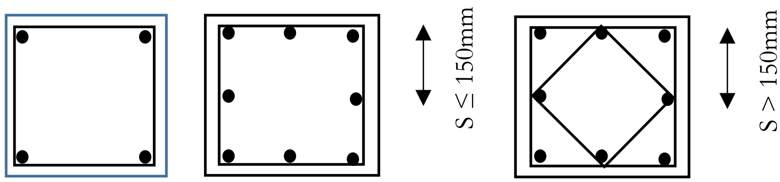


Figure 9.7: Some reinforcement arrangement in column

i. Longitudinal Reinforcement

i. Size of bars ≥ 12 mm

ii. Minimum number of bars = 4 for rectangular and 6 for circular cross section

iii. Areas of reinforcement

The code recommends that for columns with a gross cross-sectional area A_{col}, the area of longitudinal reinforcement (A_{sc}) should lie within the following limits:

$0.4\%A_{col} \leq Asc \leq 6\% A_{col}$ (for vertically cast column) and

$0.4\%A_{col} \leq Asc \leq 8\% A_{col}$ (for horizontally cast column.

ii. Links

Axial loads in columns can cause buckling of the longitudinal reinforcement and subsequent cracking and spalling of the adjacent concrete cover. In order to prevent such a situation from occurring, the longitudinal steel is normally laterally restrained at regular intervals by links passing round the bars

i. Size = the greater of $\frac{1}{4}$ of largest size of longitudinal reinforcement OR 6 mm

In practice 6 mm is not available, thus a minimum bar size of 8mm is used.

ii. Maximum spacing = 12 times the size of smallest longitudinal bars OR smallest cross section dimension. Note that links are provide to prevent diagonal shear failure.

9.4 Design of Short Braced Columns

For design purposes, BS 8110 divides short-braced columns into three categories. These are:

i. columns resisting axial loads only;

ii. columns supporting an approximately symmetrical arrangement of beams;

iii. columns resisting axial load and uniaxial

iv. columns resisting axial load and biaxial bending

i. Axially-Loaded Column

Consider a column having a net cross-sectional area of concrete Ac and a total area of longitudinal reinforcement A_{sc} (Figure 9.8)

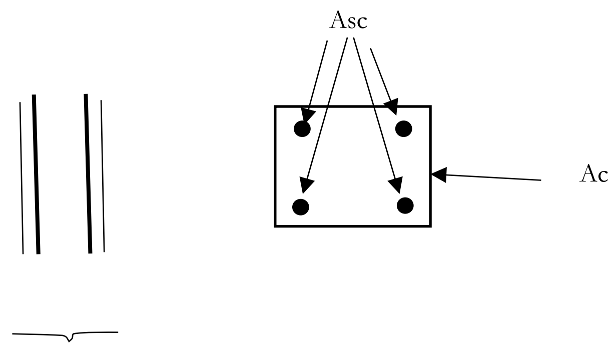


Figure 9.8: Cross section area of a concrete column

Now recall that the design stresses for concrete and steel in compression are $\frac{0.67 f_{cu}}{1.5}$ and $\frac{f_y}{1.15}$ respectively, that is

Concrete design stress = $\frac{0.67 f_{cu}}{1.5} = 0.45 f_{cu}$ **9.7**

Steel design stress = $\frac{f_y}{1.15} = 0.87 f_y$ **9.8**

Since both the concrete and reinforcement assist in carrying the load, then the ultimate load N which can be supported by the column is the sum of the loads carried by the concrete (Fc) and the reinforcement (Fs), i.e.

N= Fc + Fs **9.9**

Fc= stress × area = 0.45fcuAc **9.10**

And

Fs= stress × area = 0.87fyAsc **9.11**

Hence

N= 0.45fcuAc + 0.87fyAsc

The above equation assumes that the load is applied perfectly axially to the column. However, in practice, perfect conditions never exist. To allow for a small eccentricity of load due to inaccuracies in construction, BS 8110 reduces the design stresses in the equation by about 10 per cent, giving the following expression:

$$N = 0.4f_{cu}A_c + 0.75f_yA_{sc} \qquad \mathbf{9.12}$$

This is equation 38 in BS 8110 which can be used to design short-braced axially loaded columns.

ii. Columns supporting an approximately symmetrical arrangement of beams

When columns support an approximately symmetrical arrangement of beams and provided that:

a. the loadings on the beams are uniformly distributed, and
b. the beam spans do not differ by more than 15 per cent of the longer

The columns are subject to an axial load and 'small' moment (section 3.13.5). This small moment is taken into account simply by decreasing the design stresses in equation 38 (BS 8110) by around 10 per cent, to give the following expression for the load carrying capacity of the column:

$$N = 0.35f_{cu}A_c + 0.67f_yA_{sc} \qquad \mathbf{9.13}$$

This is equation 39 in BS 8110 and can be used to design columns supporting an approximately symmetrical arrangement of beams having uniformly distributed, and the beam spans do not differ by more than 15 per cent of the longer.

Equations 9.12 and 9.13 are not only used to determine the load-carrying capacities of short braced columns predominantly supporting axial loads but can also be used for initial sizing of these elements.

Example 9.2

A short braced reinforced column is to support an axial load of 3000KN at the ultimate. If the column is 400mm^2, determine the calculate the size of the longitudinal reinforcement required. Use fcu = 30N/mm^2, fy = 460N/mm^2.

Solution

The column does not transmit any moment, but only axial load. Using equation 9.12 (Equation 38, BS 8110)

$$N = 0.4f_{cu}A_c + 0.75f_yA_{sc}$$

$$3000 \times 10^3 = 0.4 \times 30 \times 400 \times 400 + 0.75 \times 460 \times A_{sc}$$

$$3000000 - 1920000 = 345A_{sc}$$

$$1080000 = 345A_{sc}$$

$$A_{sc} = 3130.43 \text{ mm}^2$$

Use 4Y32 (3217mm²)

Example 9.3

If the column in Example 9.2 support approximately symmetrical arrangement of beam., and the beams are all, loaded with uniformly distributed load and no span differ by more than 15% of the longer. Calculate the size of the longitudinal reinforcement required.

Solution

The column meets the conditions for the application of equation 9.13 (equation 39, BS 8110)

N = 0.35fcuAc + 0.67fyAsc

$3000 \times 10^3 = 0.35 \times 30 \times 400 \times 400 + 0.67 \times 460 \times Asc$

$3000000 - 1680000 = 308.20Asc$

$1320000 = 308.20Asc$

$Asc = 4282.93mm^2$

Use 4Y40 ($5026mm^2$)

iii. Columns resisting axial load and bending (uniaxial or biaxial)

The area of longitudinal steel for columns resisting axial loads and uniaxial or biaxial bending is normally calculated using the design charts in Part 3 of BS 8110. These charts are available for columns having a rectangular cross-section and a symmetrical arrangement of reinforcement. Though the British Standard Institution (BSI) issued these charts when the preferred grade of reinforcement was 460 not $500N/mm^2$, the charts could still be used to estimate the area of steel reinforcement required in columns but the steel areas obtained will be approximately 10 per cent greater than required. Each of these charts is particular for a selected three parameters:

i. characteristic strength of concrete, f_{cu}
ii. characteristic strength of reinforcement, f_y;
iii. $\frac{d}{h}$ ratio.

Design charts are available for concrete grades 25, 30, 35, 40, 45 and 50 and reinforcement grade $460N/mm^2$. For a specified concrete and steel strength there is a series of charts for different $\frac{d}{h}$ ratio in the range 0.75 to 0.95 in 0.05 increments. Where the actual d/h ratio for the section being designed lies between two charts, both charts may be read and the longitudinal steel area found by linear interpolation. A typical chart is shown in Figure 9.9

Design of Column under both axial load and uniaxial bending.

With columns which are subject to an axial load (N) and uni-axial moment (M), the procedure simply involves:

i) Calculate $\frac{N}{bh}$ and $\frac{M}{bh^2}$ ratios on the appropriate chart

ii) Estimate/calculate d (knowledge of the link diameter and nominal cover is necessary)

iii) Find $\frac{d}{h}$ ratio

iv) Choose appropriate chart and read off the corresponding area of reinforcement as a percentage of the gross-sectional area of concrete ($\frac{100Asc}{bh}$)

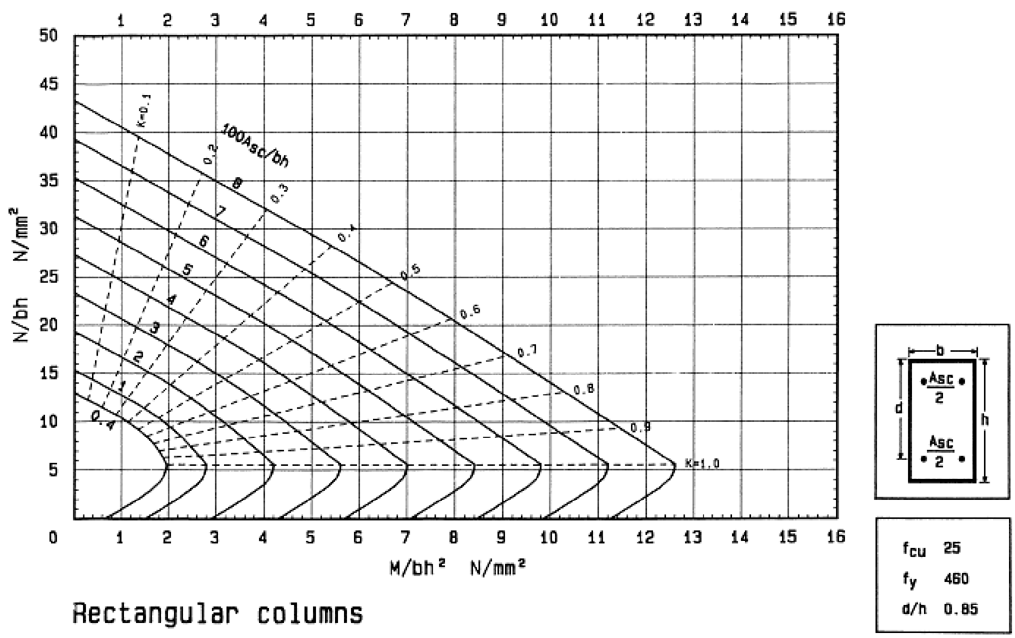


Figure 9.9: Typical Chart for Rectangular Column (Chart No 23 BS 8110 Part 3)

Example 9.4

A 300 x 350mm column, with effective height of 3m, is to support 2000KN axial load and a moment of 70KN.m about the x-x axis, design the longitudinal reinforcement. Take the fire resistance = 2hrs and moderate exposure condition. fcu = 25N/mm² and fy = 460N/mm².

Solution

The column cross section is as shown in figure 9.10

Slenderness ratio = $\frac{3000}{300}$ = 10.00 < 15

= $\frac{3000}{350}$ = 8.57 < 15

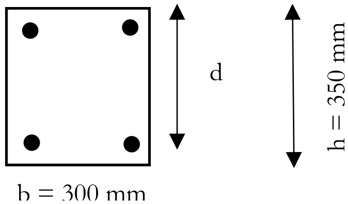


Figure 9.10: Example 9.3

Therefore, design as short column

The column support both axial load and bending moment in x-x-axis, Design graph will be used

$$\frac{N}{bh} = \frac{2000\ x\ 1000}{300\ x\ 350} = 19.08$$

$$\frac{M}{bh^2} = \frac{70\ x\ 1000000}{300\ x\ 350\ x\ 350} = 1.91$$

Assuming 16 mm bar diameter, and 8mm diameter link

Cover

i. Condition of exposure (Table 3.3)

C = 35 mm (to main reinforcement including link)

= 35 – 8 = 27 mm

ii. Fire condition (Table 3.4)

C = 25 mm (to main reinforcement including links)

= 25 – 8 = 17 mm

The larger is 27 mm

But for ease of construction, take cover = 35mm

Thus

d = h – half bar diameter – link diameter - cover

= 350 – 8 – 8 - 35 = 299 mm

Therefore

$$\frac{d}{h} = \frac{299}{350} = 0.85$$

From chart 23 (BS 8110 Part 3)

$$\frac{100Asc}{bh} = 3.1$$

That is,

$$Asc = \frac{3.1bh}{100} = \frac{3.1\ x\ 300\ x\ 350}{100}$$

= 3255mm^2.

Use 8Y25 (3927mm^2)

Design of Short Column under both axial load and Biaxial bending (clause 3.8.4.5, BS 8110).

Where the column is subject to biaxial bending, the problem is reduced to one of uniaxial bending simply by increasing the moment about one of the axes using the procedure outlined in the code as below. Referring to figure 9.11

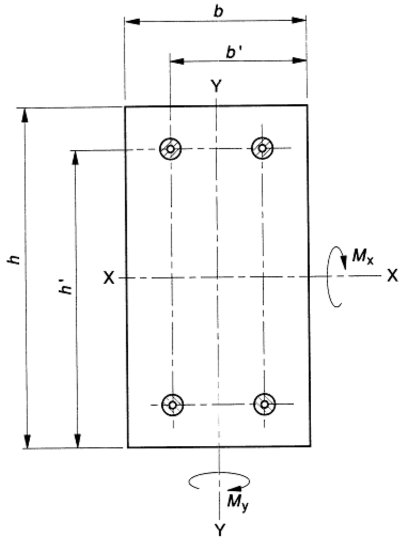


Figure 9.11: Biaxially Bent column (Figure 3.22 BS 8110)

IF

$$\frac{M_x}{M_y} \geq \frac{h'}{b'} \text{ then}$$

The enhanced design moment about the x-x axis M'x

$$M'x = Mx + \frac{\beta h}{b} M_y \quad \text{(about x-x axis)}$$

But IF

$$\frac{M_x}{M_y} \leq \frac{h'}{b'} \text{ then}$$

The enhanced design moment about the y-y axis M'y

$$M'y = My + \frac{\beta h}{b} M_x \quad \text{(about y-y axis)}$$

Where

b' and h' = effective depth

β = The enhancement coefficient for biaxial bending obtained from is given in Table 9.3 *(Table 3.22, BS 8110)*

The steel is then calculated by using the enhanced moment (Mx' or My',) in the same way as that described for uniaxial bending.

Table 9.3: The enhancement coefficient for biaxial bending (Table 3.22 BS 8110)

$\frac{N}{bhf_{cu}}$	0	0.1	0.2	0.3	0.4	0.5	≥0.6
β	1.00	0.88	0.77	0.65	0.53	0.42	0.30

Example 9.5

A 300 x 350mm column, with effective height of 3m, is to support 2000KN axial load and a moment of 70KN.m about the x-x axis, and 50KN.m about y-y- axis. Design the longitudinal reinforcement. Take the fire resistance = 2hrs and moderate exposure condition. fcu = 30N/mm² and fy = 460N/mm².

Solution

The column is shown in Figure 9. 12.

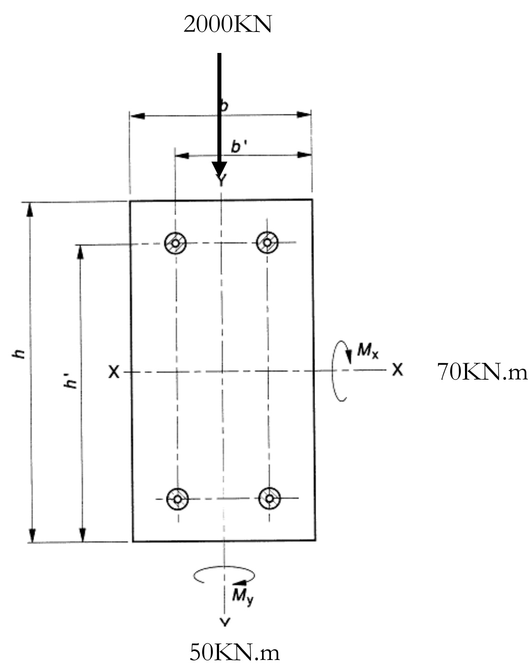


Figure 9.12: Example 9.5

Slenderness ratio = $\frac{3000}{300}$ = 10.00 < 15

= $\frac{3000}{350}$ = 8.57 < 15

Therefore, design as short column but bi-axially

Cover

i. Condition of exposure (Table 3.3)

C = 35 mm (to main reinforcement including link)

= 35 – 8 = 27 mm

ii. Fire condition (Table 3.4)

C = 25 mm (to main reinforcement including links)

= 25 – 8 = 17 mm

The larger is 27 mm

But for ease of construction, take cover = 35mm

Assuming 16 mm bar diameter, and 8 mm diameter link

b' = b – half bar diameter – link – cover

= 300 – 8 – 8 – 35 = 249 mm

Also

h' = h – half bar diameter – link – cover

= 350 – 8 – 8 – 35 = 299mm

Now

$$\frac{M_x}{M_y} = \frac{70}{50} = 1.40$$

And

$$\frac{h'}{b'} = \frac{299}{249} = 1.20$$

That is

$$\frac{M_x}{M_y} \geq \frac{h'}{b'}$$

Thus, the enhanced design moment about the x-x axis M'x

$$M'x = Mx + \frac{\beta h}{b} M_y \text{ (about x-x axis)}$$

$$\frac{N}{bhfcu} = \frac{2000 \, x \, 1000}{300 \, x \, 350 \, x \, 30} = 0.64$$

From Table 3.22 (BS 8110), β = 0.30, t:hen

$$M'x = Mx + \frac{\beta h}{b} M_y \text{ (about x-x axis)}$$

$$= 70 + \frac{0.3 \, x \, 350}{300} 50 = 70 + 17.5$$

= 87. 50KN.m

Therefore

$$\frac{N}{bh} = \frac{2000 \, x \, 1000}{300 \, x \, 350} = 19.08$$

$$\frac{M'x}{bh^2} = \frac{87.5 \, x \, 1000000}{300 \, x \, 350 \, x \, 350} = 2.38$$

And

$$\frac{d}{h} = \frac{299}{350} = 0.85$$

From chart 23 (BS 8110 Part 3)

$$\frac{100Asc}{bh} = 3.0$$

That is,

$$Asc = \frac{3.0bh}{100} = \frac{3.0 \; x \; 300 \; x \; 350}{100}$$

$= 3150mm^2$.

Use 4Y32 (3217mm²)

9.5 Design of Slender Columns

As discussed in earlier, slender column results, when

$\frac{l_{ex}}{h} > 15$ and $\frac{l_{ey}}{b} > 15$ (for braced columns)

$\frac{l_{ex}}{h} > 10$ and $\frac{l_{ey}}{b} > 10$ (for unbraced columns)

However, in most structures, the columns will be short, but occasionally slender columns will have to be designed. In that case, the bending moments will need to be increased above those calculated from the applied load. This is because additional moment will be induced by the slenderness which cause the column to deflect, so that the axial force is applied at eccentricity "e" from the true center line of the column (Figure 9. 13).

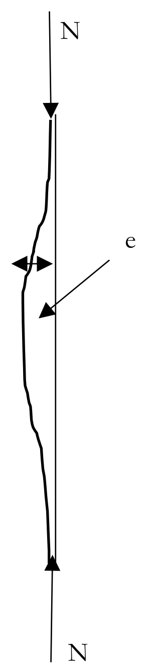


Figure 9. 13: Induced eccentricity in Slender Columns

The greater the slenderness of a column, the greater the deflection and the greater is the additional bending moment. Thus, the bending moment that should be used in design of slender column is given by

$$Mt = Mi + Madd$$

The values of Mi and Madd are as defined in the code, based on the nature of the bending. Three configurations of slender column could result. These are:

i. Slender column bent about a minor axis

   This should be designed for the ultimate load N together with Mt as given by the code

ii. Slender column bent about a major axis

   In addition to the ultimate load N, it should be designed for Mt as given by the code

iii. Slender column bent about both axis

   The slender column bent about both axes should be designed for its ultimate load N, together with the moment M_{ux} about its major axis and moment M_{uy} about the minor axis.

Chapter 10- Design of Reinforced Concrete Foundations

10.1 Introduction

Foundation is required to transmit and distribute loads from the components of the structure, namely, slabs, beams, walls, columns, etc. safely to the ground. In so doing:

i. The safe bearing capacity of the soil must not be exceeded which can cause the failure of the structure.
ii. Excessive settlement must be prevented so as not to damage structure and its service facilities such as water mains, gas mains, and other utilities mains.
iii. The foundation itself must not fail to ensure overall stability of the structure against sliding, overturning, and vertical uplift.

Because foundation failures have enormous financial, environmental and safety implications. It is essential, therefore, that much attention be paid to the design of foundations. Generally, foundation can be broadly classified into two, namely, shallow and deep foundations

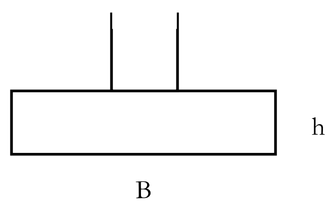


Figure 10. 1 – Foundations dimension for classification

Shallow Foundations

Shallow foundations are those foundations having depth less than or equal to the breadth ($h \leq B$). This is shown in figure 10.1. The foundations in this category are:

i. Strip Foundation
ii. Pad Foundation
iii. Combined Foundation
iv. Strap Foundation
v. Raft Foundation

Deep Foundation

These are foundations having depth greater than the breadth ($h > B$), and they include

i. Pile foundations
ii. Diaphragm foundations
iii. Caisson

All these types of foundation are in use in Nigeria. Analysis of relative usage of each of this type of foundation, is presented in Table 10.1.

Table 10.1 - Some Statistic on Foundation types usage in Nigeria (Oyenuga, 2008)

	Foundation Type	Usage in %
1	Strip	70
2	Wide Strip	8
3	Pad	16
4	Raft	5
5	Pile	1

The choice of foundation depends on:

i. Form of building
ii. The building loads
iii. The depth to a suitable bearing stratum
iv. The bearing capacity of the stratum
v. The proximity of other structures and foundations
vi. Access for equipment and machines
vii. Economy

10.2 Importance of Soil Investigations in Structural Design of Foundations

It is important for structural engineers not to be ignorant of the soil on which his/her design is to be founded. The results of soil investigations is important for the following reasons:

i. For the confirmation of bearing stratum
ii. For the confirmation of the bearing capacity of the soil
iii. For the confirmation of the settlement rate of the soil
iv. Gives an insight into the materials to be used. For example, if the report shows the presence of high concentration of some chemicals, then suitable cement may be recommended

Several foundation options are usually given from the results of soil investigations, it is however the work and responsibility of the structural engineer to choose the most appropriate based on factors like cost, ease of construction, etc.

10.3 Strip Foundation

Where small loads are to be transmitted to the ground, strip foundation are used. This type of foundation is used under walls or under a line of closely spaced columns. It is the most widely-used form of foundation for Bungalow houses. It can be narrow strip foundation or wide strip foundation.

Narrow Strip foundation

From the wall the loading is spread on the foundation at 45° as shown in Figure 10.2a. The planes through which the loading is distributed are called the shear planes. The foundation should be designed in such a way that the shear planes pass through the lower corners of the strip. If the designed foundation width is too wide, as is the case in weaker soils, plain concrete strip may bend and crack as shown in Figure 10.2b. Concrete may be made stronger in tension by providing steel reinforcement in the tension zone.

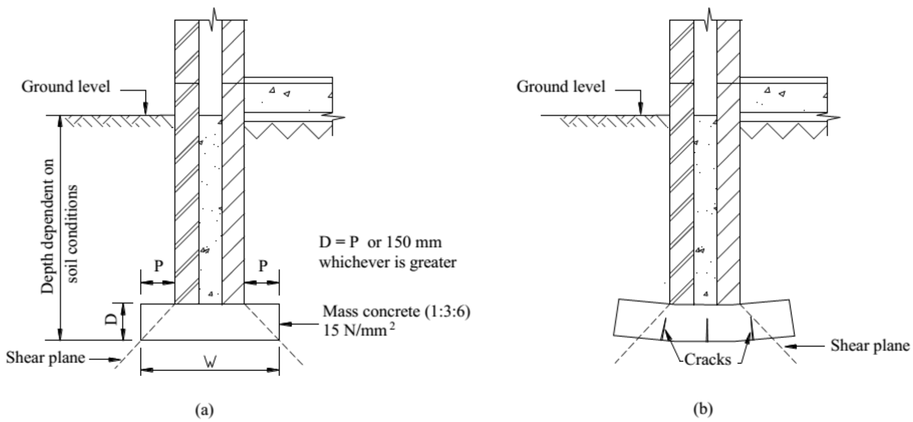


Figure 10.2. A typical Strip Foundation

Building regulations expect the design of a strip foundation should satisfy the following conditions.

i. The projections (P) of the concrete strip on either side of the wall should be equal.
ii. The minimum thickness of strip foundation should be taken as 150 mm. This value may be increase to the value of toe projection.
iii. For practical consideration, on the basis of dispersion of 45^0 from the face of the wall, B should be at least 3H for narrow strip foundation

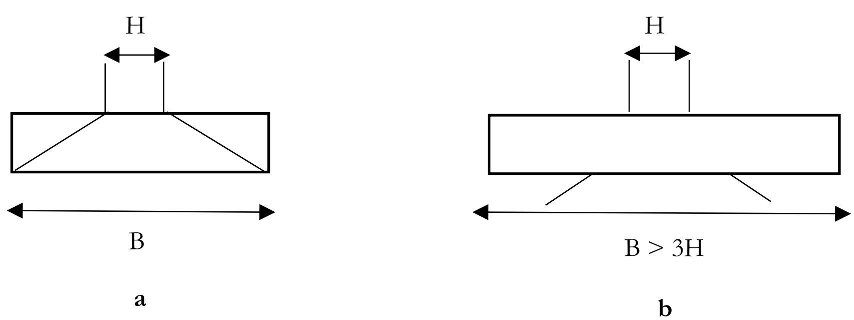


Figure 10.3. Relationship between the width of the base and thickness of wall

If B is greater than 3H, concrete base can fail in tension. Therefore, such should be designed as wide strip foundation.

iv. For Strip foundations, safe bearing pressure is used to obtain the dimensions of the foundations on the basis of serviceability limit state followed by a detailed structural design based on the ultimate limit state.

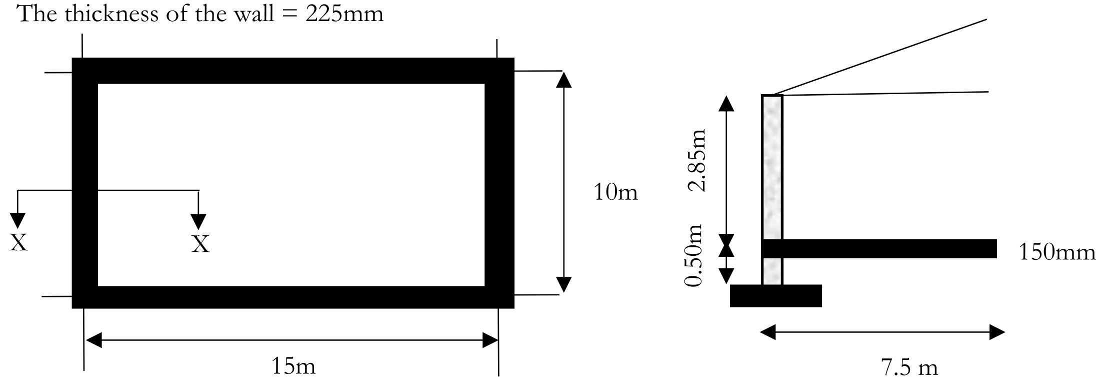
Example 10.1

Design a strip foundation for the perimeter block-wall of a bungalow, rectangular in plan, on a soil having safe bearing capacity of $75KN/m^2$. Take the blockwork load = $2.87KN/m^2$, rendering = $0.25KN/m^2$, floor finishes = 0.5KN/m2, partitions = $1.2KN/m^2$. The live load is $2KN/m^2$, roof load = $2.0KN/m^2$.

The ground floor is 150mm concrete slab.

Solution

The thickness of the wall = 225mm



(a) Plan **(b)** Section X-X

Figure 10.4: Example 10.1

Because, the walls and the foundation under the walls are very long, the calculations are based on 1 m length of the wall/foundation.

Using the longer length, L = 15 m.

The width of the area supported by the footing = 7.5 – 0.225 = 7.275m

Loading

Floor

i. The Ground floor = 2400 x 0.15 = $3.60KN/m^2$

ii. Floor finishes = $0.5KN/m^2$

iii. Partitions = $1.2KN/m^2$

Total = 5.3KN/m^2

For 1meter strip = 5.3KN/m

Wall load

i. Blockwall = 2.87 x (2.85 + 0.5) = 9.62KN/m

ii. Rendering (2 sides) = 0.25 x 2 x (2.85 + 0.5) = 1.68KN/m

Roof load

i. Roof load = 2.0 x 7.275 = 14.55KN/m

Live load = 2.0KN/m^2

= 2.0KN/m

Design Load at Serviceability = 1.0g + 1.0qk

= 1.0 (5.3 + 9.62 + 1.68 + 14.55) + 1.0 (2.0)

= 31.15 + 2.0

= 33.15KN/m

The area (A) of the foundation can be determined from the formula:

$$A = \frac{Design\ \ Load\ at\ Serviceability}{Safe\ Bearing\ Capacity}$$

$$A = \frac{33.15}{75} = 0.442\text{m}^2$$

That is,

Area of strip foundation = Width × 1 m length = 0.442 m^2

Thus, the width of the Strip

= 442mm

This is the minimum required width of the strip foundation.

However, since the thickness of the wall (H) is 225mm,

Use width B = 3H

B = 675mm.

Although reinforcement is not required for narrow strip footing, in order to control thermal and shrinkage cracking, **nominal reinforcement**, in the form of wire mesh is necessary. Thus, it is not a good structural practice to place block-wall directly on the blinding, as this practice will not result in durable construction.

Wide Strip Foundation

For a weak soil, and when the base of the foundation is more 3times thickness of the wall, concrete base can fail in tension. Therefore, such should be designed as wide strip foundation. Reinforcement for wide Strip foundations is designed for transverse BM computed from the ground bearing stresses under the footing (Figure 10.5)

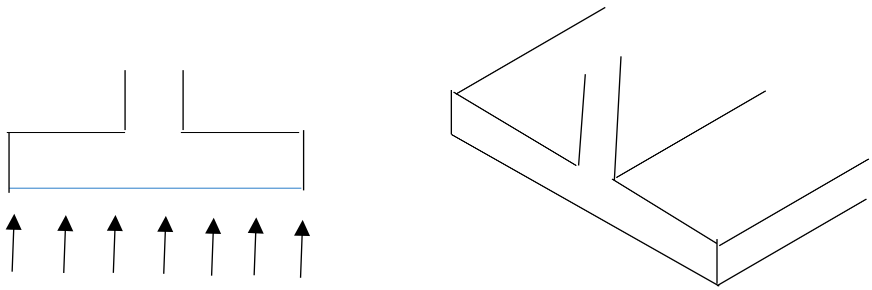


Figure 10.5 – Strip Foundation

Safe bearing pressure is used to obtain the dimensions of the foundations on the basis of serviceability limit state followed by a detailed structural design based on the ultimate limit state.

To calculate the bending moment, the footing is considered to be a cantilever, projecting out of the face of the wall in the transverse directions.

The rules for detailing and calculating for the transverse and secondary reinforcements are as for one-way solid slab.

Example 10.2

The building in the example 10.1 is to be founded on soil with bearing capacity of 40KN/m². Design a suitable strip foundation for the wall.

Solution

Design Load at Serviceability = 33.15KN/m

The area (A) of the foundation can be determined from the formula:

$$A = \frac{33.15}{40} = 0.83m^2$$

That is,

Area of strip foundation = Width × 1 m length = 0.83 m²

Thus, the width of the Strip = 830.mm

This is greater than 3H (675)

Therefore, design as wide strip footing

The design load at ultimate = 1.4gk + 1.6qk

= 1.4 (5.3 + 9.62 + 1.68 + 14.55) + 1.6 (2.0)

= 43.61 + 3.2 = 46.81KN/m

The soil pressure, p

$$q = \frac{design\ load\ at\ ultimate}{width\ of\ the\ base} = \frac{46.81}{0.83} = 56.40KN/m^2$$

for 1-meter run

$q = 56.40KN/m$

Treating the footing as a cantilever from the face of the wall

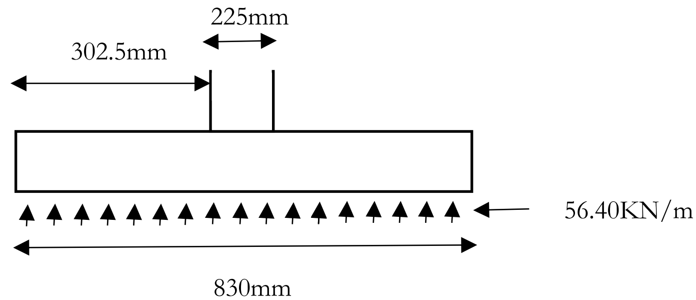


Figure 10.6: Example 10.2

Maximum Bending moment = $\frac{wl^2}{2} = \frac{56.40 \ x \ 0.3025 \ x \ 0.3025}{2}$ = 2. 58KN.m

Taking the depth (D) to be equal to the width of the wall = 0.225mm, f_{cu} = 25N/mm², f_y = 250N/mm², cover = 50mm, bar diameter = 12mm

Effective depth d = 225 – 50 – 6 = 169mm

Mu = 0.156fcubd² = 0.156 x 20 x 1000 x 169² = 111. 39KN.m

Since Mu > M, no compression reinforcement is required

$K = \frac{M}{f_{cu}bd^2} = \frac{2.58 \ x \ 1000000}{20 \ x \ 1000 \ x \ 169 \ x \ 169} = 0.05$

$z = d[0.5 + \sqrt{(0}.25 - \frac{K}{0.9})]$

= d [0.5 + √(0.25 – 0.006] = d [0.5 + 0.49]

= 0.95d

z = 0.95d = 0.95 x 169 = 160.55 mm

Thus

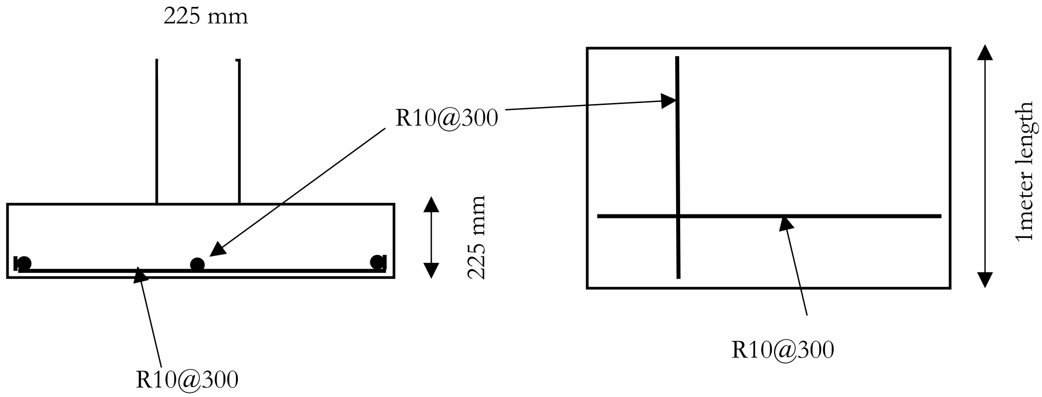
As = $\frac{M}{0.87f_yz} = \frac{2.58 \ x \ 1000000}{0.87 \ x \ 250 \ x \ 160.55}$

= 73.88mm²

Use R8@300 (168mm²)

Also use R8@300 for distribution

The sketch of reinforcement arrangement is shown in Figure 10.7



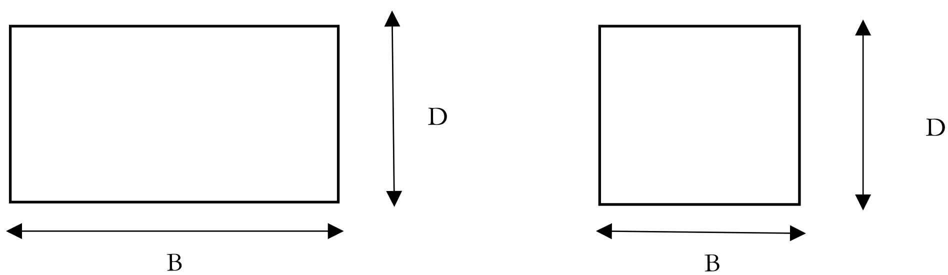
Section

Plan

Figure 10.7: Sketch of reinforcement arrangement

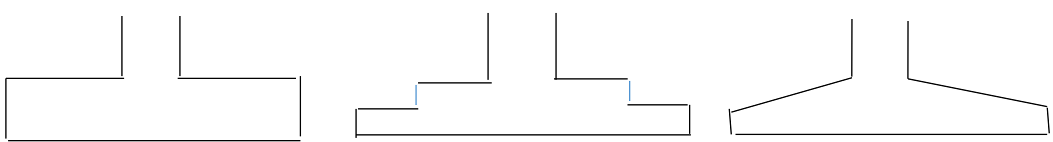
10.4 Pad Foundation

Pad footings (also known as spread footing or independent base) are usually rectangular or square in plan. It can be stepped or slopped in section (Figure 10.8)



a. Footing: Rectangular in Plan

b. Footing: Square in Plan (B =



c. Typical Cross section of pad Footing

Figure 10.8: Possible configuration of Pad foundation

When designing pad foundation, the followings should be noted.

1. Single column footings represent cantilever projections (Figure 10.9) out of the column in both directions and loaded upwards by the soil pressure. Corresponding tensile stresses are caused in both directions at the bottom surface.

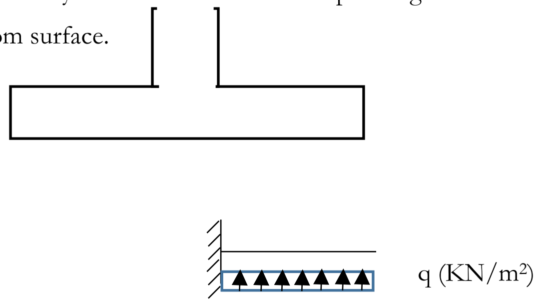


Figure 10. 9

2. In computing BM and SF, only the upward pressure q (KN/m^2) which is caused by the factored column load is considered (Figure 10.10). The self-weight of the footing does not cause moment or shear force.

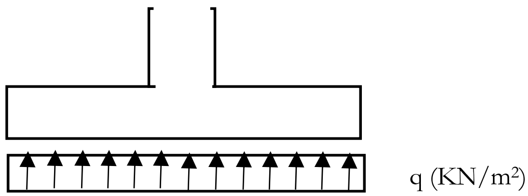


Figure 10. 10

3. Reinforcement for Bending is calculated on the basis of moments along the two faces of the columns as for cantilever solid slabs (Figure 10.11)

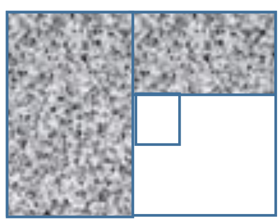


Figure 10. 11

4. For stepped footings, BM should be calculated for each step.
5. safe bearing pressure is used to obtain the base dimensions of the foundations on the basis of serviceability limit state (i.e. 1.0Gk + 1.0Qk).
6. Calculation to determine the depth is based on the ultimate limit state (i.e. l.4Gk + 1.6Qk), taking into consideration bond length for starter bars.

7. As much as possible, the base should be proportioned to achieve uniform stress or nearly uniform stress on the soil.

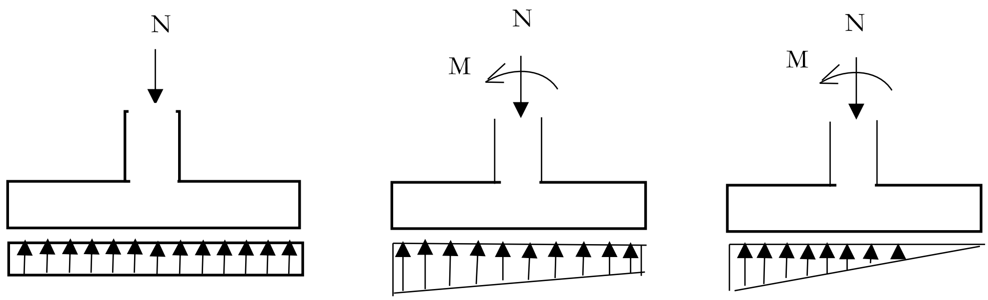
10.4.1 Pressure Distribution under Pad Footing

In pad foundation, calculations to determine the structural strength, that is, the thickness of the base and the area of reinforcement is based on the loadings and the resultant ground pressures corresponding to the ultimate limit state. For the purpose of determining the soil stress, two assumptions are made:

i. That the soil behaves elastically
ii. That the stress-strain distribution in the soil immediately under the base is linear

These assumptions allow the theory of bending to be used to determine the soil stress distribution for the axial load and the moment, provided that the stresses are always compressive. **This is because the soil-concrete interface cannot carry tension and therefore must always be compressive**.

Thus, assuming a linear stress-strain distribution, the bearing pressure across the base of the column will take any of the three forms shown in figure 10.12, according to the magnitude of axial load N and the moments M acting on the base.



(a) $e = 0$, $p = \frac{N}{BD}$

(b) $e < \frac{D}{6}$, $p = \frac{N}{BD} \pm \frac{6M}{BD^2}$

(c) $e > \frac{D}{6}$, $p = \frac{2N}{BY}$. [where $Y = 3(\frac{D}{2} . - e)$]

Figure 10. 12 – Pressure distribution under pad footing

Case 1 → the column transmits Axial Load (N) only

When the column transmits only axial load, the pressure will be uniform throughout the base of the footing as shown in figure b. The pressure "p" will be:

$$p = \frac{N}{BD} \qquad \textbf{10.1}$$

where p = pressure (KN/m^2), B and D base dimensions

Case 2 → the column transmits Axial Load (N) and Moment (M)

When the column transmits not only axial load N, but also bending moment M, then the pressures are governed (from mechanics) by equation for axial force plus bending.

$$p = \frac{N}{BD} \pm \frac{M}{Z} \quad \textbf{10.2}$$

where Z = base section modulus $(\frac{I}{y}) = \frac{BD^2}{6}$, ...thus the pressure equation becomes:

$$p = \frac{N}{BD} \pm \frac{6M}{BD^2} \quad \textbf{10.3}$$

that is

$$\text{maximum } p = \frac{N}{BD} + \frac{6M}{BD^2} \quad \text{and} \quad \textbf{10.4}$$

$$\text{minimum } p = \frac{N}{BD} - \frac{6M}{BD^2} \quad \textbf{10.5}$$

In order to ensure that no negative pressure exists on the soil (that is, no tension on the soil), the minimum value of the minimum pressure would be zero. That is:

$$\frac{N}{BD} = \frac{6M}{BD^2}$$

$$ND = 6M$$

$$\frac{M}{N} = \frac{D}{6} \quad (\frac{M}{N} = eccentricity\ e) \quad \textbf{10.6}$$

Thus, for the minimum pressure to always be positive, then $\frac{M}{N} = eccentricity\ e$ must never be greater than $\frac{D}{6}$ $(that\ is\ e < \frac{D}{6})$. **In this case, the eccentricity of the loading is said to lie within the "middle third" of the base.**

However, for large a large bending moment, the eccentricity e will be greater than $\frac{D}{6}$. In this case there would be no positive across the entire length, and the pressure diagram will be triangular ("c").

Thus, balancing the downward load and the upward pressure, (and taking Y to be the length of the positive contact)

$$\frac{1}{6}.pBY = N \quad \textbf{10.7}$$

So that

$$p = \frac{2N}{BY}. \quad \textbf{10.8}$$

Now for the load and reaction to be equal and opposite, the centroid of the pressure diagram must coincide with the eccentricity of loading. That is:

$$\frac{Y}{3}. = \frac{D}{2} - e$$

$$Y = 3(\frac{D}{2}. - e)$$

So that

$$p = \frac{2N}{3B(\frac{D}{2}.-e)}. \qquad (\text{where } e = \frac{M}{N}) \quad \textbf{10.9}$$

This is the case of e > $\frac{D}{6}$.)

10.4.2 Shear in Pad Footing

Shear must be checked in two critical situations

Shear along the vertical section

The footing slab may fail in shear across the full width of the base, and this can generate crack starting at about 1.0d from the face of the column at ultimate limit state (Figure 10.13)

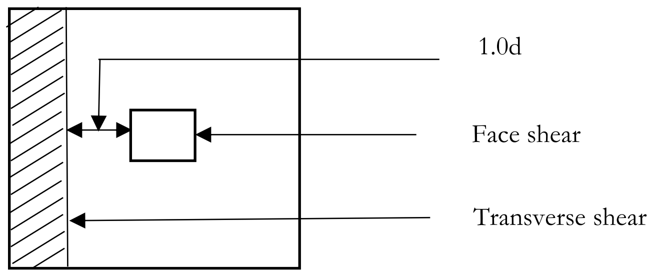


Figure 10.13: Sections to be considered for Face Shear

Punching Shear Around the column

A column base may also fail in shear at the ultimate limit state along a line at 1.5d from the face of the column (Figure 10.14).

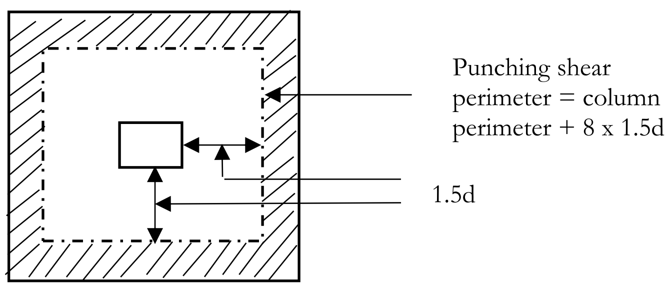


Figure 10.14: Critical Perimeter for punching shear

The shear force producing failure is the volume of the pressure diagram outside the perimeter.

Example 10. 3

A 350 x 350 mm column is required to transmit a dead load of 1000KN and live load of 500KN. If the safe bearing capacity of the soil is 200KN/m^2, design a suitable footing for the column. Use fcu = 30N/mm^2 and fy = 460N/m^2.

Solution

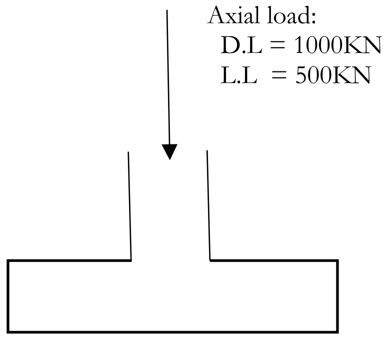


Figure 10.15: Example 10.3

Loadings

i. The Dead Load = 1000KN

   a. The self-weight of the footing (assume 10% of Dead load for estimate) = 100KN

   The total dead load = 1100KN

ii. Live load = 500KN

Loading for determination of plan area

Design Load at Serviceability = 1.0Gk + 1.0 Qk = 1.0(1100) + 1.0(500)

= 1100 + 500

= 1600KN.

Area of Footing A

$$A = \frac{Design\ Load\ at\ Serviceability}{bearing\ capacity\ of\ the\ Soil} = \frac{1600}{200} = 8m^2$$

Therefore

Choose a square base of 3 x 3m (9m^2)

Thus, the depth h of the base

$$h = \frac{weight\ of\ the\ base}{area\ of\ the\ base\ x\ density\ of\ concrete} = \frac{100}{9\ x\ 24}$$

= 0.463m

Use h = 450 mm (with this h, the weight of the base is 97.2 < 100KN)

Loading for calculation for bending and reinforcement

The earth pressure (p_e) will have to be determined based on the ultimate limit load.

Loading at the ultimate **(excluding the self-weight)** = 1.4Gk + 1.6Qk

= 1.4 (1000) + 1.6(500) = 1400 + 800

= 2200KN.

The earth pressure (p_e) = $\frac{\textit{Design Load at Ultimate}}{\textit{the chosen Area of the Base}}$

= $\frac{2200}{9}$

= 244.44KN/m (for 1 meter width of footing slab)

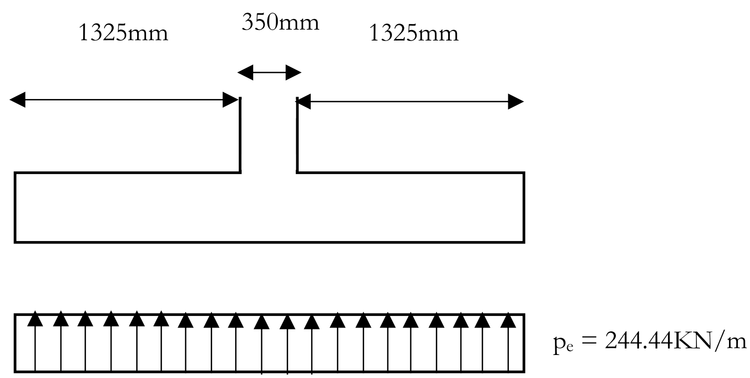


Figure 10.16: Soil pressure distribution under the footing

The maximum moment at the face of the column, M (treating as cantilever out of the column face)

$$M = \frac{p_e l^2}{2} = \frac{244.44 \ x\ 1.325 \ x\ 1.325}{2}$$

= 214. 57KN.m

Assuming 20mm diameter, and that the base is to be cast against blinding, take cover = 50mm

NOTE

The code recommendation for concrete in the ground (e.g. foundations) which may be subject to chemical attack, possibly due to the presence of sulphates, magnesium or acids in the soil and/or groundwater. Where concrete is cast directly against the earth the nominal depth of concrete cover should be at least 75 mm whereas for concrete cast against blinding it should be at least 50 mm (Arya, 2008)

Thus,

d = h – cover – half bar diameter = 450 – 50 – 10 = 390 mm

The moment of the section, Mu = 0.156fcubd^2 = 0.156 x 30x 1000 x 390 x 390

= 711. 83KN.m

Since Mu > M, no compression reinforcement is required

Thus

$$K = \frac{M}{f_{cu}bd^2} = \frac{214.57 \ x\ 1000000}{30\ x\ 1000\ x\ 390\ x\ 390} = 0.05$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$$

= d[0.5 + √(0.25 – 0.056]   = d[0.5 + 0.44] = 0.94d < 0.95d

= 0.94d = 0.94 x 390 = 366.60 mm

Thus

$$\textbf{As} = \frac{M}{0.87f_y z} = \frac{214.57\ x\ 1000000}{0.87\ x\ 460\ x\ 366.60}$$

= 1462.51mm^2

Use Y20@200 (1570mm^2) parallel to both x-x and y-y axes (Cl. 3.11.3.2)

Critical Shear Check

The Punching Shear

Figure 10.17 is the arrangement for punching shear check.

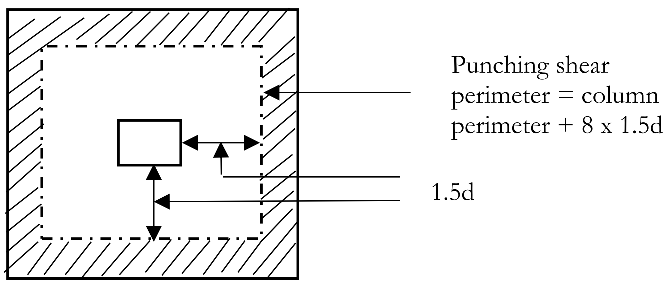


Figure 10.17: Punching shear check

The critical perimeter, p_{cr} is:

p_{cr} = column perimeter + 8 x 1.5d

= 350 x 4 + 8 x 1.5 x 390

= 6080mm

Area within the perimeter = (350 + 3d)^2

= (350 + 1170)^2 = 2310400mm^2 (2.31m^2)

The ultimate Punching force, V = earth pressure x (total plan area – area within critical perimeter)

$= 244.44\ (9 - 2.31)$

$= 1635.30KN$

The punching shear stress, v = $\frac{Ultimate\ punching\ shear}{critical\ perimeter\ x\ effective\ depth}$

$= \frac{1635.30\ x\ 1000}{6080\ x\ 390}$

$= 0.69\ N/mm^2$

But

$\frac{100As}{bd} = \frac{100\ x\ 1570}{1000\ x\ 390} = 0.401N/mm^2$

From Table 3.8

$v_c = 0.49\ x\ (\frac{30}{25})^{0.333} = 0.52N/mm^2$

Since $v_c < v$, punching failure is likely. One of the solution is to increase the total depth of the slab.

Therefore, increase the depth to 600 mm

So that d

d = h – cover – half bar diameter = 600 – 50 – 10 = 540 mm

Moment (Mu) of the section is now

$Mu = 0.156fcubd^2 = 0.156\ x\ 30x\ 1000\ x\ 540\ x\ 540 = 1364.69KN.m$

Since Mu > M, no compression reinforcement is required

Thus

$K = \frac{M}{f_{cu}bd^2} = \frac{214.57\ \ x\ 1000000}{30\ x\ 1000\ x\ 540\ x\ 540} = 0.030$

$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}]$

$= d[0.5 + \sqrt{}(0.25 - 0.033] = d[0.5 + 0.47] = 0.97d > 0.95d$

$= 0.95d$

$z = 0.95d = 0.95\ x\ 390 = 370.50\ mm$

Thus

As = $\frac{M}{0.87f_yz}$ **=** $\frac{214.57\ x\ 1000000}{0.87\ x\ 460\ x\ 370.5}$

$= 1447.\ 12mm^2$

Use Y20@200 ($1570mm^2$) parallel to both x-x and y-y axes (Cl. 3.11.3.2)

Check for Punching shear

p_{cr} = column perimeter + 8 x 1.5d

$= 350\ x\ 4 + 8\ x\ 1.5\ x\ 540$

$= 7880mm$

Area within the perimeter = $(350 + 3d)^2$

$= (350 + 1620)^2 = 3880900mm^2\ (3.88m^2)$

The ultimate Punching force, V = earth pressure x (total plan area – area within critical perimeter)

= 220 (9 – 3.88)

= 1331.20KN

The punching shear stress, $v = \frac{Ultimate\ punching\ shear}{critical\ perimeter\ x\ effective\ depth}$

$= \frac{1331.20\ x\ 1000}{7880\ x\ 540}$

= 0.32 N/mm²

But

$$\frac{100As}{bd} = \frac{100\ x\ 1570}{1000\ x\ 540} = 0.29\text{N/mm}^2$$

From Table 3.8

$$v_c = 0.41\ x\ (\frac{30}{25})^{0.333} = 0.44\text{N/mm}^2$$

Since $v_c > v$, punching failure is unlikely, thus the depth of 600 mm is acceptable

Face Shear

The maximum shear stress occurs at the face of the column

$$v_{max} = \frac{Ultimate\ design\ load}{column\ perimeter\ x\ effective\ depth} = \frac{2200\ x\ 1000}{4\ x\ 350\ x\ 540} = 2.91\text{N/mm}^2$$

But the permissible

$v = 0.8\sqrt{30} =$ 4.38N/mm². Since $v_{max} < v_{per}$, shear face failure at the face is not likely

Transverse Shear

The ultimate shear force (V) to be considered is that on the shaded area.

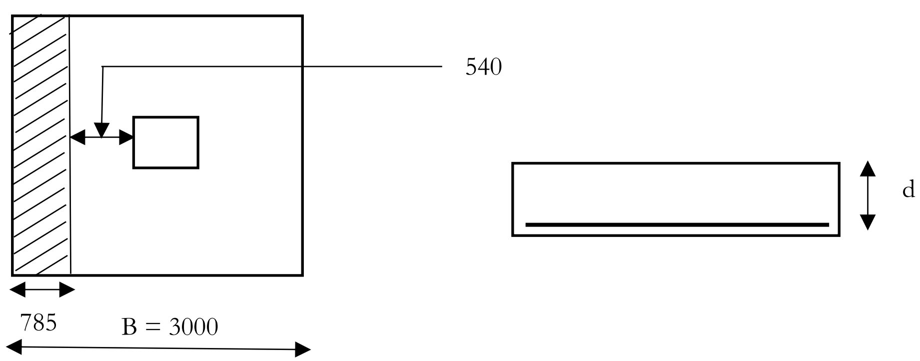


Figure 10.17: Transverse shear check

V = earth pressure x A

= 244.44 x 3 x 0.785 = 575.66KN

Design shear stress,

$$v = \frac{V}{Bd} \quad \frac{575.66 \; x \; 1000}{3000 \; x \; 540} = 0.36\text{N/mm}^2$$

Since $v < v_c$ no shear reinforcement is required.

The sketch of reinforcement arrangement is presented in figure 10.18.

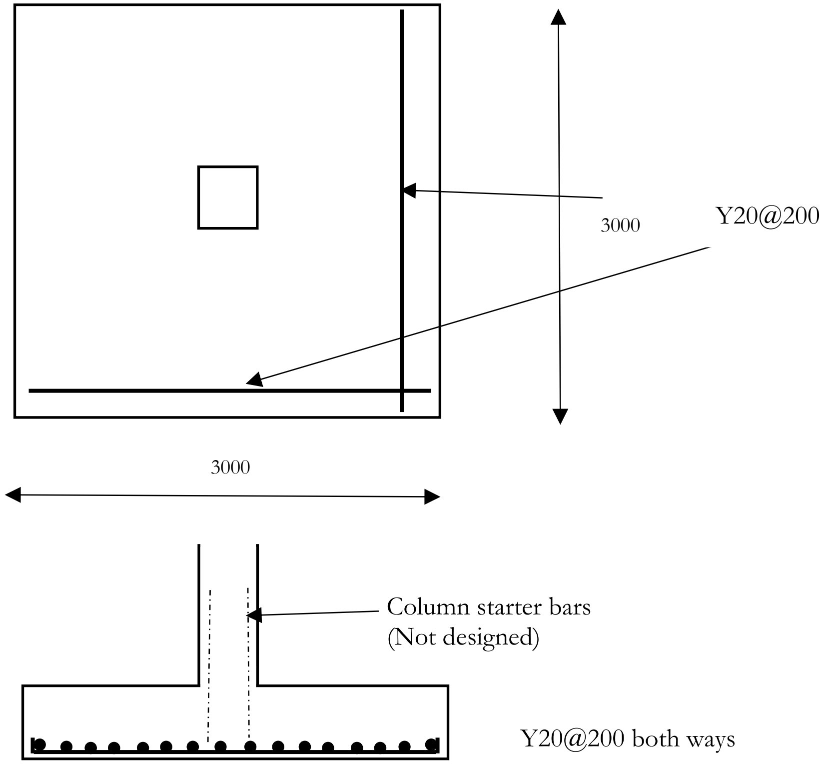


Figure 10.18: Sketch of reinforcement arrangement

Example 10.4

If the footing in Example 10.3, is to transmit additional clockwise moment of 1000KN.m, obtain an appropriate dimension for the base of the footing.

Solution

The sketch of the footing is shown in Figure 10.19.

For the footing to be wholly compressive, the resultant of soil pressure – due to the axial force and the bending moment - must fall within the middle third. If B is taken in the direction to resist the moment, an optimal rectangular dimensions (BD) is obtained.

From equation 10.3 - 5, the pressure equation is given by:

$$p = \frac{N}{BD} \pm \frac{6M}{BD^2} \qquad \textbf{10.3}$$

We can obtain the minimum B required for the minimum p to be equal to zero

That is,

$$e = \frac{M}{V} = \frac{1000}{1600} = 0.625 \quad (V = 1600KN \text{ from Example 10.3})$$

$$= \frac{B}{6} \rightarrow B = 3.75m$$

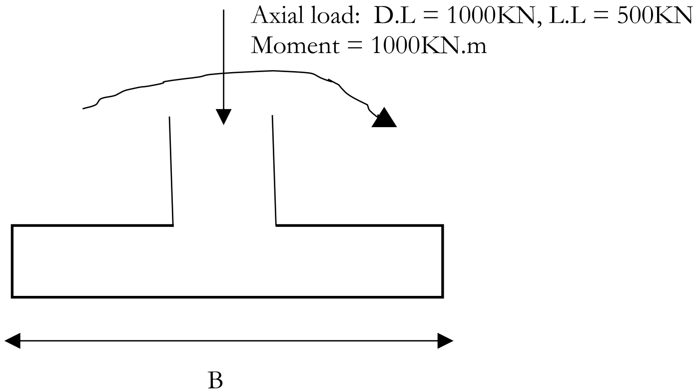


Figure 10.19: The Footing with Axial load and Mending Moment

This is the minimum B to obtain triangular pressure distribution under the footing. Once minimum B is known, values more than this minimum can be used in equation 10.4 to obtain D using the safe bearing capacity of the soil. Now, equation 10.3 can be re-arranged as follows as:

$$p = \frac{N}{BD} \pm \frac{6M}{BD^2} = \frac{N}{BD}\left(1 \pm \frac{6M}{DN}\right) = \frac{N}{BD}\left(1 \pm \frac{6e}{B}\right)$$

So that

$$\text{maximum } p = \frac{N}{BD}\left(1 + \frac{6e}{D}\right) \rightarrow 200 = \frac{1600}{3.75D}\left(1 + \frac{6 \, x \, 0.625}{3.75}\right)$$

Solving gives D = 4.23m

The results of solving for other values is shown in Table 10.2.

Table 10.2: Values of D corresponding to B values

B (m)	D (m)	A (m^2)
3.75	4.23	15.863
4.00	3.88	15.52
4.25	3.54	15.05
4.50	3.25	14.63
4.75	3.01	14.30
5.00	2.8	14.0

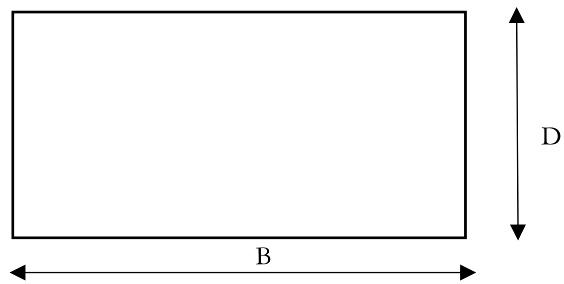


Figure 10.20: The Plan area of the footing

Depth of Footing and Anchorage bond

Another way of getting initial estimate of the depth of pad footing is to use the anchorage bond length expression of equation 6.32 with Table 6.2. That is,

$$la = \frac{0.87 f_y \varphi}{4\beta\sqrt{f_{cu}}} \qquad \textbf{6.32}$$

The minimum depth is obtained

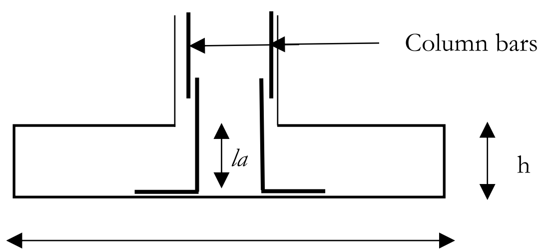


Figure 10.21: The depth/thickness of the base of pad footing

The minimum depth/thickness, h of the footing is:

h = anchorage bond length + cover + 2 x diameter of main reinforcement

$$= l_a + \text{cover} + 2\varphi \qquad \textbf{10.4}$$

When the dimensions of the base are large, it may be more economical in concrete to adopt a stepped base as shown in Figure 10.22.

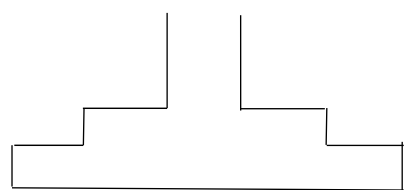


Figure 10.22: Stepped Base

Assignment

Re-design the pad footing of Example 10.3 by using equation 10.4 to determine the thickness of the base.

10.5 Combined Foundation

When columns are closed spaced together, it is always convenient or necessary to combine their footings in the form of continuous base,

Case A – for two columns only. The shape may rectangular or trapezoidal (Figure 10.23)

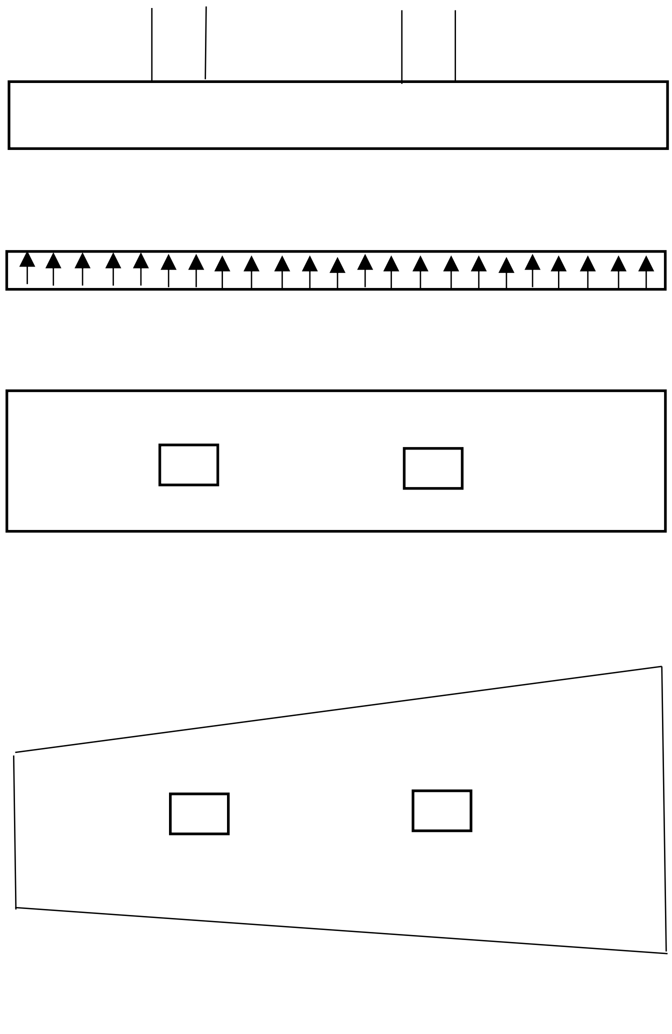


Figure 10.23: Compound footing for two columns

Case B – for more than two columns

Typical of foundation on weak soil. Can also be trapezoidal in plan.

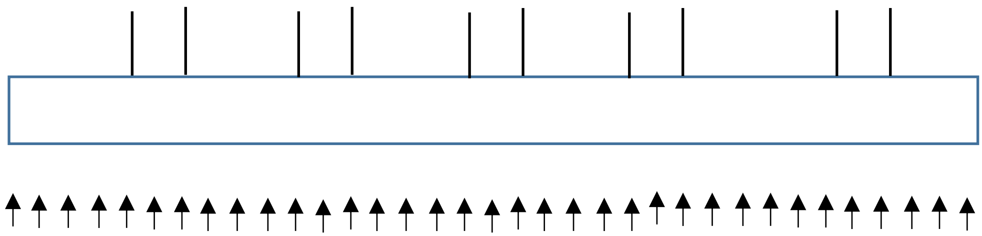
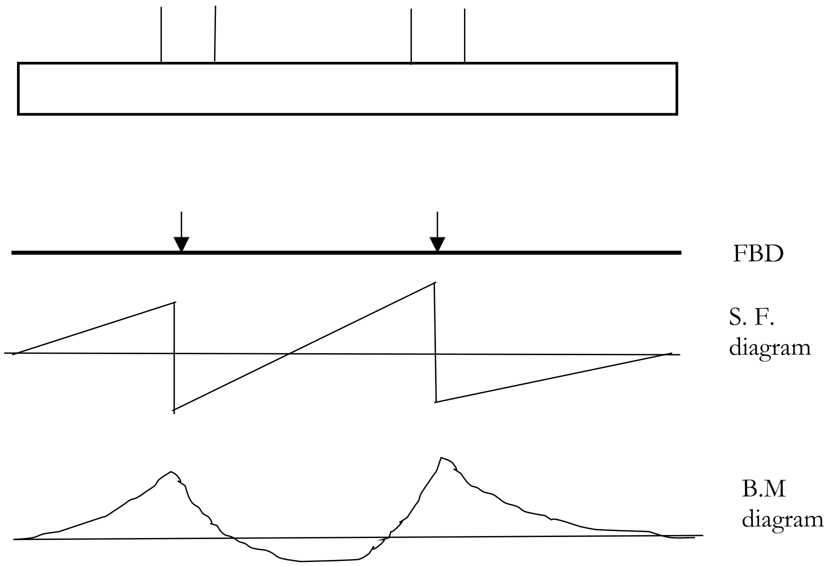


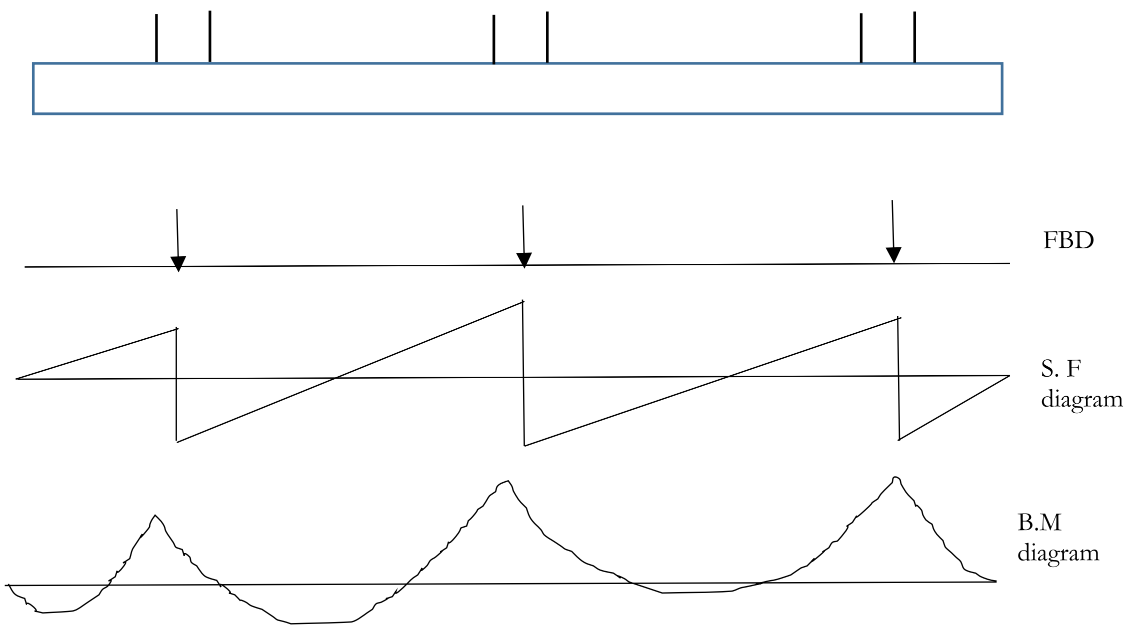
Figure 10.24: Compound footing for more than two columns

When determining the size of the foundation, it is necessary to keep the soil stress as uniform as possible. For combined footing, this is very important so as to minimize the possibility of differential settlement and the possible tilting of the structure. To achieve this, the resultant of all column forces to be transmitted by the foundation should act as nearly as possible through the centroid of the plan area of the foundation. The soli stress is then determined in exactly the same way as for a footing supporting a single column, by treating the resultant Ras a single column load.

The bending moments and shear force can now be calculated, by reinstating the original load configuration, by principles of statics. The loading diagram may now be visualized as turned upside down so that the columns appear as supports to the beam for which the reactive forces are already known and the distributed soil bearing stress appears as the load. Typical bending moments an shear force diagrams for combined footing are shown in figure 10.25.



a: for 2 columns



b: for more than 2 columns

Figure 10.25: Bending moments and shear forces for compound footing

Example 10.5

A footing is to support two columns 3.5m center-to center apart. The dimension of each of the column is 300 x 300mm. the loads on the column 1 is gk = 1500KN and qk = 2000KN, while that of column 2 is gk = 1000KN and qk = 1500KN. Design a suitable combine footing for the columns if projection from column 1 is 0.5m. Materials properties are: f_{cu} = 30N/mm², f_y = 460N/mm², the safe bearing capacity of the soil = 200KN/m².

Solution

Loadings

Column 1 → gk = 500KN, qk = 500KN, → SLS = 1000KN

Column 2 → gk = 1000KN, qk = 700KN, → SLS = 1700KN

Serviceability design load (for the 2 columns) = 1.0gk + 1.0qk

= 1.0 (500 + 500) + 1.0(1000 + 700) = 2700KN

Now, it is usual for the initial estimate to assume the self-weight of the base. It is usual to take the self-weight as between 10 – 15% of the serviceability load. For this example, it is taken as 10% (that is, 270KN)

The serviceability load = 2700 + 270 = 2970KN

Area (A) of the base

$$A = \frac{serviceability\ design\ load}{bearing\ capacity\ of\ the\ soil} = \frac{2970}{200} = 14.85m^2$$

Take A

A = 15m²

Location of the resultant R of loads from the center of column 1 (Figure 10.26).

Moment about column 1 (C1) and resultant R about column 2 (C2). R is at a distance x from column 2 (C2)

3.50 x 1000 = 3000x

x = 1.17m from column 2

x = 2.33m from column 1

To make the resultant R of the load fall on the centroid of the base (that is, at 0.5L), then

0.5L = 0.5 + 2.37

L = 5.66m

Then, with A = 15m² → L x B = 15

$$B = \frac{15}{5.66} = 2.65m$$

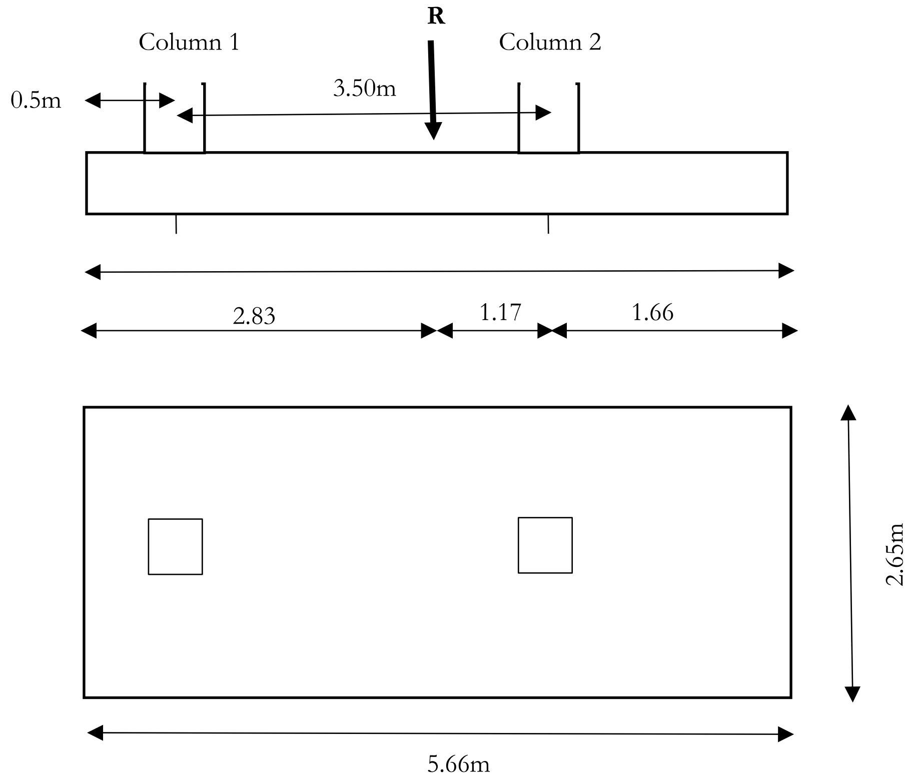


Figure 10.26: Example 10.5

Now the Design Load at Ultimate

= 1.4gk + 1.6qk

Column 1 = 1.4 (500) + 1.6 (500) = 1500KN

Column 2 = 1.4 (1000) + 1.6 (700) = 2520KN

Both columns = 1.4(500 + 1000) + 1.6(500 + 700)

= 4020KN

Therefore, the earth pressure (q_e) at the Ultimate limit

$$\mathbf{q_e} = \frac{\textit{Design Load at Ultimate}}{\textit{area of the base}} = \frac{4020}{15}$$

= 268KN/m^2

The longitudinal bending moment and shear force diagrams are shown in figure 10.27

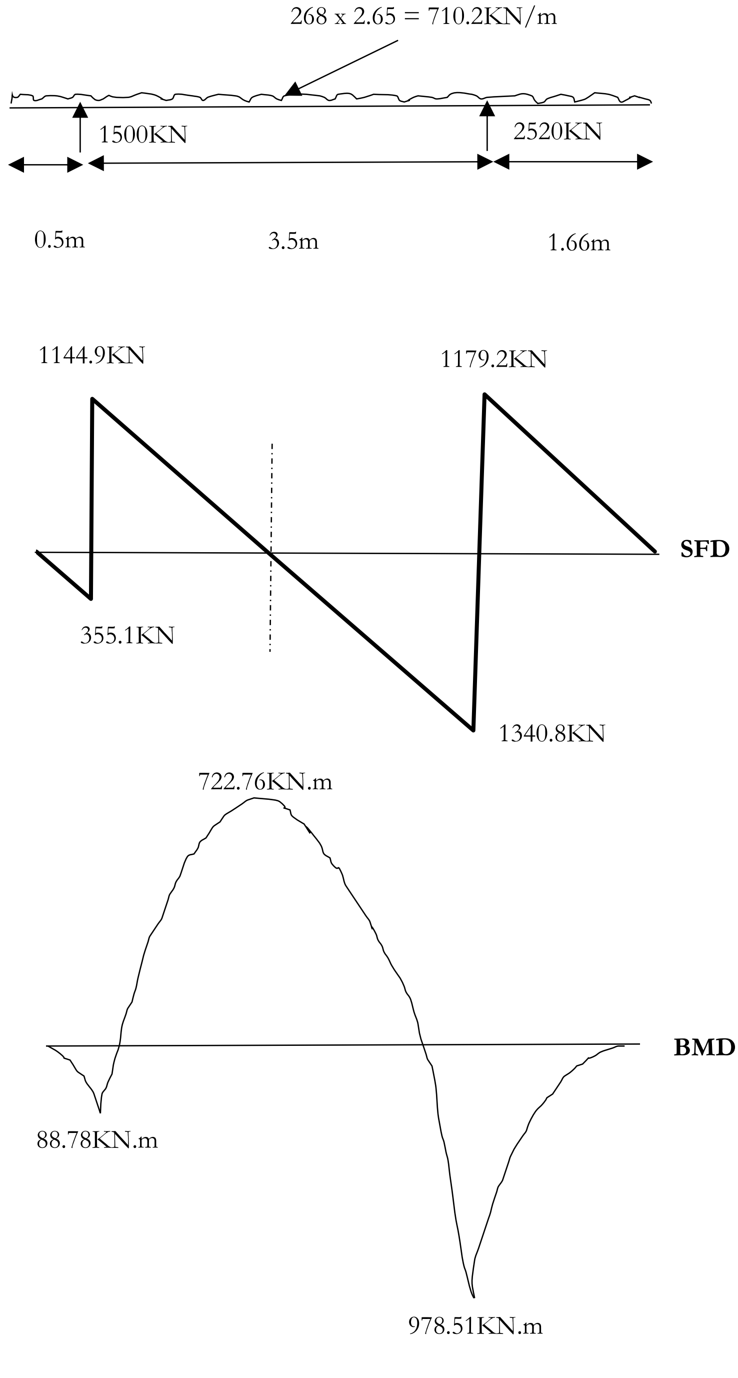


Figure 10.27: Longitudinal Shear Force and Bending Moment Diagrams

The depth of the footing can be estimated from equation 6.32

$$l = \frac{0.87 f_y \varphi}{4\beta\sqrt{f_{cu}}}$$

For deformed bars in compression, $\beta = 0.63$ (Table 6.2) and assuming 20 mm diameter bar

$$l = \frac{0.87 \ x\ 460\ x\ 20}{4\ x\ 0.63\ x\ \sqrt{30}} = 579.60 \text{ mm}$$

Assuming that concreting is cast against blinding, the cover = 50 mm (see note in Example 10.3)

Thus, from equation 10.4, the minimum depth of the footing is:

h = anchorage bond length + cover + 2 x diameter of main reinforcement

= 579.60 + 50 + 2 x 20 = 669. 60 mm

Therefore, take

h = 750 mm

d = 690 mm. (in the longitudinal direction)

Longitudinal Bending

At the Left Overhang, M = 88. 78KN.m

$$K = \frac{M}{f_{cu}bd^2} = \frac{88.78\ x\ 1000000}{30\ x\ 2650\ x\ 690\ x\ 690} = 0.002$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{0.003}{0.9})}] = 0.99d$$

> 0.95d

= 0.95d

z = 0.95d = 0.95 x 690 = 655.5 mm

Thus

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{88.78\ x\ 1000000}{0.87\ x\ 460\ x\ 655.5}$$

= 338.43 mm²

Use Y12@300 (377mm²)

But minimum reinforcement = $\frac{0.13}{100}\ bh = = \frac{0.13}{100}$ x 2650 x 750 = 2583.75 mm²

Therefore

Provide Y20@100 (3140 mm²)

At the Right Overhang, M = 978. 51KN.m

$$K = \frac{M}{f_{cu}bd^2} = \frac{978.51\ x\ 1000000}{30\ x\ 2650\ x\ 690\ x\ 690} = 0.03$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{0.03}{0.9})}] = 0.97d$$

> 0.95d

= 0.95d

$z = 0.95d = 0.95 \times 690 = 655.5$.mm

Thus

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{978.51 \times 1000000}{0.87 \times 460 \times 655.5}$$

$= 3730.06 \text{ mm}^2$

Use Y20@75 (4190mm^2)

At the Midspan, M = 722. 76KN.m

$$K = \frac{M}{f_{cu} b d^2} = \frac{722.76 \times 1000000}{30 \times 2650 \times 690 \times 690} = 0.02$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{0.02}{0.9})}] = 0.98d$$

$> 0.95d$

$= 0.95d$

$z = 0.95d = 0.95 \times 690 = 655.5$ mm

Thus

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{722.76 \times 1000000}{0.87 \times 460 \times 655.5}$$

$= 2755.15 \text{ mm}^2$

Use Y20@100 (3140mm^2)

The Transverse Bending

Near the columns, bending takes place in both longitudinal and transverse directions. The transverse steel has to be provided to take care of the transverse moment.

In the transverse bending, d = 750 – 50 – 30 = 670 mm

Transverse Moment

$$Mt = 268 \times \frac{1.325^2}{2} = \textbf{235.25KN.m}$$

$$K = \frac{M}{f_{cu} b d^2} = \frac{235.25 \times 1000000}{30 \times 1000 \times 670 \times 670} = 0.008$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{0.008}{0.9})}] = 0.99d$$

$> 0.95d$

$= 0.95d$

$z = 0.95d = 0.95 \times 620 = 589$ mm

Thus

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{235.25 \times 1000000}{0.87 \times 460 \times 589}$$

$= 998.02 \text{ mm}^2$

Use Y20@300 (1050mm^2)

But minimum reinforcement = $\frac{0.13}{100}\ bh$ = = $\frac{0.13}{100}$ x 1000 x 750 =975 mm^2

Therefore, the provided Y20@ 300 (1050 mm^2) is OK.

It is usual to place the transverse steel over a minimum width extending to the equivalent of the effective depth from the face of the column as shown in Figure 10.28.

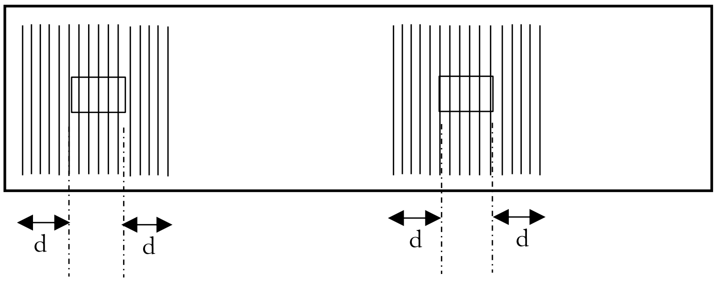


Figure 10. 28: Placement of transverse reinforcement

Shear Design (Shear Stress (5N/mm^2 or 0.8√fcu, that is, 4.38N/mm^2, whichever is less)

According to BS 8110, punching shear is checked at the critical perimeter which is at 1.5h from column face. From figure 10.29, the critical perimeter (1.5h = 1.5 x 0.75 = 1.125) for column 1 lies outside the base area.

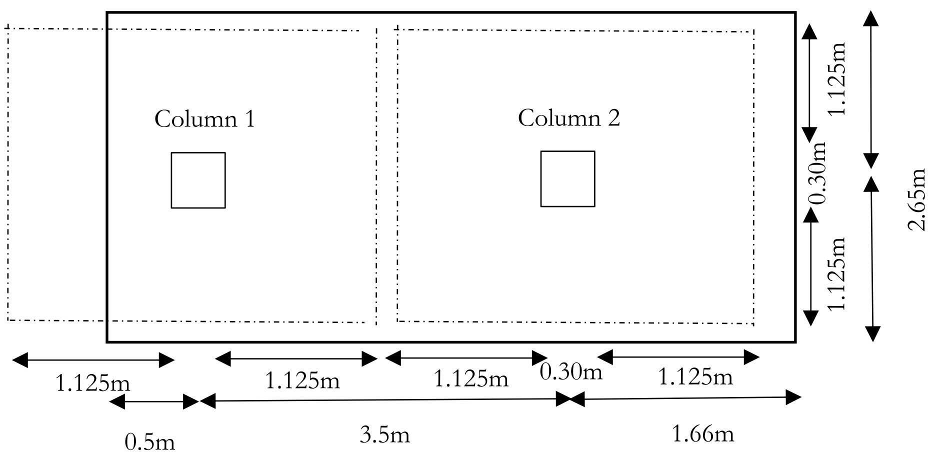


Figure 10. 29: Critical perimeter for punching shear check

Thus, punching shear cannot be checked for column 1. However, since the footing is a thick slab with bending in both directions, the critical sections for shear can be taken as 1.5d from the face of the column. Using the maximum shear force

V = 1144.90 –1.185 x 710.2 (Note: 1.5 x d + $\frac{1}{2}$ x column dimension = 1.5 x 690+150 = 1.185m)

$V = 303.43KN.$

Thus,

$$v = \frac{V}{bd} = v = \frac{303.43 \times 1000}{2650 \times 690}$$

$= 0.17 \text{ N/mm}^2$

Also

$$\frac{100As}{bd} = \frac{100 \times 3140}{1000 \times 690} = 0.46\text{N/mm}^2$$

From Table 3.8

$$v_c = 0.484 \text{ x } (\frac{30}{25})^{0.333} = 0.51\text{N/mm}^2$$

$> 0.17\text{N/mm}^2$. Hence OK.

For column 2

The critical perimeter, p_{cr} is:

p_{cr} = column perimeter + 8 x 1.5d

= 300 x 4 + 8 x 1.5 x 690 (Figure 10.29)

= 9480 mm (9.48m)

Area within the perimeter = $(300 + 3d)^2$

$= (300 + 2070)^2 = 5.62\text{m}^2$

The ultimate Punching force, V = earth pressure x (total plan area – area within critical perimeter)

With the earth pressure in this case

$$\frac{design\ load\ at\ ultimate}{the\ are\ of\ the\ base} = \frac{1.4 \times 1000 + 1.6 \times 700}{3.41 \times 2.65} = 278.87 \text{ KN/m per 1 m strip}$$

Then,

The Punching force, V = 278.87(9.04 – 5.62)

= 953.74KN

The punching shear stress, $v = \frac{Ultimate\ punching\ shear}{critical\ perimeter\ x\ effective\ depth}$

$$= \frac{953.74 \times 1000}{9480 \times 690}$$

$= 0.15 \text{ N/mm}^2$

But

$$\frac{100As}{bd} = \frac{100 \times 3730}{1000 \times 690} = 0.541\text{N/mm}^2$$

From Table 3.8

$$v_c = 0.50 \text{ x } (\frac{30}{25})^{0.333} = 0.53\text{N/mm}^2$$

Since $v_c > v$, punching failure is not likely to occur. The total depth of the slab is OK.

Also, the critical sections for shear taken at 1.5d from the face of the column 2, as in column 1. Using the maximum shear force

$V = 1340.80 - 1.185 \times 710.2$ (Note: $1.5 \times d + \frac{1}{2} \times$ column dimension = $1.5 \times 690 + 150 = 1.185$m)

V = 499.23KN.

Thus, the shear stress is:

$$v = \frac{V}{bd} = v = \frac{499.23 \times 1000}{2650 \times 690}$$

$$= 0.273 \text{ N/mm}^2$$

Also

$$\frac{100As}{bd} = \frac{100 \times 4190}{1000 \times 690} = 0.61\text{N/mm}^2$$

From Table 3.8

$$v_c = 0.503 \times (\frac{30}{25})^{0.333} = 0.54\text{N/mm}^2$$

$> 0.273\text{N/mm}^2$. Hence OK.

Thus, shear reinforcement is not required for both columns 1 and 2. The sketch of the reinforcement arrangement is shown in figure 10.30.

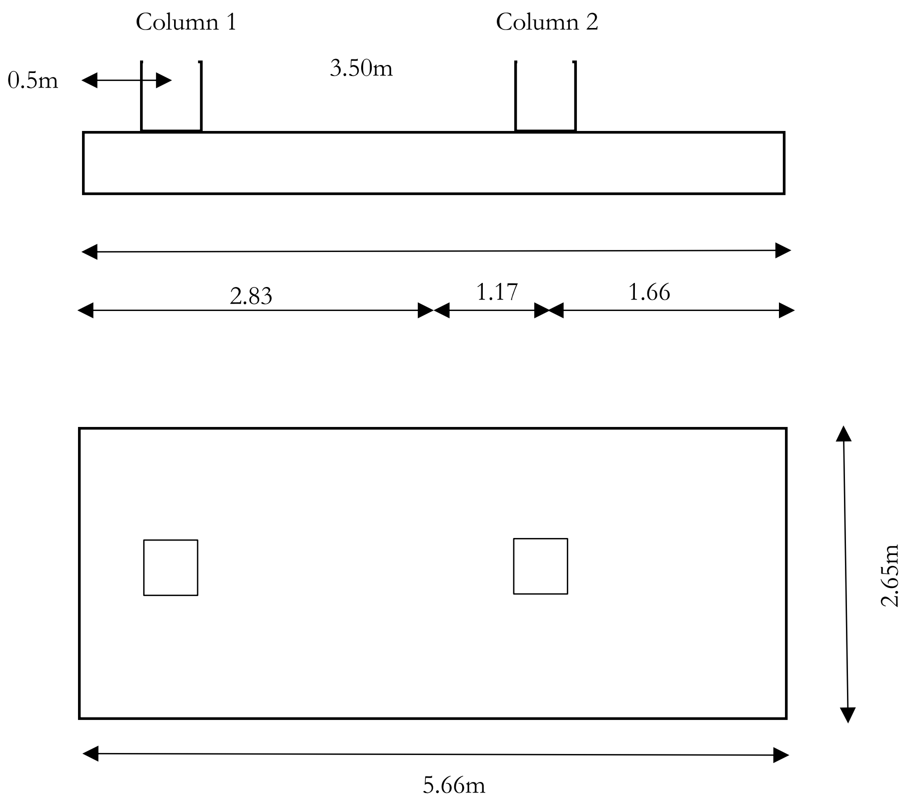


Figure 10.30: Arrangement of reinforcement

10.6 Design of others Types of Foundation

Strap Foundation

Closely related to pad footing is the strap footing (Figure 10.31). This type of foundation is used where the base for an external column must not project property line. In that situation strap foundation is constructed to link the base of the exterior column affected to the interior column. The purpose of the strap is to restrain the overturning moment due to the eccentric load on the exterior column base. In strap footing, it is necessary that resultant of the loads on the two columns pass through the centroid of the areas of the two bases. Thus, the base areas are proportioned in such a way that the bearing pressures are uniform and equal under both bases.

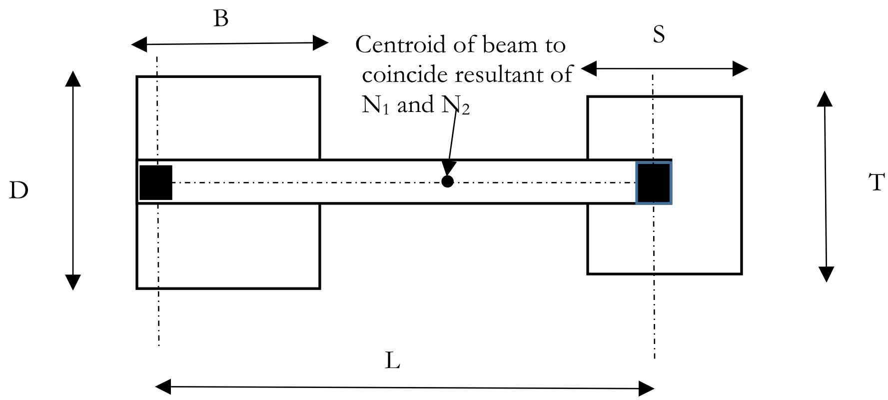


Figure 10. 31: Strap Foundation

Raft Foundation

A raft foundation (Figure 10.32) is used to transmit loads to the ground by means of reinforced concrete slab that is continuous over the base of whole structure. The slab is able to receive all the downward loads and distribute them over a wide area large enough to avoid over-stressing the soil beyond its bearing capacity. Raft foundation are used in the following cases:

1) Founding foundation over a weak soil - Thus the raft is able to span any areas of weaker soil and spread its load over a wide area
2) For heavily loaded structure → one continuous base in preference to many closely-spaced separate footings is often provided
3) Situations where settlement is a problem → raft foundation is used. In this case, if settlement eventually occurs, it is uniform or make the structure tilt, but do not lead to cracking.

Raft foundations which are below the level of the water table should be checked to ensure that they are able to resist the uplift forces due to the hydrostatic pressures.

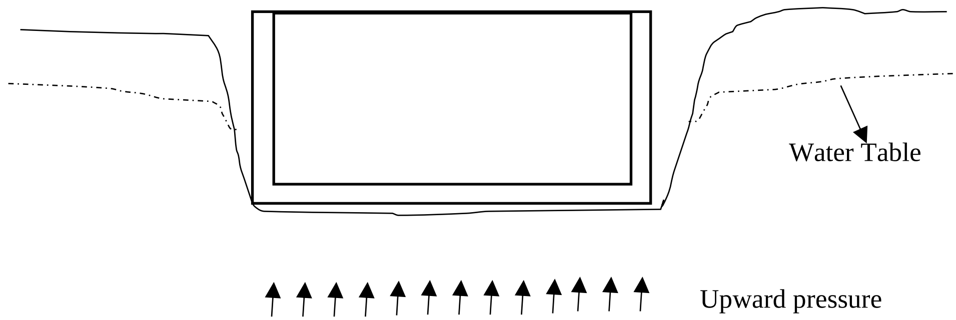


Figure 10. 32: Raft Foundation

This may be critical during construction, it may be necessary to provide extra weight to the raft or lower the water table by pumping. Alternatively, the slab can be anchored down with short tension piles.

Types of Raft Foundation

Many types of raft, namely (i) Slab raft, (ii) Slab and Beam raft, (iii) Cellular raft.

1) Slab Raft

   Slab raft is a uniform thickness of slab over the entire structure through which loads are transmitted over entire area. Slab rafts are used where single isolated pads footing are closely spaced or tend to overlapped one another. In those situations, it would be cheaper and easier to provide and construct slab raft. Slab thickness is usually between 150 and 300 mm. Thickness in excess of 300 mm are also found depending on the loads.

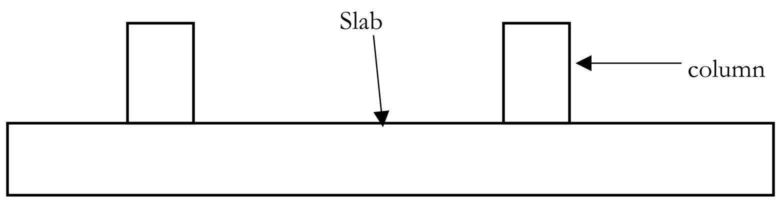


Figure 10. 33: Typical Slab Raft

   The slab raft is usually designed as an inverted flat slab. Thus, the bending moments coefficient are applicable as in ordinary flat slab.

2) Slab and Beam Raft

   The slab and beam raft are employed where loads due to the building is such that it will result in very thick slab, if slab raft were to be used. The slab and beam raft can be with **downstand** beam or upstand **beams.** However, the slab and beam raft with downstand beams are favored since the slab can also be used as oversite concrete (slab and beam raft with upstand beams require additional oversite concrete). The slab and beam raft with downstand beams are commonly used in Nigeria, and depths ranges from 1050 – 1500 mm.

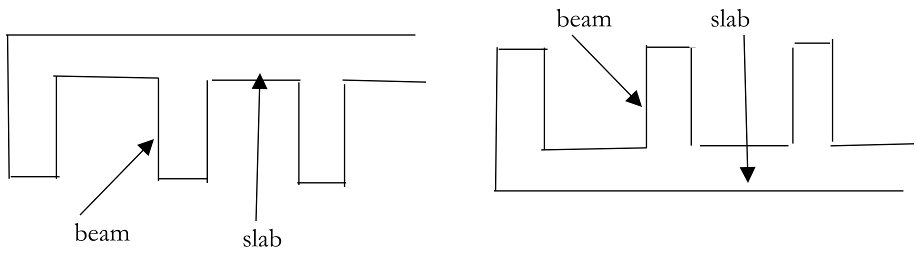


Figure 10.34: (Slab and Beam Raft (a) downstand beam (b) Upstand beam

The slab and beam raft are designed as an inverted slab and beam system. It usual during construction to provide blinding under the raft beam of between 50 and 75 mm.

3) Cellular Raft

When a very weak soil is encountered, cellular raft can be employed. This type is rarely used because it is expensive.

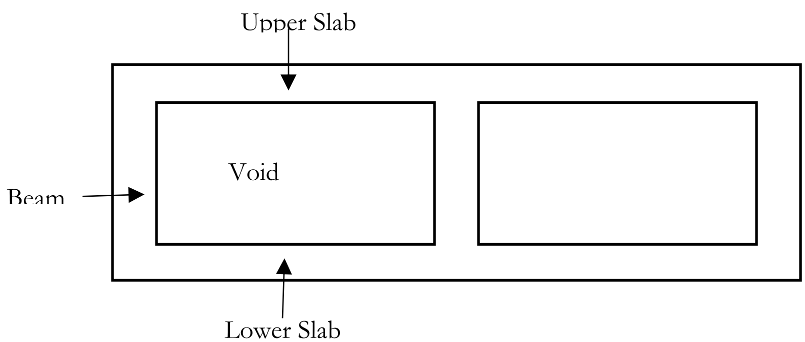


Figure 10. 35: Cellular Raft

Generally, the design of raft foundation involves the estimation of all, loads on the building, and then location the centroid of these loads. The center of the raft foundation is made to coincide with the centroid of the loads in order to achieve a uniform pressure distribution on the soil underlying soil. When site conditions do not allow this, the eccentricity e must be calculated, and the pressure calculated for each corner of the building.

Pile Foundation

Pile can be defined as a column of concrete or steel filled with concrete driven into the ground or cast in-situ below the surface of the soil for the purpose of transmission of loads from the structure to a firmer soil stratum below. Pile foundations are used when soil conditions are so poor that it becomes uneconomical or not possible to provide adequate spread footings. Piles carried load either by

a. End bearing
b. Friction

End bearing piles are those that transmit loads by the bearing at the toe of the pile on substratum. This is useful where the subsoil on which the pile will terminate is a hard stratum. For example, rock, compact sand, etc. Friction piles are those who rely on the frictional resistance of the pile and the surrounding soil strata for load transmission. Such piles behave like suspended columns held in position by the adhesive properties of the soil.

Generally, piles can be used in the following cases

1) When the design load of the structure cannot be sufficiently spread without exceeding the ground bearing capacity.
2) When design load of the structure does not result in the ground bearing capacity being exceeded but however resulted in unacceptable settlement.
3) Where a more firmer and more reliable stratum can easily be reached easily and economically instead of locating the foundation on shallow depth with unpredictable levels of settlement behavior
4) When the structure has to be built on soil strata liable to swelling and shrinkage. For example, areas previously used as landfills, etc.
5) When the structure is to be built over water such as bridges, jetties, etc.
6) Where lateral loads on the structure predominate. For example, tall structures, berthing vessels, arches, etc.
7) For underpinning works to arrest a failing foundation.

In addition to structural classification (end bearing and friction piles), piles can be classified according in the followings

1) **Classification based on modes of Construction**

   Two types exist: either (i) driven piles or (ii) bored piles.

   Driven piles are precast piles and driven to the prescribed depth

   Bored piles are constructed through the use of boring implement to the desired depth, and the hole then filled with concrete. Precautions must however be taken to prevent collapse of the hole during boring. Also, the end of the bore piles can also be enlarged to increase its load carrying capacity.

2) **Classification based on materials of construction**

On this basis, three types exist, namely, Concrete piles, Sheet piles and timber piles.

Pile Caps

Piles are usually designed and constructed in groups, except cases when large diameter bored single piles are used to support loads. A group usually consists of more than 2 piles. These piles are joined together by concrete structure referred to as pile caps. Pile caps are designed for moments and shear. They are however checked for punching shear. In order to resist busting action, lateral bars are usually provided. The sizes of the pile caps vary depending on the diameter of the pipe to be capped. Typical pile caps are shown in figure 10.36.

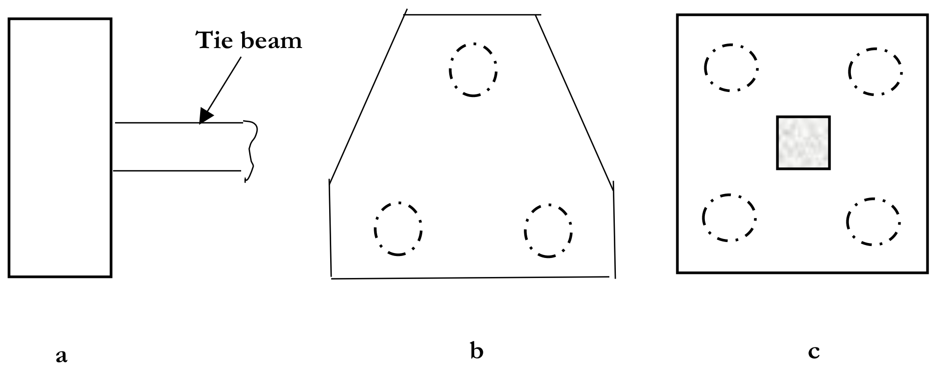


Figure 10. 36: Typical plans of pile caps: (a) Two pile cap, (b) Three pile cap and (c) Four pile cap

Depth of Pile Cap

Except for large diameter bored piles, where a single pile can be used to support or transmit the proposed load, piles are always design in groups of two (2) or more.

There are two approaches to the determination of the depth of pile caps

i. D = 2 x pile diameter + 100 mm
ii. $d = \sqrt{\frac{M}{0.156fb}}$ (d = effective depth)

Design of Pile Caps (BS 8110 cl. 3.1.14)

Pile caps are principally designed as beams with emphasis on tension reinforcement and shear. In order to prevent failure by bursting due to high tension, a cage of reinforcement (burst bars) in all the sides and in three dimensions are provided.

It is to be noted that the load-carrying capacity of group of piles is not necessarily a multiple of that for the single pile – usually less. For group of friction piles, the reduction can be in one-third. However, for a group of ends bearing piles, the load-carrying capacity is substantially the sum total of the resistance of each individual pile.

The design for shear is according to the provisions of Section 3.5.5. and 3.7.7 of BS 8110 (1997)

Chapter 11– Design of Reinforced Concrete Retaining Walls

11.1 Introduction

Retaining walls are structures that are used to provide stability for earth or other materials where topological condition disallows the mass of the material to assume natural shape. It is a term used to refer to a freestanding structure without lateral support at its top.

They are commonly used to hold back or support soil banks, oil or ore pipes, and in some cases water. Retaining walls can be grouped under three categories. These are:

1. Free-standing retaining wall
2. Retaining wall for basement construction
3. Retaining wall for bridge abutment

Free-standing retaining walls will be discussed in this section

11.2 Free Standing Retaining wall

Free-staining walls are classified into three basic types, based on the method employed to achieve stability, namely: (i) gravity walls and (ii) cantilever wall, and (iii) counterfort wall.

Gravity Walls

These are walls that depend on its weight, not only for strength, but also to satisfy stability requirements in relation to overturning and sliding. They are usually constructed of mass concrete, using a mix design that do not generate high hydration temperature with sound construction procedures and efficient curing techniques. Distribution reinforcements are included in the faces to control thermal and shrinkage cracking. Other construction materials for gravity walls include masonry and stone.

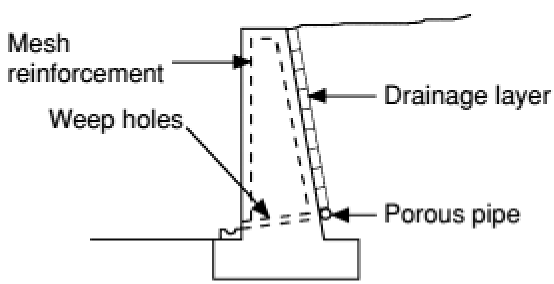


Figure 11: 1. A typical gravity wall

The dimensions of gravity wall are such that the width of base is about a third of the height of the retained material. Gravity walls are designed so that the resultant force on the wall due to the dead weight and the earth pressures is kept within the middle third of the base. It is usual to include a granular layer behind the wall and weep holes, in form of porous pipes, (Figure 11.1) near the base to minimize hydrostatic pressure behind the wall. The main advantages with this type of wall are simplicity of construction and ease of maintenance. Where walls up to 2 m in height are required, it is generally economical to choose a gravity retaining wall.

Cantilever Wall

This is the most common type of retaining wall, and it is used when the height of soil to be retained is up to 6m. Cantilever walls use cantilever to achieve structural stability. These are designed as vertical cantilever spanning from a large rigid base which often relies on the weight of backfill on the base for stability. A key is sometimes incorporated at the base of the wall in order to prevent sliding failure of the wall. The key can be under the stem or at the heel (Figure 11.2)

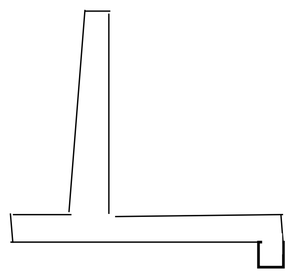


Figure 11. 2: A typical cantilever wall with key

Counterfort Retaining walls

Similar to cantilever wall, except that it is used where the cantilever stem is too long (above 6m), or when there are very high pressures behind the wall. It has a tie – called counterfort - which binds the base and the wall together built at interval to reduce the bending moments and the shear force (Figure 11.3). Also, the weight of the structure and the earth is used for stability.

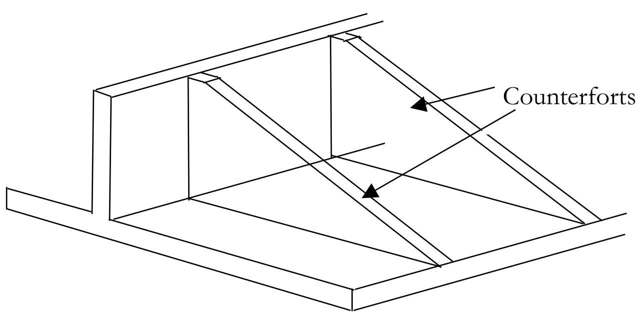


Figure 11. 3: A Counterfort wall

Generally, retaining wall must be of adequate proportion to be able to withstand or resist overturning, sliding in addition to being structurally sound.

Some terms used in a typical cantilever retaining wall is shown in Figure 11.4

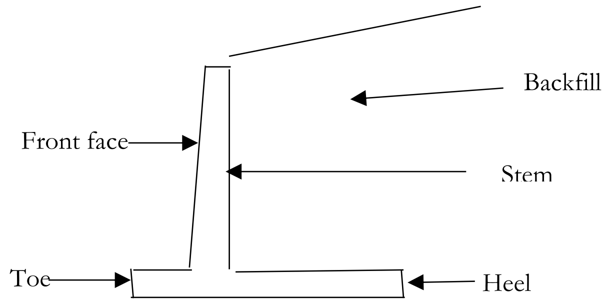
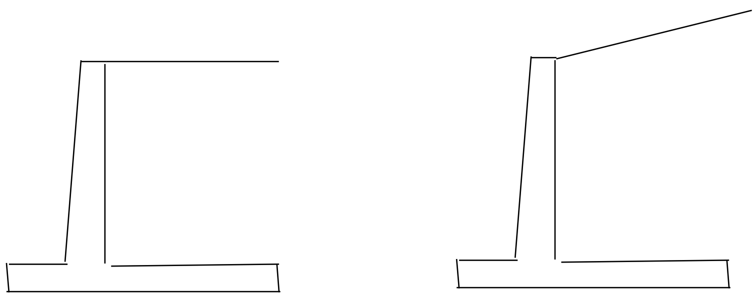


Figure 11.4: Parts of a typical retaining wall

The Backfill

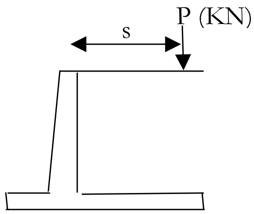
The material to be retained behind the wall is called the backfill. It can be a granular material, cohesive material, or both. If it is level, it is called a level backfill, and sloppy backfill, if it is inclined at an angle to the horizontal (Figure 11. 5).



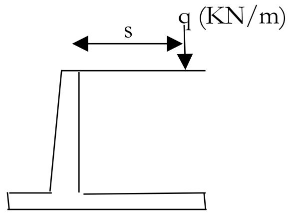
a. Level Backfill **b.** Sloppy Backfill

Figure 11. 5: Typical form of backfill: (a) Level backfill, (b) Sloppy backfill

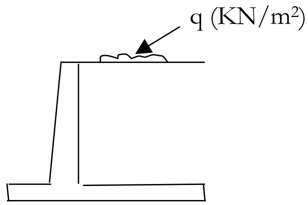
There are however instances, where the retaining wall will have to retain other form of material in addition to the backfill. These materials are called surcharge. Surcharge occurs in many forms. For example, a load (say a tower) may rest on the backfill so as to act as a point load. This is called **point load surcharge**. If a fence or concrete blockwall or conduit pipe is constructed and rest on the backfill, this is called **line load surcharge**. **Strip surcharge** results when a roadway or railway is constructed on top of the backfill. This is loading intensity over a finite width. Houses, when constructed on the backfill, are regarded as **area load backfill**. This is shown in Figure 11. 6. Each surcharge constitute additional load to the retaining wall and should be assessed appropriately.



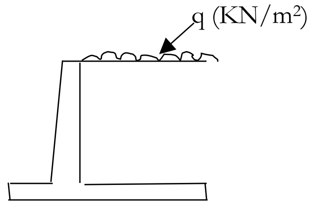
a. Point load surcharge



b. Line load surcharge



c. Strip load surcharge



d. Area load surcharge

Figure 11.6: Forms of surcharge

11.3 Soil Pressures in Retaining Walls

In the design of retaining wall, wall-soil interaction must be considered. The method most commonly used for determining the soil pressures on the wall, is based on Rankin's formula. Although conservative, it is straightforward to apply. The pressure on the wall resulting from the retained fill has a destabilizing effect on the wall, trying to push it away from the backfill, and is normally termed active pressure. This pressure increases with the depth and it is maximum at the base of the wall. The earth in front of the wall resists the destabilizing forces, trying to push toward backward, and is termed passive pressure. It also increases with the depth.

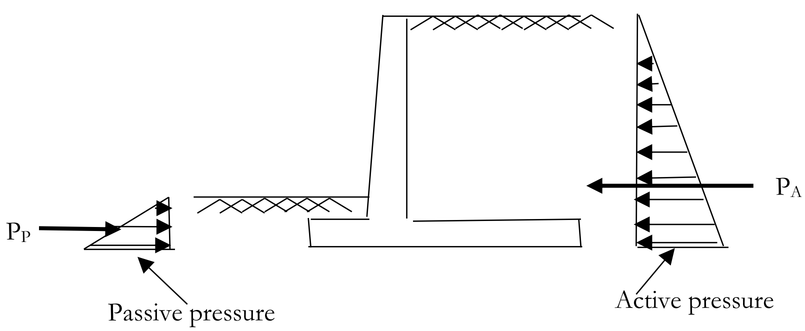


Figure 11. 7: Active and Passive pressures acting on the wall

If:

φ = angle of repose of the backfill (^{0})

ka = coefficient of active earth pressure

kp = coefficient of passive earth pressure

ko = coefficient of earth pressures

h = height of the retained backfill

From soil mechanics

$$k_a = \frac{1-Sin\emptyset}{1+Sin\emptyset} \qquad 1$$

$$k_p = \frac{1+Sin\emptyset}{1-Sin\emptyset} \qquad 2$$

$$ka = \frac{1}{kp} \qquad 3$$

When the soil moves away from the wall, that is, active earth failure pressure, this pressure (P_A) at a depth "h" is

$$P_A = ka\,\gamma h \qquad 4$$

Also, when the soil moves towards the wall, that is, passive earth failure pressure (P_F), then at a depth "h",

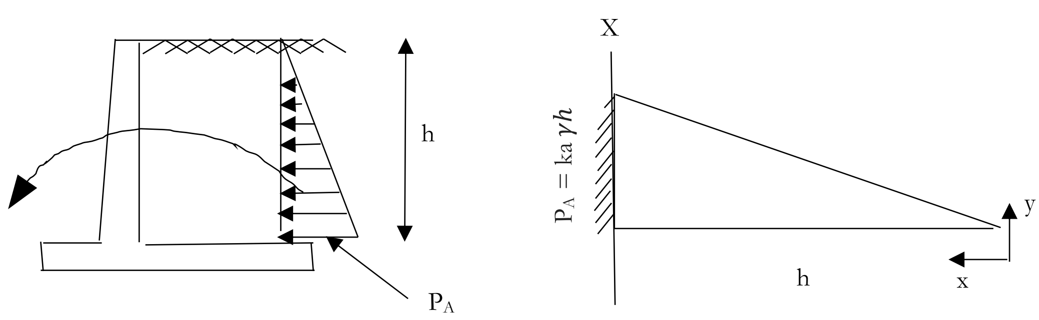
$$P_P = kp\,\gamma h \qquad 5$$

11.4 Bending Moments and Shear Force Calculation in Retaining Wall

Most walls are designed for active earth pressures, and the value of P_H varies with the height of the wall as shown in Figure 11.8.

The maximum active earth pressure P_A is at the depth h,

$$P_A = ka\,\gamma h$$



(a) **(b)**

Figure 11.8: Shear force and Bending moment calculation

The SF is the area of the triangular portion (Figure 11.8b) and the BM is the moment of this area about its centroid. That is:

$$\text{Mxx} = \frac{P_A h}{2} \text{ x } \frac{h}{3} \;\; = \frac{\text{ka}\,\gamma h.h}{2} \text{ x } \frac{h}{3} \qquad \textbf{6}$$

$$= \frac{\text{k}_\text{a}\gamma h^3}{6} \qquad \textbf{7a}$$

$$= (\frac{1-Sin\emptyset}{1+Sin\emptyset})(\frac{\gamma h^3}{6}) \qquad \textbf{7b}$$

So, the most critical section is the point at which the stem meets the base wall

11.5 Structural Design of Retaining Walls

As in the case of slabs, the design of retaining walls is usually based on a 1 m width of section. Structural failure of the wall may arise if the base and stem are unable to resist the vertical and horizontal forces due to the retained soil. The area of steel reinforcement needed in the wall can be calculated by considering the ultimate limit states of bending and shear. For the purpose of design, cantilever retaining walls can be regarded as consisting of three cantilever beams. That is, the stem, the heel and the toe are all considered to be cantilever beams.

The area of reinforcement can be calculated as for beams and slabs using the equation

$$\mathbf{As} = \frac{M}{0.87 f_y z}$$

Where

M = design moment

fy = grade of reinforcement

$z = d[0.5 + \sqrt{(0}.25 - \frac{K}{0.9})]$

$\text{K} = \frac{M}{f_{cu} b d^2}$

The area of distribution steel is based on the minimum steel area (A_s) given in Table 3.25 of BS 8110. That is:

As = 0.13%Ac for high yield/grade steel

As = 0.24%Ac for low yield/grade steel

Where Ac is the total cross section area of concrete.

11.6 Stability Analysis in Retaining Wall

In stability analysis, the wall is checked to ensure its safety against three modes of failures, namely:

a. Sliding

b. Overturning

c. Bearing failure – beneath the base due to excessive pressure on the soil.

Consider the arrangement of forces in figure 11.9

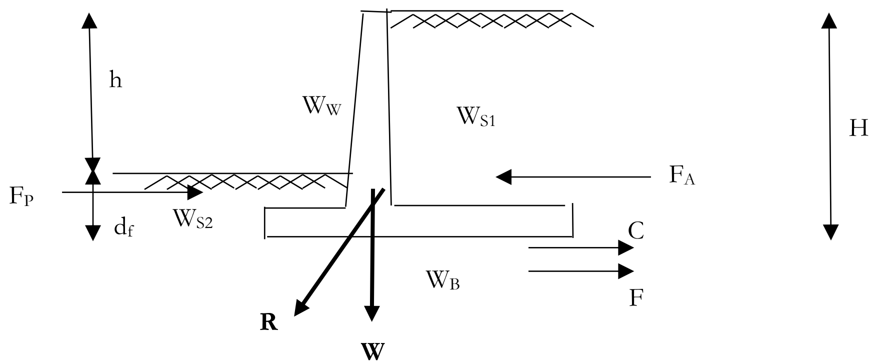


Figure 11.9: Arrangement of forces for stability analysis

F_A	= active force
F_P	= passive force
W_W	= weight of wall
W_B	= weight of base
W_{S1}	= weight of backfill material
W_{S2}	= weight of material on the toe
W	= the weight of all vertical forces
C	= cohesive force
F	= friction force
R	= resultant of all forces
d_f	= depth of foundation

$$= \frac{P}{\gamma}\left(\frac{1-Sin\phi}{1+Sin\phi}\right)^2 \quad 8$$

Where:

γ = unit weight/density of the backfill (KN/m^3) and P = bearing capacity of the soil

h = height above the ground level.

H = $h + d_f$

Sliding

Sliding failure will occur if the active pressure force (F_A) exceeds the passive pressure force (F_P) plus the friction force (F_F) arising at the base/ground interface

This is given in terms of factor of safety, FS

$$FS = \frac{Sum\ of\ Resisting\ Forces}{Sum\ of\ Driving\ Forces} = \frac{F + F_p}{F_A} \qquad \mathbf{9}$$

The contribution of forces due to passive earth pressure at the toe is usually ignored. In that case, equation 9 becomes

$$FS = \frac{Sum\ of\ Resisting\ Forces}{Sum\ of\ Driving\ Forces} = \frac{F}{F_A} \qquad \mathbf{10}$$

≥ 2.0 for cohesive soil backfill

≥ 1.5 for cohesionless soil backfill

Where:

F = $\mu R = \mu \sum W \downarrow$

μ = coefficient of friction between the base and the soil

W = all the vertical forces

Overturning

Overturning usually occur the overturning effect of the active pressure about the toe of the wall. This type of failure can be checked by taking moments about the toe of the foundation and ensuring that the ratio of sum of restoring moments ($\sum M_r$) and sum of overturning moments ($\sum M_o$) is expressed in terms of factor of safety (FS). The Factor of Safety FS is

$$FS = \frac{Sum\ of\ Resisting/Restoring\ Moments\ (M_r)}{Sum\ of\ Overturning\ Moments\ (M_o)} \qquad \mathbf{11}$$

≥ 2.0 for cohesive soil backfill

≥ 1.5 for cohesionless soil backfill

M_r → Weight of soil + Self weight + Passive force (usually neglected)

Mo → Pa

Bearing Failure

This is when the bearing pressure of the soil is exceeded. Bearing failure is checked by considering the interplay of the resultant R of all the vertical forces at the base and the mode of intensity of stress distribution of the soil at the base.

Now if "x" is the position of the distance of the resultant R of all the vertical forces from the toe of the wall, then,

$Rx = Mww + Mws1 + Mws2 + \ldots\ldots$

$$x = \frac{Mww + Mws1 + Mws2 + \ldots\ldots}{R} \quad (12)$$

For stability, the resultant R must lie within a region in the base so that the stresses/pressures everywhere are compressive. This region is called the "the middle third".

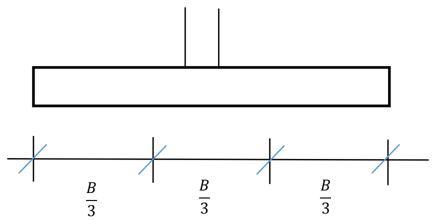


Figure 11.10: Middle third requirement for stability of walls

Also, the resultant R will have an eccentricity of "e" with respect to the geometrical center of the base, and the relationship is given by:

$$e = \frac{B}{2} - x \qquad \text{(where B = base length)} \quad (13)$$

Further, recall that the pressure intensity q, at the base of any foundation is given by the expression

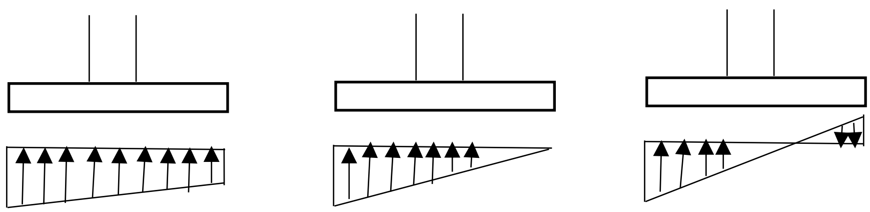
$$q = \frac{R}{B}\left(1 \pm \frac{6e}{B}\right) \quad (14)$$

having two values – minimum and maximum

$$q_{max} = \frac{R}{B}\left(1 + \frac{6e}{B}\right) \quad (15)$$

$$q_{min} = \frac{R}{B}\left(1 - \frac{6e}{B}\right) \quad (16)$$

Stability analysis involves consideration of three cases of the soil pressure equation in order to have arrangement of base dimensions that will result in compression over the whole base



$e < \frac{B}{6}$ **(a)** $\qquad e = \frac{B}{6}$ **(b)** $\qquad e > \frac{B}{6}$ **(c)**

Figure 11. 11: Pressure distribution under the wall

The pressure distribution under the retaining wall can be any of the form shown in Figure 11.11.

Case (a)

$e < \frac{B}{6}$, both q_{max} and q_{min} have positive values, so that the resultant falls within the middle third

Case (b)

$e = \frac{B}{6}$, $q_{max} = \frac{2R}{B}$, and $q_{min} = 0$, so that the resultant falls just at the middle third

Case (c)

$e > \frac{B}{6}$. q_{max} = positive and q_{min} = negative, so that the resultant falls outside the middle third

Case (a) is the obvious solution, that is, the resultant of R must fall within the middle third of the base to avert bearing failure. The base width must be adjusted until the condition is satisfied

11.7 Construction Consideration

i. Drainage consideration

1. Unless the wall is designed to retain water, it is important to have proper drainage behind the wall in order to limit the pressure to the wall's design value. Drainage materials will reduce or eliminate the hydrostatic pressure and improve the stability of the material behind the wall
2. Provision of Weep Holes

ii. Stability enhancement

Stability of retaining wall is enhanced through the provision of shear Key at the base.

11.8 Design Procedures for Cantilever Retaining Wall

1. Initial dimension – Practical guides

   The followings can be used for initial dimensioning when designing a cantilever wall.

   a. Determine the depth of the foundation from the soil parameter using equation

   $$d_f = \frac{P}{\gamma}\left(\frac{1-Sin\phi}{1+Sin\phi}\right)^2$$

   where

   ϕ = angle of repose of the soil

   γ = unit weight of the soil

   P = bearing capacity of the soil

   b. Determine the coefficients – active (ka) and passive (kp)

   c. Top width of the stem not less than 200mm

   d. The bottom width of the stem wall – design to suit the maximum BM arising out of the horizontal pressure of the earth

   e. Width B of the base foundation slab → depends on the height H of the retaining wall

i. B = 0.5 H to 0.6 H → without surcharge

ii. B = 0.7 H to 0.75 H → with surcharge

f. Toe projection, $t_p = \frac{B}{3}$

g. The thickness of the base slab $\frac{H}{12}$

2. Calculation of Soil Pressures on the Wall from the properties of the soil
3. Structural Design
   a. Stem wall
   b. Heel slab
   c. Toe slab
4. Stability Analysis
5. Detailing
6. Constructional specifications

Example 11.1

Design a cantilever type of retaining wall to retain earth and embankment of 4 m height about the ground level. The density of the soil is 18 KN/m^3, the angle of repose ϕ of the soil to be retained is 30^0. The embankment is level at the top. The safe bearing capacity of the soil is 200 KN/m^2. And the coefficient of friction between the soil and the foundation m is 0.50. Adopt a concrete grade of 25 N/mm^2 and steel grade of 460 N/mm^2. Density of Concrete is 2400 kg/m^3.

Solution

Dimensioning

i. The Depth of the foundation – measured from the top of the soil above the toe

$$d_f = \frac{P}{\gamma}\left(\frac{1-Sin\phi}{1+Sin\phi}\right)^2$$

$$d_f = \frac{200}{18}\left(\frac{1-Sin30}{1+Sin30}\right)^2$$

$$= 1.2m$$

ii. The coefficient

a. $Ka = \frac{1}{3}$

b. $Kp = 3$

iii. The overall height H

$H = 4 + 1.2 = 5.2m$

iv. The top of the Stem

Try 200mm (the minimum value)

v. The width B of the foundation base

$B = 0.5H$ to $0.6H$ → since no surcharge

$= 5.2 * 0.5$ to $5.2 * 0.6$

$= 2.6 - 3.12$

Take B = 3m

vi. The toe projection

$t_p = \frac{B}{3}$

$= 1$ m

vii. The thickness of the base slab

$t_b = \frac{H}{12}$

$= \frac{5200}{12}$

$= 433.33$ mm

Use 450 mm

viii. The height of the Heel

$5.2 - 0.45$

$= 4.75$m

The sketch of the Retaining Wall

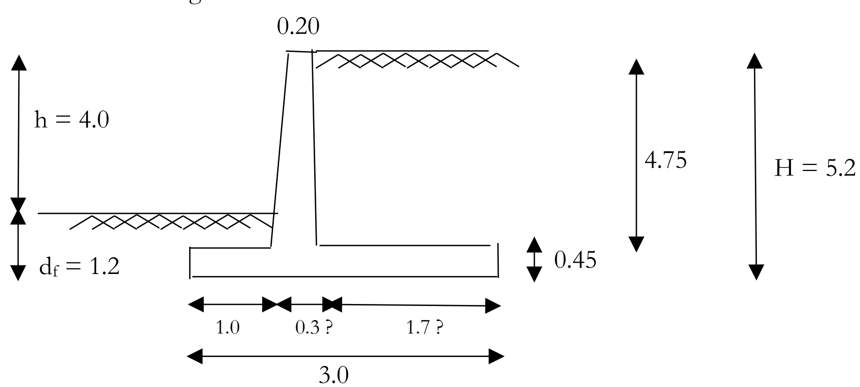


Figure 11.12: All the dimensions of the Retaining wall

Structural Design of the Elements

i. The Stem Wall
ii. Heel Slab
iii. Toe Slab

The Stem Wall - Design

The design for Max BM due to the horizontal soil pressure over the height 4.75m.

The BM at the junction of the stem and the base

$$\text{Mmax} = = \frac{k_a \gamma h^3}{6} \quad \text{(there is no surcharge)}$$

And

$$\text{ka} = (\frac{1 - Sin\emptyset}{1 + Sin\emptyset} = \frac{1}{3}$$

thus

$$\text{Mmax} = \frac{\frac{1}{3} \times 18 \times 4.75^3}{6}$$

= 107.17 Kn-m

The ultimate moment is now obtained by applying the factor of safety (for inaccuracies in soil, concrete, steel, imperfection in workmanship, etc.) treating the soil to be retained as **Imposed load (s**ince occasion may demand its removal and replacement in future). So a factor of safety of 1.6 is used. That is:

Mult = 107.17 x 1.6

= 171.472 Kn-m

Recall that the moment of resistance of a rectangular section is given by:

$\text{Mu} = 0.156\text{fcubd}^2$

That is

$\text{Mu} = 0.156\text{fcubd}^2 = 171.472 \times 10^6$

For 1meter strip b = 1m = 1000mm

Thus, the minimum effective depth can be obtained by equating the ultimate moment with the moment of the section. That is,

$\text{Mu} = 0.156\text{fcubd}^2 = 171.472 \times 10^6$

From where we get

$d^2 = 43967.18$

minimum d = 209. 68 mm (at the base of the stem)

By making allowance for cover and reinforcement take **h = 300 mm**, and d = 250mm

The dimension is now complete **(the question marks in Figure 11.12 are now resolved)**

With this effective depth

$\text{Mu} = 0.156\text{fcubd}^2 = 0.156 \times 25 \times 1000 \times 250 \times 250 = 243.75\text{KN.m}$

Since Mu > M, no compression reinforcement is required.

$$K = \frac{M}{f_{cu} b d^2} = \frac{171.472 \times 1000000}{25 \times 1000 \times 250 \times 250}$$

= 0.009

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})] = d[0.5 + 0.35)]$$

= 0.85d = 0.85 x 250

= 212.50 mm

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{171.472 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 212.5}$$

$= 2016.31mm^2$.

Provide Y20 @125 ($2510mm^2$)

But the BM of the Stem will reduce along the length, so reinforcement ought to be reduced, so as not to be wasteful to use Y20@125 for the whole length. Thus, the reinforcement must be cut off at a point called the critical cut off point. This is usually at the point where the BM is half. That is:

$$= \frac{k_a \gamma h^3}{6} = \text{half of } 171.472 \text{ Kn-m}$$

$$= \frac{0.3333 \ x \ 18 \ x \ h^3}{6} = 85.74$$

Solving gives h = 4.44 m

Half of the BM = 85.74, and this value will be attained at a distance h = 4.44m from the top.

Only 50% of the reinforcement will extend beyond this point. That is, Y20@250

Secondary reinforcement is given according to BS 8110, which for 460N/mm2 grade of steel, it is:

As (secondary) = 0.0013 x bh

= 0.0013 x 1000 x 250 (average of top and bottom width)

= 325 mm^2

Provide Y10@250 mm (314 mm^2)

Preliminary Sketch (Figure 11.13)

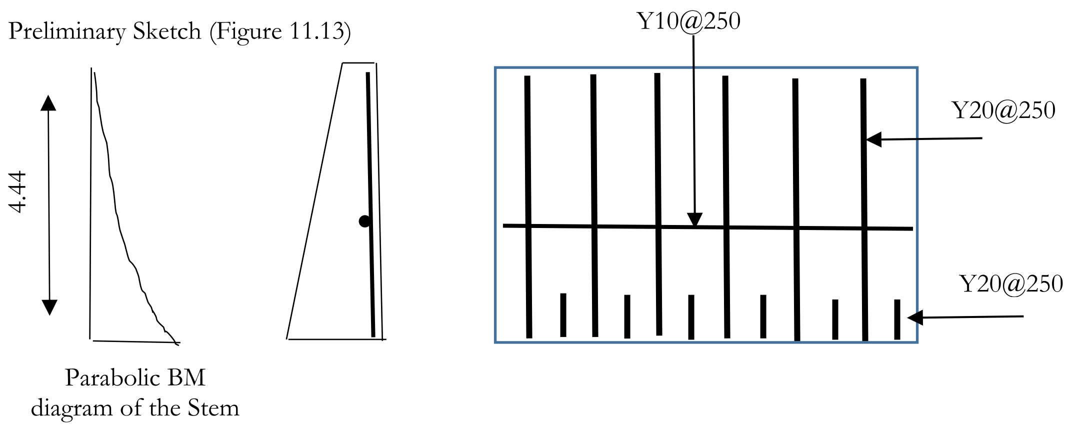


Figure 11.13: The Preliminary Sketch of the Stem Reinforcement

Stability Analysis – Bearing Failure

The diagram of the wall showing all the forces is shown in the figure 11.14.

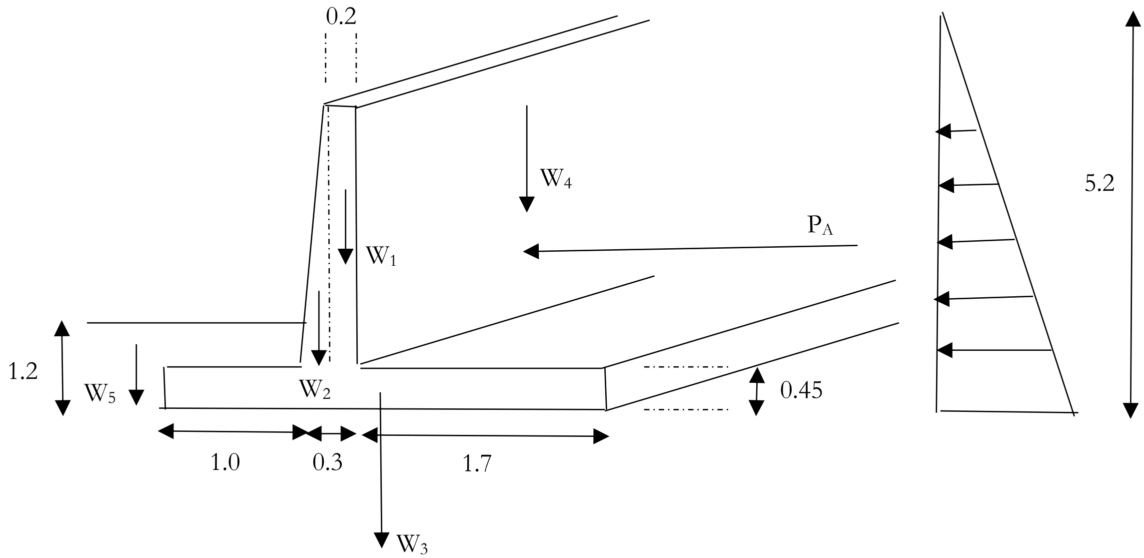


Figure 11. 14: Arrangement for stability analysis

Table 11.1: The Reactions and Moments for the Retaining Walls

Load Designation	Vertical Load (KN)	Horizontal Load (KN)	Location of load from the toe (m)	Bending Moment (KN.m)
W_1 (Stem Wall)	0.2 x 1.0 x 4.75 x 24 = 22.8		1.2	27.36
W_2 (Stem Wall)	(0.1 x 4.75)/2 x 1.0 x 24 = 5.7		1.067	6.08
W_3 (Base slab)	3.0 x 0.45 x 1.0 x 24 = 32.4		1.5	48.60
W_4 (Backfill)	1.7 x 4.75 x 1.0 x 18 = 145.35		2.15	312.50
W_5 (Toe Soil)	1.0 x 0.75 x 1.0 x 18 = 13.5		0.5	6.75
P_A (active pressure)		$\frac{k_a \gamma h^2}{2} = \frac{1}{3} \frac{18(5.2)^2}{2} =$ 81.18	$\frac{H}{3} = \frac{5.2}{3} = 1.733$	- 140.58
Total	$\sum W = R = 219.75$	$\sum = 81.18$		$\sum 260.71$

Now the resultant R of all the vertical forces acts at a distance x from the toe, equals to

$$x = \frac{M}{\sum W} = \frac{260.71}{219.75}$$

$$= 1.19\text{m}$$

Now the eccentricity "e" is given by

$$e = \frac{B}{2} - x \quad = \frac{3}{2} - 1.19 \quad = 0.31$$

But the ratio $\frac{B}{6} = \frac{3}{6} = 0.5$

Since $e < \frac{B}{6}$ then the resultant is within the middle third of the base. It is thus safe against bearing failure.

The foundation base dimension is OK.

The Heel and Toe Slab – Design

We need to have an idea of the pressure distribution at the base of the slab. Now recall the pressure intensity equation given by:

$$q = \frac{R}{B}\left(1 \pm \frac{6e}{B}\right)$$

with the maximum value of:

$$q_{max} = \frac{R}{B}\left(1 + \frac{6e}{B}\right) = \frac{219.75}{3}\left(1 + \frac{6 \, x \, 0.31}{3}\right) = 118.67 \text{ KN/m2}$$

and minimum value of:

$$q_{min} = \frac{R}{B}\left(1 - \frac{6e}{B}\right) = \frac{219.75}{3}\left(1 - \frac{6 \, x \, 0.31}{3}\right) = 27.84 \text{ KN/m2}$$

These two values fell below the allowable soil pressure of 200 KN/m2

The pressure distribution at critical section of the base slab is shown in Figure 11.15.

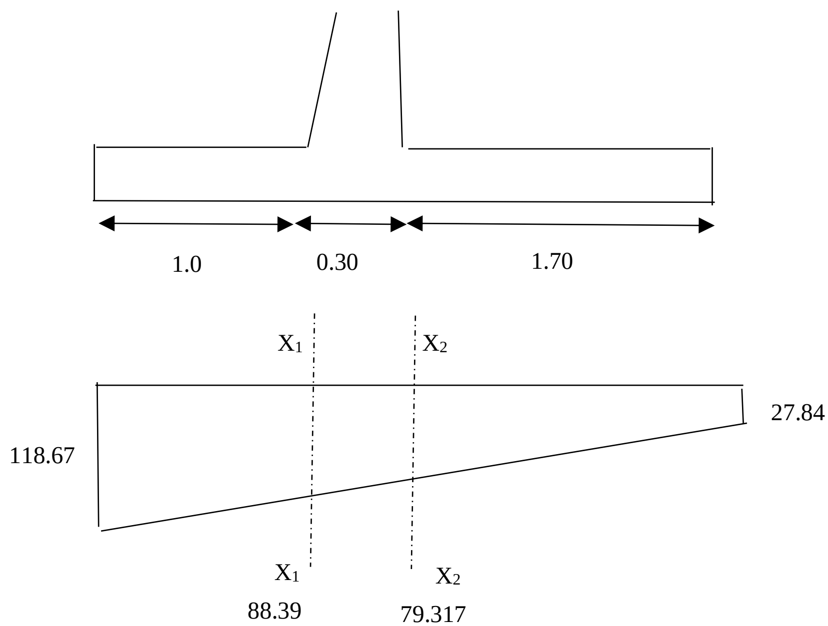


Figure 11.15: Soil Pressure Calculation at the base of the retaining wall

The pressure at X_1-X_1 is:

$$= 27.84 + \frac{2.0(118.67-27.84)}{3} = 88.39 \text{ KN/m2}$$

And the pressure at X_2-X_2 is:

$$= 27.84 + \frac{1.7(118.67-27.84}{3} = 79.31\text{KN/m2}$$

Design of Heel Slab

The slab will be designed as a cantilever fixed X_2-X_2 at subjected to the loading as shown

i. Load due to self-weight of the soil on the heel → acting down

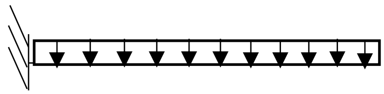


Figure 11.16: Loading due to the self-weight of the soil on the Heel

= 1 x 18 x 4.75 = 85.5 KN/m

ii. Load due to self-weight of concrete → acting down

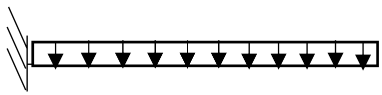


Figure 11.17: Loading due to the self-weight of the concrete on the Heel

= 1 x 24 x 0.45 = 10.80KN/m

iii. Load due to soil distribution → acting upward (from Figure 11.15)

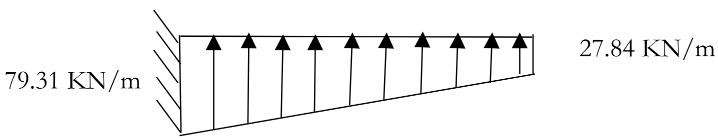


Figure 11.18: Loading due to the soil pressures on the Heel

Superimposing the loadings (i), (ii) and (iii), then we have a trapezoidal loading arrangement acting downward.

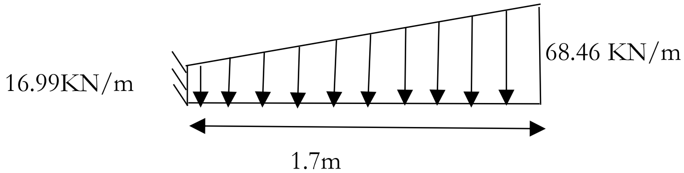


Figure 11.19: Resultant of all loads on the Heel

The Maximum BM. = 16.99 x 1.7 x $\frac{1.7}{2}$ + $\frac{1}{2}$ x 1.7 x (68.46 − 16.99)x $\frac{2}{3}$ x 1.7 = 24.55 + 49.58

= 74.13 KN-m

The Ultimate BM, Mu

Mu = 1.6 x 74. 13 = 118.61 KN.m

The depth of the Heel = 450 mm, assuming d = 400 mm

Then

Mu = 0.156fcubd² = 0.156 x 25 x 1000 x 400 x 400 = 624KN.m

Since Mu > M, no compression reinforcement is required.

$$K = \frac{M}{f_{cu}bd^2} = \frac{118.61\ x\ 1000000}{25\ x\ 1000\ x\ 400\ x\ 400}$$

= 0.03

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}] = d[0.5 + 0.47)] = 0.97d$$

= 0.95d = 0.95 x 400

= 380 mm

$$\mathbf{As} = \frac{M}{0.87 f_y z} = \frac{118.61\ x\ 1000000}{0.87\ x\ 460\ x\ 3805}$$

= 779.94mm².

Provide Y16 @250 (804 mm²)

Secondary reinforcement is given according to BS 8110, which for 460N/mm2 grade of steel, it is:

As (secondary) = 0.0013 x bh

= 0.0013 x 1000 x 450 (average of top and bottom width)

= 585 mm²

Provide Y10@125 mm (628 mm²)

Position of reinforcement based on the bending effects of the resultant loadings is shown in Figure 11.20.

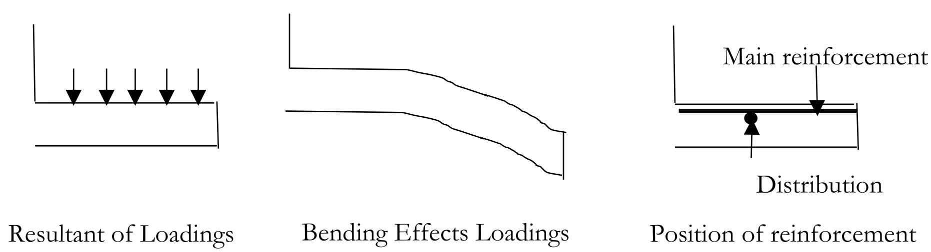


Figure 11. 20: Position of reinforcement for the resistance of the resultant of all Loads on Heel

Design of Toe Slab

The slab will be designed as a cantilever fixed X_2-X_2 at subjected to the loading as shown

i. Load due to self-weight of the soil on the Toe → acting down

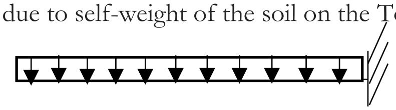


Figure 11.21: Loading due to the self-weight of the soil on the Toe

= 1 x 0.75 x 18 = 13.5KN/m

ii. Load due to self-weight of concrete → acting down

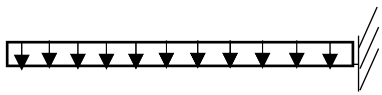


Figure 11.22: Loading due to the self-weight of the concrete on the Toe

= 1 x 0.45 x 24 = 10.80KN/m

iii. Load due to soil distribution → acting upward

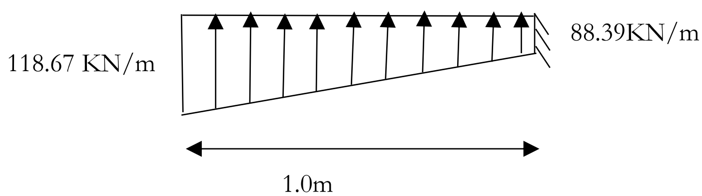


Figure 11.23: Loading due to soil pressures on the Toe

Superimposing the loadings (i), (ii) and (iii), then we have a trapezoidal loading arrangement acting upwards

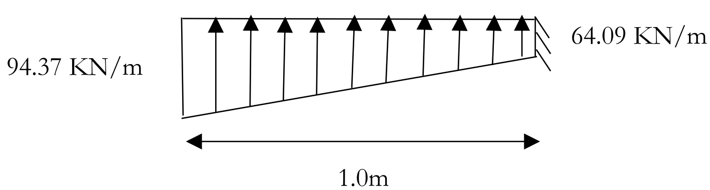


Figure 11.24: Resultant of all loads on the Toe

The Maximum BM. = 64.09 x 1.0 x $\frac{1.0}{2}$ + $\frac{1}{2}\ x\ 1.0\ x\ (94.37 - 64.09) x\ \frac{2}{3}$ x 1.0 = 32.05 + 10.09

= 42.13 KN.m

The ultimate BM = 1.6 x 42.14

= 67.43 KN.m

As for the Heel, the depth of the toe = 450 mm, assuming d = 400 mm

Then

As for the Heel, so for the Toe, Mu > M, no compression reinforcement is required.

$K = \frac{M}{f_{cu}bd^2} = \frac{67.43\ x\ 1000000}{25\ x\ 1000\ x\ 400\ x\ 400}$

= 0.02

$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9}})] = d[0.5 + 0.48)]$ = 0.98d

= 0.95d = 0.95 x 400

= 380 mm

As = $\frac{M}{0.87 f_y z} = \frac{67.61\ x\ 1000000}{0.87\ x\ 460\ x\ 3805}$

= 443.83 mm^2.

Provide Y16 @300 (672 mm^2)

Secondary reinforcement is given according to BS 8110, which for 460N/mm2 grade of steel, it is:

As (secondary) = 0.0013 x bh

= 0.0013 x 1000 x 450 (average of top and bottom width)

= 585 mm^2

Provide Y10@175 mm (646 mm^2)

The position of the reinforcement to effectively resist the bending effects of the resultant loadings is shown in Figure 11.25.

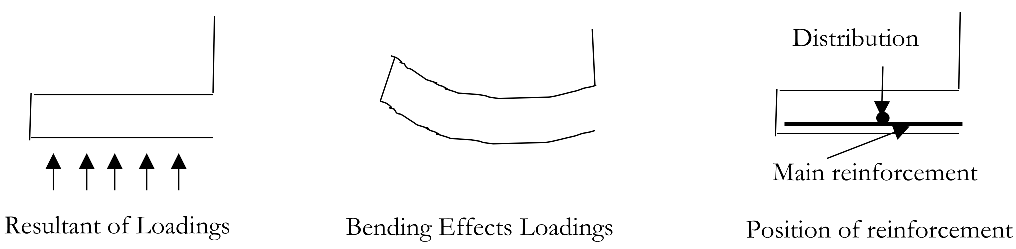


Figure 11.25: Position of reinforcement for the resistance of the resultant of all Loads on Toe

The reinforcement arrangement for the whole retaining wall is shown in Figure 11.26. To prevent shrinkage crack,

i. Steel should be provided at the front face of the wall, the Heel and the toe to prevent cracking. This is based on the minimum steel area.

ii. The heel reinforcement should extend to the toe and the toe to the heel, but subject to anchorage bond length as contain BS 8110.

iii. The reinforcement from the stem should be made to extend to the Key under it.

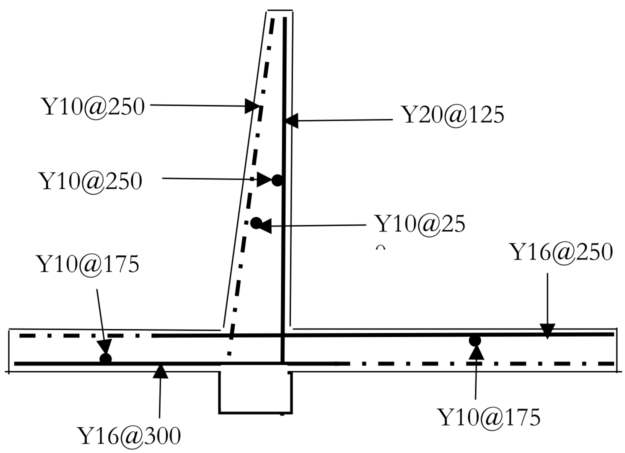


Figure 11.26: The Sketch of Reinforcement arrangement for all the parts of the Retaining wall

Stability Analysis

1. Sliding

From equation 9 (that is, ignoring the passive forces)

$$FS = \frac{Sum\ of\ Resisting\ Forces}{Sum\ of\ Driving\ Forces} = \frac{F}{F_A} \qquad \textbf{9}$$

$$= \frac{\mu\Sigma W\downarrow}{F_A}$$

f = coefficient of friction between the base and the soil

W = all the vertical forces

Given that the coefficient of friction, $\mu = 0.50$

From Table 11.1

Sum of horizontal forces $\sum F_A = 219.75$ KN

Sum of vertical Forces $\sum W = 81.18\ KN$

Now for sliding safety, FS = $\frac{\mu\sum W}{\sum P} \geq 1.5$

$$= \frac{0.5\ x\ 219.75}{81.18} = 1.35 < 1.5 \quad \text{(that is, for cohesionless soil)}$$

Therefore, it is not safe against sliding

Options

i. Re-design by increasing the base width so that $\sum W\ \ increases$ and $\sum P$ remains the same
ii. Design a base key (either under the Stem or at the end of the Heel)

Let us design the Key under the Stem

Take the depth of the key to be 400mm below the Stem

The Key will be subject to, from Figure 11.15:

a. Passive force

= kp x q x h = 3 x 88.39 x 0.4 = 106 KN/m

b. Active force

= ka x q x h = 1/3 x 79.31 x 0.4 = 10.56 KN/m

Thus, the total resistance force

$$= \frac{\sum key + \sum W}{\sum P}$$

$$= \frac{95.55 + 0.5\ x\ 219.75}{81.18}$$

$$= 2.53$$

$$> 1.5$$

Thus, the wall is safe against sliding. This is shown in Figure 11.26.

2. Overturning

$$\text{FS} = \frac{Sum\ of\ REsisting\ Moments\ \ (M_r)}{Sum\ of\ Overturning\ Moments\ (M_O)}$$

$$= \frac{401.29}{140.58)} \quad \text{(from Table 11.1)}$$

$$= 2.86 > 1.5$$

Thus, the wall is safe against overturning

Constructional Specifications

Weep holes to discharge rainwater during raining season.

11.9 Design of Counterfort Retaining Wall

When the height of the cantilever retaining wall is more than 6m, the stem thickness will be large and the need to design the wall as counterfort. A typical counterfort wall is shown in Figure 11. 27.

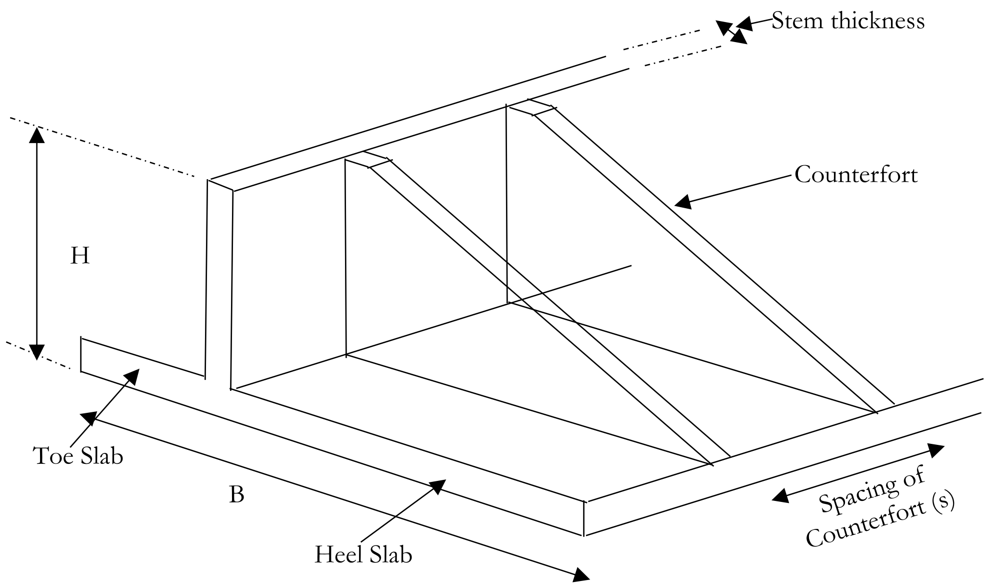


Figure 11.27: A typical Counterfort Wall

Rules for Design

1) Preliminary dimensioning
2) Design Principles
    a. The Stem Wall
    b. The Toe Slab
    c. The Heel Slab
    d. Design of the Counterforts
    e. Detailing
3) Stability checks

Preliminary Dimension

Some guides for preliminary of counterfort walls are given below

a. Spacing s of counterforts → 3.0 to 3.5 m
b. Overall wall height = height of soil retained + depth of foundation
c. Base width B → 0.6H to 0.7H

d. Toe projection $t_p = \frac{B}{4}$

e. Thickness of base slab $t_b = 0.02sH$

Design Principles

1) Stem Wall

The stem wall is designed as a continuous slab resting over (or held by) a series of counterforts, with moments as shown in Figure 11.28.

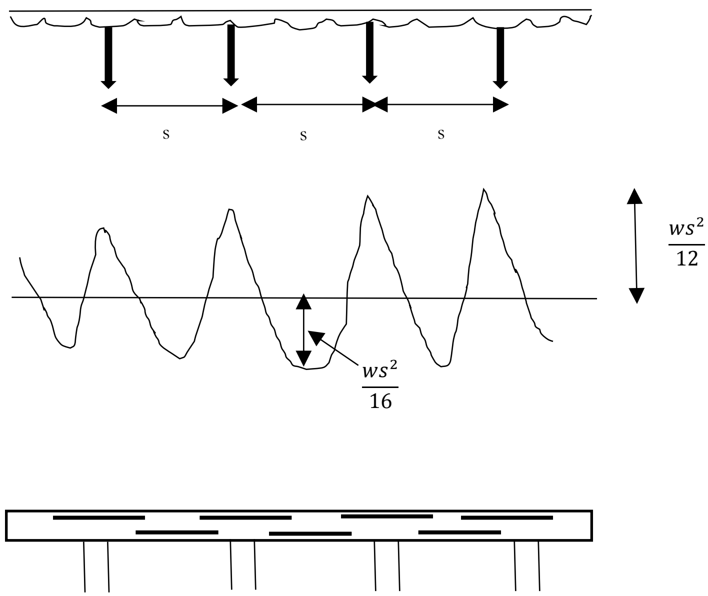


Figure 11.28: Bending Moments and position of reinforcement for Stem Wall Design

The maximum bending moment is taken as

$$M_{max} = \frac{P_h s^2}{12} \qquad \textbf{17}$$

Where

P_h = horizontal (active) pressure at any depth

s = spacing of the counterforts

2) Toe Slab

The toe slab is designed as cantilever to resist upward soil pressure of the foundation together with the down ward self-weight of the base slab plus the weight of the soil supported above it (identical to the principle of design as in the case of cantilever type of Retaining wall)

3) Heel Slab

The heel slab is designed as a continuous slab held by or supported by a series of counterforts to resist the soil pressure at the foundation together with the downward self-weight of the slab plus the weight of the soil supported above the heel slab. The maximum bending diagram is similar to that of stem, with the maximum bending moments, taken as:

$$M_{max} = \frac{Ps^2}{12} \qquad \mathbf{17}$$

4) Design of Counterfort

The counterforts are designed as cantilever fixed at the base slab, subjected to a varying pressure given by

$$P_h s \qquad \mathbf{18}$$

The design of the connections between the counterforts and stem wall as well as the counterfort and the heel are designed for respective forces and reinforcement are designed to resist this force.

Example 11.2

Design a counterfort type of retaining wall to retain an earth to a height of 6 m. the safe bearing capacity of soil is 160 kN/m^2, the angle of repose is 30^0, the density (unit weight) of the soil is 16 Kn/m^3 and coefficient of internal friction μ of 0.50. Take the spacing between counterfort to be 3 m, f_{cu} = 30 N/mm^2, f_y = 460 N/mm^2, and density of concrete = 24 KN/m^3.

Solution

i. Depth of foundation $d_f = \frac{P}{\gamma}(\frac{1-Sin\phi}{1+Sin\phi})^2$

$$d_f = \frac{160}{16}(\frac{1-Sin30}{1+Sin30})^2$$

$$= 1.11 \text{ m}$$

$$= 1.2 \text{ m}$$

ii. The overall depth H

$$H = 1.2 + 6 = 7.2 \text{ m}$$

iii. Thickness of the slab

$$= 0.02sH$$

$$= 0.02 \text{ x } 3 \text{ x } 7.2 = 0.43.2 \text{ m}$$

$$= 0.45 \text{ m}$$

iv. The base width

$$= 0.6H – 0.7 H$$

$$= 0.6 \text{ x } 7.2 \text{ to } 0.7 \text{ x } 7.2$$

$$= 4.32 – 5.04$$

Take = 4.5 m

v. The Toe Projection = $\frac{B}{4}$

$= \frac{4.5}{4} = 1.1$ m

Use = 1.0

The diagram of the dimensions

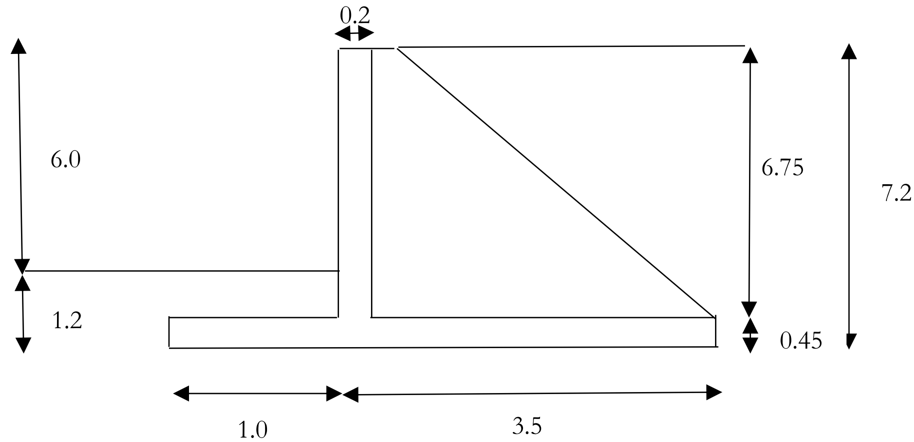


Figure 11. 29: The Counterfort Retaining Wall showing all the dimensions

The Stem Wall

The stem wall is designed for horizontal earth pressure and the stem will be designed as continuous slab supported by series of counterforts, just like a slab of 1 m strip in the horizontal direction

Now the maximum pressure intensity at the junction of the stem wall and base slab is:

$$P_h = k_a w H$$

and

$$k_a = \frac{1-Sin\theta}{1+Sin\theta} = = \frac{1-30}{1+30} = 0.33333$$

therefore

$$P_h = 0.3333 \text{ x } 16 \text{ x } 6.75$$

$$= 36 \text{ KN/m}^2$$

From Figure 11.28, at the Support

$$\text{Mmax} = \frac{P_h s^2}{12}$$

$$= \frac{36 \; x \; 3^2}{12} = 27\text{KN.m}$$

At the ultimate

Mu = 1.6 x 27 = 43. 20 KN. m

At the Span

Mmax = $\frac{P_h s^2}{16}$

= $\frac{36\ x\ 3^2}{16}$ = 20.25 KN. M

And at the ultimate,

Mu = 1.6 x 20.25 = 32.40 KN. M

Now the thickness of the stem wall is based on **the support momen**t rather than the **span moment**. That is,

Mu = 0.156fcubd²

43.2 = 0.156 x 30 x 1000 x d²

d = 96.07 mm

But the code recommends minimum thickness of 200 mm, so take h = 200 mm.

Making allowance for cover, conditions of exposure and reinforcement diameter, take d = 150mm

d = 150mm

Now the moment of resistance of the section at d = 150mm

Mu = 0.156 x 30 x 1000 x 150 x 150

= 105.30 KN.m

Since

Mu > M, then no compression reinforcement is needed

K = $\frac{M}{f_{cu}\ bd^2}$ = $\frac{43.20\ x\ 1000000}{30\ x\ 1000\ x\ 150\ x\ 150}$ = 0.064

Z = d[0.5 + $\sqrt{(0.25 - \frac{k}{0.9})}$] = 0.92d

= 0.92 x 150 = 138mm

Thus

As = $\frac{M}{0.87fyz}$ = $\frac{43.20\ x\ 1000000}{0.87\ x\ 460\ x\ 138}$

= 782.22 mm²

Use Y16@250 (804 mm) → at inner face

The Span Reinforcement

M = 32. 4KN.m (Note, the same d, k, and z as for the Support)

As = $\frac{32.4}{0.87fyz}$ = $\frac{32.40\ x\ 1000000}{0.87\ x\ 460\ x\ 138}$

= 586.66 mm²

= Y12@175 (646 mm²)

Distribution steel

$$Asd = \frac{0.13}{100} bh = \frac{0.13}{100} \text{ x } 1000 \text{ x } 200$$

$$= 260 \text{ mm}^2$$

Use **Y10@250 (314 mm²)**

Check for Shear F (at the support)

$$F = \frac{P_h s}{2} = \frac{36 \ x \ 3}{2} = 54KN$$

The shear stress v (at ultimate) $= \frac{Shear\ Force\ (F)}{area\ \ (bd)}$

$$= \frac{1.6 \ x \ 54}{1000 \ x \ 160}$$

$$= 0.54 \text{ N/mm}^2$$

From Table 3.8 (BS 8110), $v_c = 0.63$

Since $f_{cu} > 25$ N/mm², that is 30N/mm², this value is to be multiplied by $\sqrt[3]{(\frac{fcu}{25})}$

That is:

$v_c = 0.63 \text{ x } 1.17 = 0.74$

Since $v_c > v$

Shear is OK.

Sketch of reinforcement arrangement

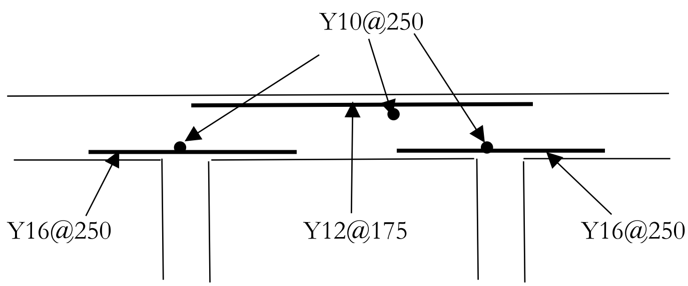


Figure 11. 30: Sketch of reinforcement arrangement in the Stem

Stability Analysis

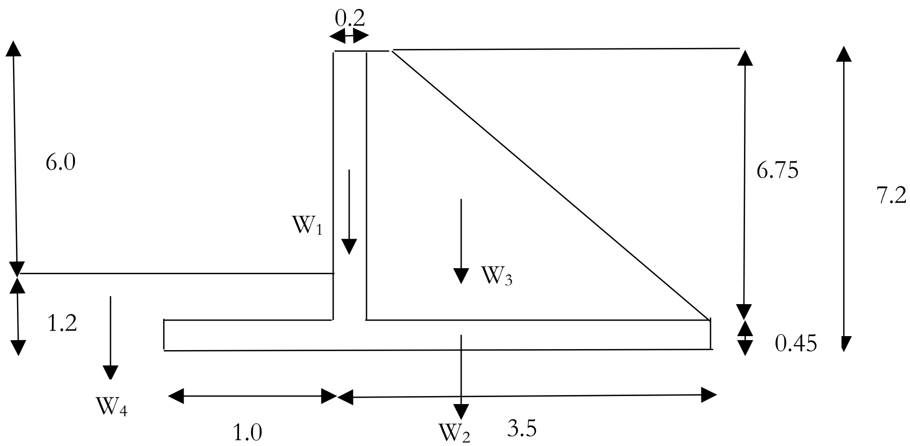


Figure 11. 31: Arrangement for stability analysis

Table 11.2: The Reactions and Moments for the Retaining Walls

Load Designation	Vertical Load (KN)	Horizontal Load (KN)	Location of load from the Heel (m)	Bending Moment (KN.m)
W_1 (Stem Wall)	0.20 x 6.75 x 1 x 24 = 32.40		3.4	110.16
W_2 (Base Slab)	0.45 x 4.5 x 1 x 24 = 48.60		2.25	109.35
W_3 (Backfill)	6.75 x 3.3 x 1 16 = 354.24		1.65	584.50
W_4 (Toe Soil)	Ignored			-
P_H (active pressure)		$\frac{k_a\gamma h^2}{2} = \frac{1}{3}\ \frac{16(7.2)^2}{2} =$ 138.24 Note the "h" used	$\frac{H}{3} = \frac{7.2}{3} = 2.4$	331.78
	$\sum W = R$ = 435.40	$\sum =$ 138.24		1136.79

The lever arm (x) of the resultant

$$x = \frac{\sum M}{\sum W} = \frac{1136.79}{435.40}$$

$$= 2.60 \text{ m (it is within the middle third)}$$

Thus

$$e = x - \frac{B}{2} = 2.60 - \frac{4.50}{2} = 2.60 - 2.25 = 0.35 \text{ m}$$

and

$\frac{B}{6} = \frac{4.5}{6} = 0.75$

Since

0.35 < 0.75. Thus, it is safe against overturning

Sliding

Now the coefficient of friction is 0.5, then

Sliding = $\frac{\mu \sum W}{\sum H} = \frac{0.5 \ x \ 437.4}{138.24}$

= 1.58 > 1.5

Thus, safe against sliding. It does not need the provision of shear key at the base.

The Heel and Toe Slab – Design

In order to design the Heel and the Toe, an idea of the pressure distribution at the base of the slab is necessary. Now recall the pressure intensity equation given by:

$$q = \frac{R}{B}(1 \pm \frac{6e}{B})$$

with the maximum value of:

$$q_{max} = \frac{R}{B}(1 + \frac{6e}{B}) = \frac{437.40}{4.5}(1 + \frac{6 \ x \ 0.35}{4.5}) = 97.53 \text{ KN/m2}$$

and minimum value of:

$$q_{min} = \frac{R}{B}(1 - \frac{6e}{B}) = \frac{437.40}{4.5}(1 - \frac{6 \ x \ 0.35}{4.5}) = 51.84 \text{ KN/m2}$$

These two values fell below the allowable soil pressure of 160 KN/m2

The pressure distribution at critical section of the base slab is as given below

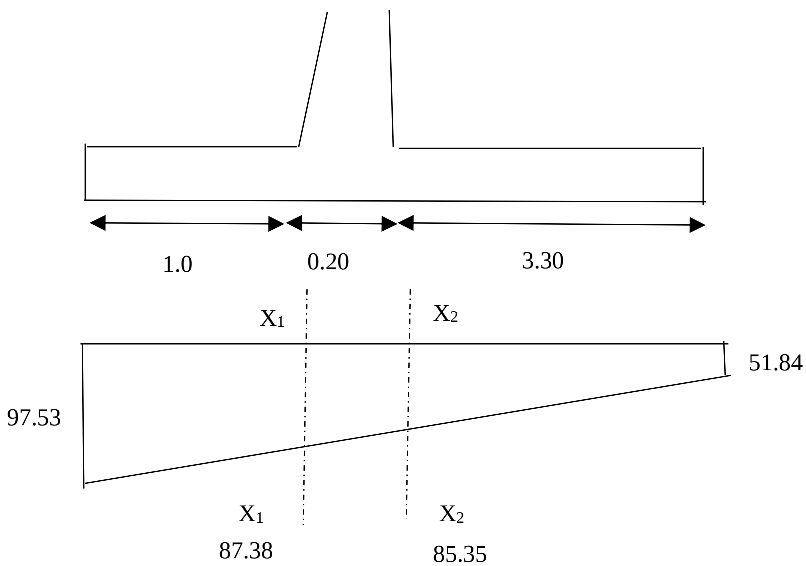


Figure 11.32: Pressure distribution

The pressure at X_1-X_1 is:

$$= 51.84 + \frac{3.50(97.53-51.84)}{4.5} = 87.38 \text{ KN/m2}$$

And the pressure at X_2-X_2 is:

$$= 51.84 + \frac{3.30(97.53-51.84)}{4.5} = 85.35\text{KN/m2}$$

Heel Slab Design

The slab will be designed as a continuous slab held by the counterfort, subjected to the loading (see explanation for Heel design in Cantilever retaining wall)

i. Load due to self-weight of the soil on the heel → acting down

= 6.75 x 16 = 108KN/m2

ii. Load due to self-weight of concrete slab → acting down

= 0.45 x 24 = 10.8

iii. Load due to soil distribution → acting upward **(Figure 11.32)**

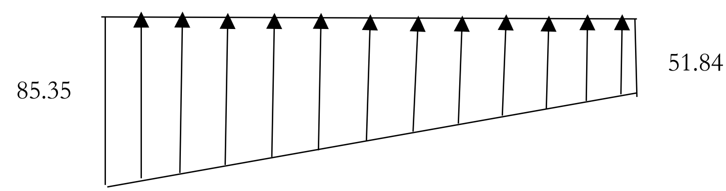


Figure 11.33: Soil pressure at the Heel

The total summation of pressure distribution is downward.

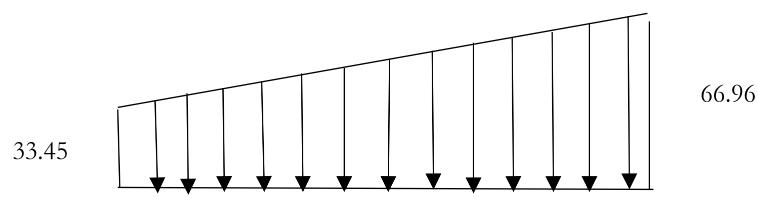


Figure 11. 34: The total summation of all pressures

The maximum BM for the Heel

i. At the Support

$$\text{Mmax} = \frac{P_h s^2}{12}$$

$= \frac{66.96 \ x \ 3^2}{12} = 50.21$ KN.m

At the ultimate

M = 1.6 x 50.21= 80.34 KN. m

ii. At the Span

$Mmax = \frac{P_h s^2}{16}$

$= \frac{66.96 \ x \ 3^2}{16} = 37.65$ KN. M

And

M = 1.6 x 37.65 = 60.24 KN. M

The depth h of the Heel is 450mm, take d = 400mm (taking into consideration cover reinforcement diameter)

d = 400 mm

Therefore

Mu = 0.156fcubd^2 = 0.156 x 30 x 1000 x 400 x 400 = 748.8 KN.m

Since Mu > M, no compression reinforcement is needed

At the Support

$K = \frac{M}{f_{cu} \ bd^2} \qquad = \frac{80.34 \ x \ 1000000}{30 \ x \ 1000 \ x \ 400 \ x \ 400}$

= 0.02

And

$Z = d[0.5 + \sqrt{(0.25 - \frac{0.02}{0.9})}]$

= d(0.5 + 48] = 0.98d

Take z = 0.95d

= 0.95 x 400

= 380mm

Thus

$As = \frac{M}{0.87fyz} \quad = \frac{80.34 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 380}$

= 528.29 mm^2

Use = **Y10@125 (628 mm^2)**

At the mid Span (with same k and z for the Support

$As = \frac{M}{0.87fyz} \quad = \frac{60.24 \ x \ 1000000}{0.87 \ x \ 460 \ x \ 380}$

= 396.12 mm^2

Use Y10@175 (449 mm^2)

Distribution steel

$$Asd = \frac{0.13}{100} bh = \frac{0.13}{100} \times 1000 \times 450$$

$$= 585 \text{ mm}^2$$

Use **Y10@125 (628 mm²)**

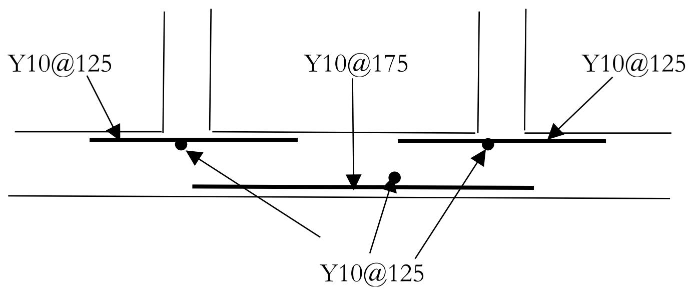


Figure 11.35: Sketch of the Heel reinforcement

Toe Slab Design

The toe slab will be designed as cantilever just as the toe slab in the cantilever retaining wall. The toe slab will be subjected to the following loadings

i. Load due to self-weight of the soil on the heel → acting down

$= 1 \times 1.2 \times 18 = 21.6 KN/m^2$

ii. Load due to self-weight of concrete slab → acting down

$= 1 \times 0.45 \times 24 = 10.8 KN/m^2$

iii. Load due to soil distribution → acting upward **(Figure 11.32)**

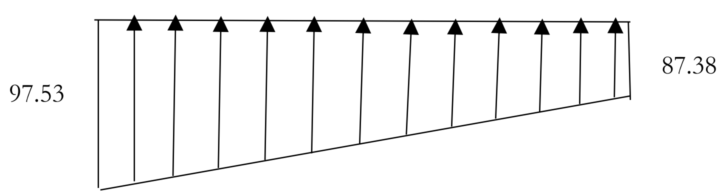


Figure 11.36: Soil pressure at the Toe

The total summation of pressure distribution is upward

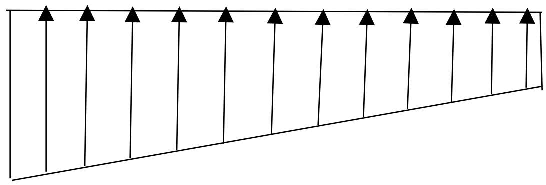


Figure 11. 37: The total summation of all pressures

The Maximum Moment Mmax

$$Mmax = 54.98 \text{ x } 1 \text{ x } \frac{1}{2} + \frac{1}{2} \text{ x } 1 \text{ x } (65.13 - 54.98) \text{ x } \frac{2}{3} = 30.87KN.m$$

Ultimate Moment Mu

$$Mu = 1.6 \text{ x } 30.87 = 49.39KN.m$$

The depth of the toe slab = 450 mm, assuming d = 400 mm

Then

As for the Heel, so for the Toe, Mu > M, no compression reinforcement is required.

$$K = \frac{M}{f_{cu}bd^2} = \frac{49.39 \; x \; 1000000}{30 \; x \; 1000 \; x \; 400 \; x \; 400}$$

$$= 0.01$$

$$z = d[0.5 + \sqrt{(0.25 - \frac{K}{0.9})}] = d[0.5 + 0.49)] = 0.99d$$

$$= 0.95d = 0.95 \text{ x } 400$$

$$= 380 \text{ mm}$$

$$\textbf{As} = \frac{M}{0.87 f_y z} = \frac{49.39 \; x \; 1000000}{0.87 \; x \; 460 \; x \; 380}$$

$$= 324.77mm^2.$$

Provide Y12 @300 ($377mm^2$)

Secondary reinforcement is given according to BS 8110, which for $460N/mm^2$ grade of steel, it is:

As (secondary) = 0.0013 x bh

= 0.0013 x 1000 x 450 (average of top and bottom width)

= 585 mm^2

Provide Y10@175 mm (646 mm^2)

The arrangement of reinforcement is similar to the toe in cantilever retaining wall.

Design of Counterfort

The counterfort is designed as a cantilever subjected to the earth pressure and it is assumed to be fixed at the base slab

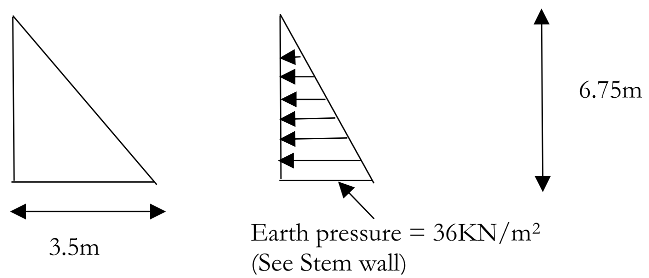


Figure 11.38: Loadings on the Counterfort

The loading on the counterfort is

= the soil pressure x the spacing of the counterfort (Figure 11.38)

= 36 x 3 = 108 KN/m

The maximum BM

= area of the pressure diagram x the centroid

$= \frac{1}{2}$ bh $\frac{h}{3}$ $= \frac{1}{2}$ x 108 x 6.75 x $\frac{6.75}{3}$

= 820.12KN-m

The ultimate design moment Mu

Mu = 1.6 x 820.12 = 1312.19 KN.m

Assume that width of counterfort = 350 mm (usually between 0.3 → 0.4m)

The counterfort is assumed to be a T-beam as shown in Figure 11.39

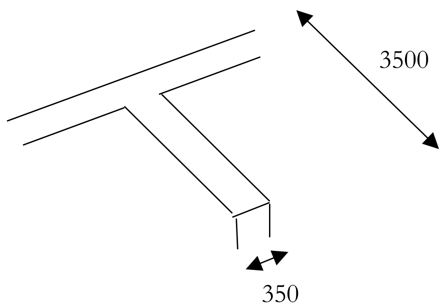


Figure 11. 39: Counterfort modelled as T-Beam

b_w = 350 mm

b = 350 + $\frac{1}{5}$ x 0.7 x 3000 = 770mm

h = 3500 mm, making allowance for cover and reinforcement of 100mm

d = 3400mm

Thus,

$$K = \frac{M}{f_{cu}\, bd^2} \qquad = \frac{1312.19 \times 1000000}{30 \times 770 \times 3400 \times 3400}$$

$$= 0.005$$

And

$$Z = d[0.5 + \sqrt{(0.25 - \frac{k}{0.9})}]$$

$$= d(0.5 + \sqrt{(0.25 - \frac{0.00}{0.9})}]$$

$$= d(0.5 + 0.49) = 0.99d$$

Take z = 0.95d

$$= 0.9 \times 3400$$

$$= 3230 \text{ mm}$$

Thus

$$As = \frac{M}{0.87 fyz} \quad = \frac{1312.19 \times 1000000}{0.87 \times 460 \times 3230}$$

$$= 1015.\ 12\text{mm}^2$$

The reinforcement will be determined as if it were a beam

Provide 4Y20 (1260 mm²)

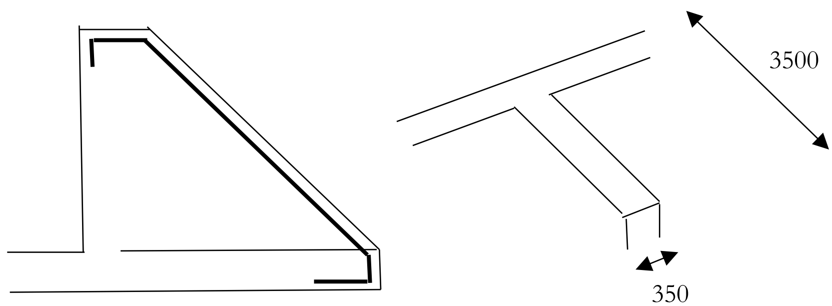


Figure 11: 40: Reinforcement arrangement of counterfort

Connections

i. Connections between the Stem Wall and the Counterfort

Spacing of counterfort = 3 m

Thickness of counterfort = 0.35 m

Pressure (for 1 m strip) = 36 $KN.m^2$

The clear distance between counterfort = 3 – 0.35 = 2.65 m

The force that the reinforcement (stirrup) are to be designed for

= pressure x spacing

= 36 x 2.65 = 95.4KN

At ultimate = 1.6 x 95.4 = 152.64 KN

Now the load decreases upwards. The forces in the stirrups is found

Area of reinforcement provided for the counterfort = 1260 mm^2

Therefore

$$\frac{100A_s}{bd} = \frac{100x\ 1260}{350\ x\ 3400} = 0.11$$

From Table 3: 8,

$v_c = 0.34$

Making adjustment for concrete grade f_c = 30N/mm^2

$v_c = 0.40$ N/mm^2

and

$$v = \frac{V}{bd} = \frac{152.64x\ 1000}{350\ x\ 3400} = 0.13\text{N/mm}^2$$

Since $v < 0.5v_c$, only nominal link is required. Assuming a 2-legged 10 mm diameter link.

Area = 157.8mm^2. Therefore,

$$\text{Spacing } S_v = \frac{0.87f_yA_{sv}}{0.4b} = \frac{0.87\ x\ 460\ x\ 157.8}{0.4\ x\ 350}$$

$= 451.08$mm

Provide

Y10@300 mm (262 mm^2)

The stirrups are to be provided horizontally and the spacing reduced upwards as the pressure decreases, but must not exceed the minimum spacing

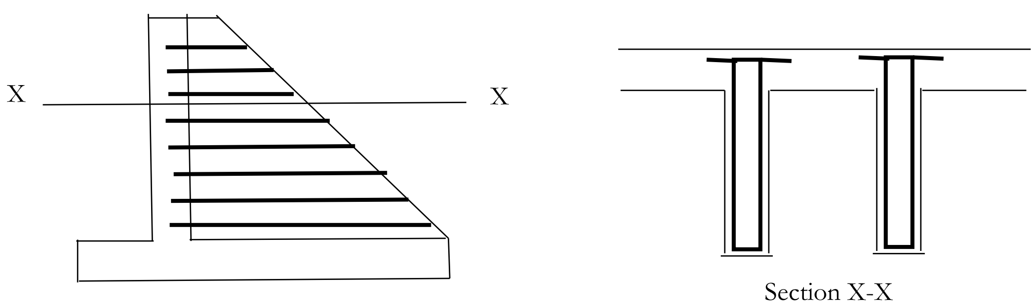


Figure 11.41: Sketch of arrangement links

ii. Connections between the Heel Slab and the Counterfort

There is tensile force to be resisted between the heel and the counterfort. From the pressure diagram in Figure 11.34

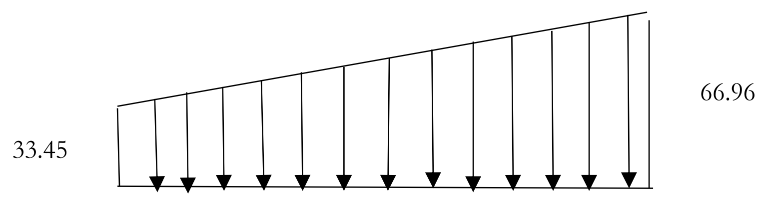


Figure 11. 34

Thus, the force for which the stirrups are to be designed (considering 1 m strip leftward) → pressure x clear spacing

= 66.96 x 2.65 = 177.45 KN

At ultimate

= 1.6 x 177.45

= 283. 91 KN

Thus,

$$v = \frac{V}{bd} = \frac{283.91 \ x \ 1000}{350 \ x \ 3400} = 0.24\text{N/mm}^2$$

Since $v < 0.5v_c$, only nominal link is required. Assuming a 2-legged 10 mm diameter link. Therefore, provide, as in the calculations for Connections between the Stem Wall and the Counterfort. That is:

Y10@300 mm (262 mm^2)

The stirrups will be provided vertically.

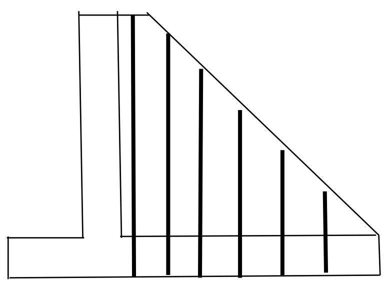


Figure 11.42: Sketch of Arrangement of Links

Chapter 12 – Presentation and Documentation of Results of Structural Design Process

12.1 Introduction

It is now becoming apparent that good presentation of processes and results of design works is not only important aspect of a good design work, but also necessary for the purpose of documentation. Documentation is important for easy reference to previous design works, and more importantly, it serves as a source of data base in a quest for improved future works. It is necessary that a structural engineer build this into his or her habit at the onset. The essential parts of the presentation and documentation are: (i) Design calculation sheet, (ii) Structural detailing, and (iii) Bar bending schedule.

12.2 Design Calculation sheets

The design sheets consist in two parts, namely the cover design sheet and calculation sheet

12.2.1 The Cover Sheet

This serves as a cover to the calculation sheets. It is like the title page of a report. The cover design sheet contains the design information at a glance. A typical cover design sheet is shown in (Table 12.1).

Table 12.1: A Typical Cover Design Sheet

Eirenikos Associates **Consulting Civil/Structural Engineers**	
Project:	**Prepared by:**
Job Ref. No.:	**Checked by**
	Date:
Design Information	
Client	
The Architect	
Project Site	
Design Engineer	
Supervising Engineer	
Relevant Codes	
Materials characteristic Strength - Concrete - Steel	
Exposure conditions	
Intended Use of the Structure	

Thus, a cover design sheet should contain relevant information about the project, at a glance.

12.2.2 Design Calculation Sheet

The calculation sheets are professionally design as in Table 12.2. For example, I want to start a design of beam that is simply supported by finding its effective depth. In the example used, it is no longer necessary to say BS 8110. This has been stated in the cover design sheet. Simply citing table implied that you are referring to BS 8110. Also, it is necessary to adopt a page numbering of the format shown. It gives the total number of pages for the calculation, and it also checks against any omission.

Table 12.2: Calculation Sheet

<table>
<tr><td colspan="3">Eirenikos Associates
Consulting Civil/Structural Engineers
Project: Prepared by:
Job Ref. No.:   Checked by
Date:</td></tr>
<tr><td colspan="3"></td></tr>
<tr><td>Reference</td><td>Calculation</td><td>Output</td></tr>
<tr><td>Table 3.9</td><td>
4m

Assuming a tension modification factor 1. For simply supported beam

$\frac{l}{d} = 20$

$d = \frac{4000}{20}$
$= 200$ mm</td><td>d = 200mm</td></tr>
<tr><td colspan="3">Page No.$\frac{1}{x}$</td></tr>
</table>

12.3 Structural detailing

The structural detailing is necessary for works on site. It is important that information that is contained on the detailing sheet agrees with the information in the design sheets. It is usual to use A2 or A3 paper as structural detailing sheet. A typical division of structural detailing sheet is shown in Table 12.3.

Table 12.3: Typical Structural Detailing Sheet

<table>
<tr><td colspan="3" rowspan="2">A</td><td colspan="2">Notes</td></tr>
<tr><td colspan="2">F</td></tr>
<tr><td>B</td><td>C</td><td colspan="2">D</td><td>E</td></tr>
</table>

Information contained in each division is as follows

Table 12.4: Information in the divisions of Typical Structural Detailing Sheet

Division	Information
A	The structural detailing
B	- The name and address of the Company that designed the project - Project code (Job Title)
C	- The name of the Project - The location of the Project
D	- Designer - Supervising designer
E	- Scale used - Date of the project - Page numbering of structural detailing sheet (because it is possible to used more than one sheet)
F	Professional notes relevant to the project

12.4 Bar bending schedule

The bar bending schedule is extracted from the structural detailing sheet. It gives every information of each bar used. A typical bar bending schedule is shown in Table 12.5

Table 12.5: Typical Bar Bending Schedule

	Location	Bar Mark	Description	Shape	Length (m)	Total No of bars	Total Length (m)
1	**Beam**	**1**	**Y16**		**1**	**6**	**6**
2	**Slab**	**2**	**R10**		**7.2**	**43**	**309.6**

For each bar, not only its location is identified, but also data on its length, shape, etc. are given.

This information is particularly important, as it allows a rough estimate of the total length for each of the diameter (size) of reinforcement required for the project to be estimated. The length is then converted to weight using the Table 12.6. Reinforcement bars are sold in weight (tonnes) in the market.

The values in Table 12.6 are in kilogram, which can be converted to tones from which the cost of reinforcement can be estimated. The weight is what is used for estimating cost.

Table 12.6: Diameter, Perimeter and Weight of Reinforcement (Mosley et al., 2013)

Bar Size (mm)	6	8	10	12	16	20	25	32	40
Perimeter (mm)	18.85	25.10	31.40	37.70	50.20	62.80	78.50	100.50	125.60
Weight (kg/m)	0.332	0.395	0.616	0.888	1.579	2.466	3.854	6.313	9.864

Note:

Bar weight based on density of 7850 kg/m^3.

It is to be noted that the column for "shape" in Table 12.5 can be placed differently in by some authors, the guiding bending rules must however conform to BS 4466:1989.

Chapter *13* – Preventing Early Deterioration and Failures in Concrete Structures

13.1 Introduction

Failures of concrete structures, both old buildings and buildings under construction, are becoming rampart. This calls for concerted efforts to formulate and implement strategies for its mitigation. At the design stage, it is expected that a building, except few structures, will last at least 50years. But for the building to last for that years after construction, it is important that the same diligence and perseverance, that were employed during the design stage be employed also at every stage of the construction process. That is, diligence and perseverance in the selection of materials, as well as in ensuring that the processes and procedures that will result in good structural concrete are adopted and followed.

13.2 Measures for strong and durable Concrete Structures

In order to achieve a strong and durable concrete structures, the followings should be the due attention.

1) Proper and correct selection of materials
2) Integrity and inerrancy in design calculations and detailing
3) Provision of adequate quality control and supervision
4) Sound procedure leading to the choice of suitable foundation
5) Healthy human habits

13.2.1 Proper and Correct selection of materials

The primary constituents of concrete are cement, sand, gravel and water. These materials should not only be carefully selected to meet strength requirement but must also be selected to meet the environmental or soil conditions where the concrete is to be placed. Thus, thorough knowledge of the environment that the structure will be domiciled, under extreme conditions, is an important part of a good structural design. For example, ordinary Portland cement (OPC) will not be suitable, as a binder, in marine environment, or for foundation in chemically aggressive soils, or construction of sewage treatment plant structures. Also, the knowledge of the temperature regimes of the environment is also desired. This is because, approaches and precautions to be adopted when concreting in hot climate or at elevated temperatures are different from concreting under normal temperatures, or cold temperatures.

13.2.2 Integrity and inerrancy in design calculations and detailing

Errors in design calculations are a major cause of concrete failures. It is thus necessary that an independent check should be made of all design calculations to ensure that the section sizes, slab thickness etc. and reinforcement sizes and spacing specified are adequate to carry the worst combination of design loads in relation to the operating standard. The check should include overall stability, robustness and serviceability and foundation design. Also, the overall arrangement of the structure should be correct, efficient and robust. The provisions specify the cover to reinforcement, minimum thicknesses for fire resistance, maximum and minimum steel areas, bar spacing limits and reinforcement to control cracking, lap lengths, anchorage of bars etc. should be adhere to.

The use of substandard materials should be avoided. Designs are carried out in accordance with the minimum requirements of a particular code. The materials available should be able to meet these minimum requirements specified. For example, BS 8110 (1997) stipulated a characteristic tensile strength of high yield steel of $460N/mm^2$. It is necessary that the steel used meets this requirement. Also, the concreting materials available, should be able to produce the minimum characteristic compressive strength of $30N/mm^2$ stipulated by BS 8110 (1997).

Incorrect detailing is one of the commonest causes of failure and cracking in concrete structures. Internal or element detailing must comply with the code requirements. In every respect, the structural detailing of reinforcement must agree with the results of the design process. There should not be mistake or oversight.

13.2.3 Provision of inadequate quality control and supervision

Early deterioration and complete collapse of structures can be prevented through the quality control of products and processes of concrete production. In most building and construction works, concrete is produced on site, thus the control of the products and processes is very important to obtain a good structural concrete. Reinforcement comes to site as a finished product. Its control is limited to ensuring good workmanship and its proper arrangement.

Quality Control of Concreting

The processes involved in concrete production are: mix design, batching, mixing, placement, compaction and curing. Each of these processes must be controlled for a good quality concrete to result.

a. Mix design

   Good quality concrete begins at the design stage. Without prejudice to other properties, strength is universally agreed to be the common indicator of good concrete. A mix design that contains the available ingredients in the right proportion with the right water/cement ratio bearing in mind the desire for workability to enhance the required strength should be used. A mix design that uses materials that can only be obtained at great cost or transported over a long distance should be avoided. A low water/cement ratio has been found to reduce porosity of the hardened cement paste at the initial stage of hydration, resulting in high strength.

b. Batching

This involves the weighing of the constituents of the concrete obtained from mix design. It is important that the followings are adhere to.

i. Avoiding undesirable materials during batching processes. For example, dust, organic matter in any form, etc. must not be allowed to find their way into the mix.

ii. The use of the prescribe material at every batching time. This means that the materials for the concrete production are available for finish the job. It will be improper, if midway in the project, a new material is brought in because the original material has been exhausted. This is especially true in aggregates. The gradation of aggregates is affected by the quarry, from which aggregates are obtained.

c. Mixing

Thorough mixing together of the ingredients is necessary obtain a cohesive (that is, concrete that is flowable or mobile for placement), stable (that is, without segregation and bleeding during transportation), homogeneous mix. Appropriate mixing procedure should be adopted.

d. Placement

Placement of concrete should not be done through dropping over a great vertical height as it could cause segregation. In such cases, chutes or pipes should be used in such cases.

e. Compaction

Wet concrete contains voids from many sources, which are: entrapped, gel pores and entrapped air. It these voids are not expelled; the concrete will be porous. It is thus important that concrete be compacted well to reduce the porosity to the minimum possible. The method of compaction employed is such that it makes adequate and full compaction of the wet concrete possible. Inadequate compaction results in two things.

i. Loss of strength. The volume of voids influences the strength of concrete. Low porosity in concrete results in high strength. If concrete is not properly compacted by ramming or vibration the result is a portion of porous honeycomb concrete.

ii. Loss of durability. Early deterioration of concrete results if it is porous. A porous concrete is a permeable concrete. Permeability of concrete largely determines the vulnerability of concrete to external agencies. Penetration of concrete by materials in solution such as, sulfates. chlorides, and other aggressive chemicals and substances affects it durability.

Thus, adequate compaction is essential to give a, strong and impermeable concrete.

f. Curing

Curing of concrete, a procedure for promoting the hydration of cement which consists of control temperature and moisture movement from and into the concrete, is necessary for the production of good concrete. Suitable curing methods should be used. Evaporation of water from the water has to be prevented. Otherwise, there will be inhibition of hydration process which will lead to reduction in strength. Also, loss of water from the surface can cause shrinkage cracking. During curing the concrete should be kept damp and covered.

Workmanship and arrangement of Reinforcement

It is important that the bending of reinforcement is of high quality and workmanship. Also, the position and the arrangement of reinforcement must conform to structural detailing.

a. Inadequate cover to steel

   Insufficient cover to reinforcement exposes the reinforcement to danger of corrosion. Inadequate cover to reinforcement permits ingress of moisture, gases and other substances and results in corrosion of the reinforcement. Corrosion of reinforcement leads to reduction in effective area and reduction in strength. Other adverse effects on concrete are cracking and spalling of the concrete.

b. Reduction in the number and spacing of provided reinforcement

   Under no circumstance should the number and spacing of the reinforcement in structural drawings be reduced.

c. Position and arrangement of reinforcement

   Efforts should be made to ensure that:

   i. Bars are placed at right position
   ii. Bars are properly aligned. This is especially true for columns or other reinforcement that have to be placed vertically. The bars must not be allowed to be out of alignment.
   iii. Bars are placed in the correct direction. For example, in slab, the distribution reinforcement should not be mistakenly arranged as the main bar.

d. Reinforcement at joints should be properly anchored to ensure rigidity. All the beam-columns joints, slab-beam joints, column-footing joint etc. should be properly anchored. Corners under closing bending moments and corners under opening bending moments should be appropriately anchored.

e. Links

   The position of the links is by no means less important. Links and stirrups should be correctly arranged and positioned. Consistency in spacing of links and stirrups in both beams and columns, should be adhere to. For columns, links should be perfectly horizontal.

13.2.4 Sound procedure leading to the choice of suitable foundation

This item of design ought to be treated separately because of its importance. While a defectively design beam or column can be salvaged at relatively little cost, a defectively foundation, if it does not lead to the demolition of the building, can only be corrected at great cost. Thus, adequate attention should be given to foundation, both at the design and construction stage. It is possible that accurate knowledge of the soil may not be known at the design stage, but at the construction, there should be no doubt as regard the true situation of the building. If necessary, confirmatory or additional soil investigations should be carried out. It is not advisable to construct on a soil without geotechnical investigations.

13.2.5 Healthy Human Habits

Some behavioral patterns of people, especially supervising engineers, owners, and users, etc., constitute important factors that can cause concrete structures to deteriorate fast and eventually to fail. Some of these are:

i. Negligence

   This is usually on the part of supervising Engineer manifested by absence of diligence, misplaced priority and vigilance. Supervising engineer must know and understand the purpose of his/her engagement on site, and thus should conduct himself or herself with high sense of professionalism. Site is not a place for making friends or social contacts. Pursuit of any of these may lead to negligence of professional duty, in a profession that expect a high degree of responsibly. Negligence of professional duty can result in deterioration and collapse of buildings

ii. Dereliction of Responsibility

   It is important that a structural engineer knows and understand his duty as well as his/her relationship with allied professionals. Only a structural engineer can do a structural engineering work. Thus, delegating any part of the job to a colleague from any of the allied professions can prove disastrous, thus it should be avoided.

iii. Overloading

   Extreme overloading will cause cracking and eventual collapse. Factors of safety in the original design allow for possible overloads but vigilance is always required to ensure that the structure is never grossly overloaded. A change in function of the building or room can lead to overloading, e.g. if a class room is changed to a library the imposed load can be greatly increased.

iv. Structural alterations

   If major structural alterations are made to a building, the members affected and the overall integrity of the building should be rechecked. Common alterations are the removal of walls or columns to give a large clear space or provide additional doors or openings. Steel beams are inserted to carry loads from above. In such cases the bearing of the new beam on the original structure should be checked and if walls are removed the overall stability may be affected. In some instance, additional floor, that is not part of the design brief, has been added to the original building, without first checking whether the existing foundation is adequate for the additional floor.

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Index

A

B

C

D

E

F

G

H

I

L