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On The Dynamic Analysis of A Simply Supported Rectangular Plate

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Abstract

The dynamic behaviour of a simply supported rectangular plate is studied. This research work is based on the theory of the orthotropic plate simply supported on two sides and free on two other sides. The plate is excited by a moving load while the dynamic response of the structure was obtained using the classical double Fourier series expansion technique, which satisfies the boundary conditions at the four edges. In the absence of the external excitation, the vibration yields free frequencies, other wise, forced frequency is produced. The results obtained from the numerical example are in agreement with the ones in the existing literatures. In addition, the effects of variations in flexural rigidity and that of the frequencies of vibrations are also presented.

Keywords: dynamic, rectangular plate, classical, frequencies.

1.0 Introduction

Over the years, the deflection of rectangular thin plates clamped at four edges under the influence of uniformly distributed loads is a problem that has provoked researches and scientific investigations by numerous researchers because of its technical importance. A plate is called thin if its thickness is at least one order of magnitude smaller than its span. This problem has received more attention from different researchers such as the mathematicians, the engineers, the physicists, and so on, applying various methods of solutions ranging from analytical to numerical solutions [1]-[9].

Some of the aforementioned Scholars used single cosine series and the superposition method as a generalization of Hencky's solution [1]. The problem of bending of a rectangular plate with two opposite edges simply supported was treated by Hutchinson [6] and obtained the deflections for uniformly loaded rectangular plates. In 1985, Burgess and Mahajerin [10] investigated a numerical method for laterally loaded thin plates. The problems and remedy for the Ritz method in determining stress resultants of corner supported rectangular plates was treated by Wang et al [11], and the solution of clamped rectangular plate problems was credited to Taylor and Govindjee [12].

However, the accuracy of the analytic solutions obtained in the literature varies. Those for simply supported plates are exact while others are approximate. Wang et al suggested approximate methods which were discovered to be inefficient due to loss of accuracy [13]. Further study and analysis of rectangular plates with fixed edges under the influence of the uniform load was credited to Imrak and Gerdemmeli [14]. They found the exact solution for the deflection of a clamped rectangular Plate under uniform load using trigonometric and hyperbolic series.

The detailed analyses of the deflections of clamped rectangular plates are carried out in this paper. A solution of the governing differential equation describing the deflection of a thin rectangular plate using classical double cosine series is presented. From the model developed, the associated expressions for the bending moments were obtained. Finally, a numerical example is presented. The results obtained are in good agreement with the existing ones in literature [14]. In addition, the effects of the variations in the flexural rigidity on the deflections were also presented.

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2.0 Fundamental Equations of a Rectangular Plate Element

2.1 Equilibrium of the plate element

Assuming that the plate is subjected to lateral forces only from the fundamental equilibrium equations the following can be used;

$$\sum M_x 0, \sum M_y = 0 \sum P_2 = 0$$

thus the external load P_2 is called by Q_x and Q_y transverse shear forces and by M_x and M_y bending moments. The significant deviation from the two dimensional grid-works action is the presence of the twisting moments M_{xy} and M_{yx}. From figure (1), if the sum of the moment of all forces around the y axes is zero, this gives;

$$\left(M_x + \frac{\partial mx}{\partial x} dx \right) dy - Mxdy + \left(My_x + \frac{\partial mx}{\partial y} dy \right) - My_x dx - (qx + \frac{\partial qx}{\partial x} dx) dy \frac{\partial x}{2} - qxdy \frac{dx}{2} = 0 \quad (2.1),$$

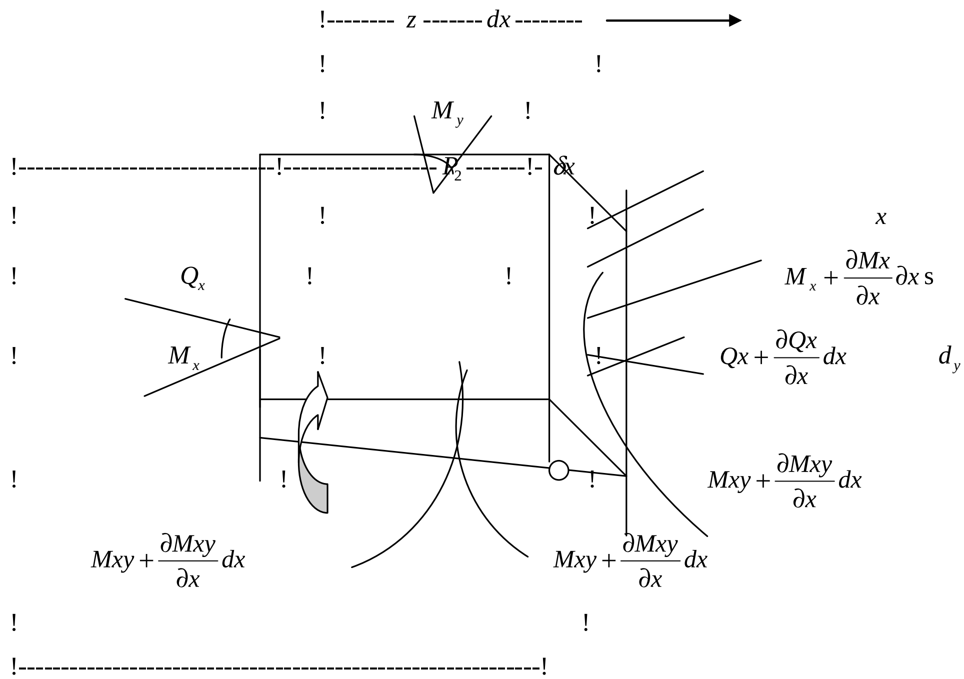

Figure 1 : A parallelepiped cut out of the plate

After simplification, we neglect the term containing $\frac{1}{2} \frac{\partial qx}{\partial x} (\partial x) dy$, since it is a small quantity of higher order, thus equation (2.1) becomes

$$\frac{\partial mx}{\partial x} dxdy + \frac{\partial My_x}{\partial y} dydx - q_x dxdy = 0$$

and after division by $dxdy$, we obtain

$$\frac{\partial m_x}{\partial x} + \frac{\partial my_x}{\partial y} = qx \quad (2.2)$$

In a similar manner the sum of the moments around the x axes gives

$$\frac{\partial m_y}{\partial y} + \frac{\partial my_x}{\partial x} = qx \quad (2.3)$$

The summation of all forces in the z direction yields third equilibrium equation.

$$\frac{\partial qx}{\partial x}dxdy + \frac{\partial qy}{\partial y}dxdy + Pzdxdy = 0$$

which after division by $dxdy$ gives

$$\frac{\partial qx}{\partial x}dxdy + \frac{\partial qy}{\partial y}dydx = -Pz \tag{2.4}$$

Substituting equations (2.2) and (2.3) into (2.4) and observing
$M_{xy} = M_{yx}$ we obtain

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{2\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -Pz(x, y) \tag{2.5}$$

The bending and twisting moments in equation (2.5) depend on the strains and the strains are functions of the displacement components (u, v, w). The next step, is to seek the relations between the internal moments and displacement components.

2.2 Relation Between Stress, Strain And Displacement

The assumption that the material is elastic permits the use of the two-dimensional Hooke's law

$$\delta x = E\varepsilon x + V\delta y \tag{2.6}$$

$$\delta y = E\varepsilon y + V\delta x \tag{2.7}$$

which relates stress and strain in a plate element. Substituting (2.2) into (2.6), we obtain

$$\delta x = \frac{E}{1-V^2}(\varepsilon x + V\delta y) \tag{2.8}$$

In similar manner;

$$\delta y = \frac{E}{1-V^2}(\varepsilon y + VEx) \tag{2.9}$$

In figure (1), the torsion moments M_{xy} and M_{yx} produce in plane shear stresses hxy and hyx which are related to the shear by the pertinent Hooke's relationship. Using assumptions (5) and (6);

$$\Rightarrow \theta - \frac{\partial w}{\partial x} \tag{2.10}$$

$$\Sigma x = \frac{\Delta dx}{dx} = \left[dx + \frac{\left(\frac{\partial \theta}{\partial x}\right)dx \;] - dx}{\partial x} \right] = z\left(\frac{\partial \theta}{\partial x^2} \right) \tag{2.11}$$

By putting (2.10) into the above expression, we obtain;

$$\Sigma x = -\frac{z\partial^2 w}{\partial x^2} \tag{2.12}$$

also, the strain due to normal stresses in the y direction is given by

$$\Sigma y = -\frac{z\partial^2 w}{\partial x^2} \tag{2.13}$$

The curvature changes in the deflection middle surface are defined by

$$\left.\begin{aligned} K_x &= -\frac{\partial^2 w}{\partial x^2} \\ K_y &= -\frac{\partial^2 w}{\partial x^2} \\ K &= -\frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\} \tag{2.14}$$

where K represent the warping of the plate

2.3 Internal Forces components

The stress component δx and δy produced bending produce moments in the plate element in a manner similar to that in elementary beam theory, thus by integration of the normal stress components, the bending moments acting on the plate element are obtained,

$$\left.\begin{aligned} Mx &= \int_{\frac{-h}{2}}^{\frac{-h}{2}} \delta xzdz \\ Mx &= \int_{\frac{h}{2}}^{\frac{-h}{2}} \delta yzdz \end{aligned}\right\} \quad (2.15)$$

Similarly, the twisting moments produced by the shear stresses
$\mathrm{T} = txy = tyx$ can be calculated from:

$$\left.\begin{aligned} Mx &= \int_{\frac{h}{2}}^{\frac{h}{2}} \tau xyzdz \\ Mx &= \int_{\frac{-h}{2}}^{\frac{h}{2}} \tau xyzdz \end{aligned}\right\} \quad (2.16)$$

but $\tau xy = \tau xyx = \tau n$, therefore, $Mxy = Myx$. If we substitute equation (2.12) and (2.13) into (2.8) and (2.9). The normal stresses δx and δx are expressed in terms of the lateral deflection W. Thus we obtain;

$$\delta x = \frac{-E_2}{1-v^2}\left(\frac{\partial^2 W}{\partial x^2} + \frac{v\partial^2 W}{\partial y^2} \right) \quad (2.17)$$

$$\delta y = \frac{-E_2}{1-v^2}\left(\frac{\partial^2 W}{\partial x^2} + v\frac{\partial^2 W}{\partial x^2} \right) \quad (2.18)$$

If we integrate equation (2.16) after the substitution of the equation (2.17) and (2.18) for δx and δy gives

$$Mx = \frac{-Eh^3}{12(1-v^2)}\left(\frac{\partial^2 W}{\partial x^2} + V\frac{\partial^2 W}{\partial y^2} \right)$$

$$= -D\left(\frac{\partial^2 W}{\partial x^2} + V\frac{\partial^2 W}{\partial y^2} \right)$$

$$D(Kx + VKy \quad (2.19)$$

$$My = -D\left(\frac{\partial^2 W}{\partial y^2} + V\frac{\partial^2 W}{\partial x^2} \right)$$

$$\Rightarrow D(Ky + VKx) \quad (2.20)$$

where $$D = \frac{Eh^3}{R(1-v^2)} \quad (2.21)$$

represents the bending or flexural rigidity of the plate in the same manner

$$Mxy = -(\ 1-v\ )D\frac{\partial^2 xy}{\partial y \partial y}$$

$$D(1-v)k \quad (2.22)$$

The substitution of equation (2.19), (2.20) and (2.22) into (2.5) yields the governing differential equation of the plate subjected to lateral load; i.e

$$\frac{\partial^4 W}{\partial x^4} + \frac{2\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4} = \frac{P_Z(x,y)}{D} \quad (2.23)$$

Where W is the deflections of the plate midsurface P_z denotes lateral pressure load on the plate and D is the constant flexural rigidity terms of the material properties, which consists of the Young's modulus of the material E and Poisson's ratio v and the plate thickness h. Thus, the governing equations expresses the relationship between the rectangular plate, lateral load and it's deflection in the case of plate with small deflection.

3.0 Solution Technique

In general, there are four types of mathematical exact solutions available for plate problems namely, closed- form solution, superimposed solution of the biharmonic equation, double trigonometric series solution, and single series solution.
However, in this paper, we shall be using double trigonometric series solution to solve the governing equation of a plate.
We solve the governing equations,

$$\frac{\partial^4 y}{\partial x^4}+\frac{2\partial^4 W}{\partial x^2 \partial y^2}+\frac{\partial^4 W}{\partial y^2}=\frac{P_z(x,y)}{D} \tag{3.1}$$

Subject to the boundary conditions of rectangular plate

$$\left.\begin{aligned} &W=0 \text{ and } \frac{\partial w}{\partial x}=0 \\ &x=a, \text{ and } y=b \end{aligned}\right\} \tag{3.2}$$

and

$$\left.\begin{aligned} &(M_x)_y=0 \quad x=a, \\ &(M_y)_x=0 \quad y=b, \end{aligned}\right\} \tag{3.3}$$

If the deflection of the clamped rectangular plates is expressed by double cosine series

$$W(x,y)=\sum_{M=1}^{M}\sum_{n=1}^{N}\left(1-\cos\frac{2\pi x}{a}\right)\left(1-\cos\frac{2\pi y}{b}\right)Wmn \tag{3.4}$$

(for $n, m = 1,2,3,...$) and the lateral load is given by

$$P_z(x,y)=\sum_{M=1}^{M}\sum_{n=1}^{N}P_{mm}\left(1-\cos\frac{m\pi x}{a}\right)\left(1-\cos\frac{n\pi y}{b}\right) \tag{3.5}$$

where W_{mn} and P_{mn} are unknown coefficients which can be derived from equation (3.2) while P_z is a lateral load.

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using equations (3.4) and (3.5) in equation (3.2), we obtain

$$W_{mm}\left(\frac{m\pi}{a}\right)^4(-)\cos\frac{m\pi x}{a}\left(1-\cos\frac{m\pi x}{b}\right)+2\left(W_{mm}\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi x}{b}\right)^2\cos\frac{n\pi x}{a}\cos\frac{n\pi x}{b}\right.$$

$$\ldots+W_{mm}\left(\frac{n\pi}{b}\right)^4(-)\cos\frac{n\pi x}{b}\left(1-\cos\frac{m\pi x}{a}\right)$$

$$=P_{mn}\left(1-\cos\frac{m\pi x}{a}\right)\left(1-\cos\frac{n\pi x}{b}\right)$$

$$\Rightarrow W_{mm}\pi^4\left[\left(\frac{m}{a}\right)^4\cos\frac{m\pi x}{a}\left(1-\cos\frac{n\pi x}{b}\right)-2\left(W_{mm}\left(\frac{m\pi}{ab}\right)^2\frac{m\pi x}{a}\cos\frac{n\pi x}{b}+\left(\frac{n}{b}\right)^4\cos\frac{n\pi x}{b}\right.\right.$$

$$1-\cos\frac{m\pi x}{a}]=\frac{P_{mn}}{D}\left(1-\cos\frac{m\pi x}{a}\right)\left(1-\cos\frac{n\pi x}{b}\right)$$

$$W_{mm}=\frac{P_{mn}(1-\cos\frac{m\pi x}{a})\left(1-\cos\frac{m\pi x}{b}\right)}{D\pi^4\left[\left(\frac{m}{a}\right)^4\cos\frac{m\pi x}{a}\left((1-\cos\frac{n\pi x}{b}\right)-2\left(\frac{mn}{ab}\right)^2\cos\frac{m\pi x}{a}\cos\frac{n\pi x}{b}+\left(\frac{n}{b}\right)^4\cos\frac{n\pi y}{b}\left(1-\cos\frac{n\pi y}{a}\right)\right.}$$

Putting W_{mn} into equation (3.5), we obtain the analytical solution for the deflection of the plate which is represented as

$$W(x,y)=\frac{1}{D\pi^4}\left[\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{P_{mn}\left(1-\cos\frac{m\pi x}{a}\right)\left(1-\cos\frac{n\pi y}{b}\right)}{\left[\left(\frac{m}{a}\right)^4\cos\frac{m\pi x}{a}\left(1-\cos\frac{n\pi y}{b}\right)-2\left(\frac{mn}{ab}\right)^2\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}+\left(\frac{n}{b}\right)^4\cos\frac{n\pi y}{b}\left(1-\cos\frac{n\pi y}{b}\right)\right]}\right)\times\left(1-\cos\frac{m\pi x}{a}\right)\left(1-\cos\frac{n\pi y}{b}\right)\right]$$

4.0 Solution of the Moments

The solution of the moment is obtained by substituting equation (3.4) into (2.19)

$$M_x = D\left(\frac{\partial^2 W}{\partial x^2}+V\frac{\partial^2 W}{\partial y^2}\right)$$

$$M_x = \pi^2 D\left(\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{m\pi}{a}\right)^2\cos\frac{m\pi x}{a}\left(1-\cos\frac{m\pi y}{b}\right)W_{mn}+V\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\right.$$

$$\left.\left(1-\cos\frac{m\pi x}{a}\right)\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\right)$$

$$M_X = \pi^2 D\left[\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\left(\frac{m}{n}\right)^2\cos\frac{m\pi x}{a}\left(1-\cos\frac{n\pi y}{b}\right)W_{mn}\right)+V\left(\left(1-\cos\frac{m\pi x}{a}\right)\cos\frac{n\pi y}{a}\right)\right]$$

And

$$M_Y = D\left(\frac{\partial^2 W}{\partial y^2}+V\frac{\partial^2 W}{\partial x^2}\right)$$

$$M_y = D\left(\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{n\pi}{b}\right)^2\cos\frac{m\pi y}{b}\left(1-\cos\frac{m\pi x}{a}\right)W_{mn}+V\left(\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{m\pi}{a}\right)^2\right.\right.$$

$$\left.\left.\cos\frac{m\pi}{a}\left(1-\cos\frac{n\pi y}{b}\right)\right)\right)$$

$$M_y = \pi^2 D\sum_{m=1}^{M}\sum_{n=1}^{N}W_{mn}\left(\frac{n}{b}\right)^2\cos\frac{n\pi x}{b}\left(1-\cos\frac{m\pi x}{a}\right)+V\left(\frac{m}{b}\right)^2\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

for some specific value of m and n.

5.0 Illustrative Example

In order to demonstrate the application of the method , let us examine a simply supported rectangular plate Subjected to a uniformly distributed load, given that $P_{mm} = 8P_0\pi^2{}_{mn}$ and m,n are positive odd integers $m = 1,3,5....$ and $n = 1,3,5...$

In the first step, the uniformly distributed lateral load is expanded into double Cosine Series

Recall that

$$P_z = \sum_{m-1}^{M}\sum_{n=1}^{N}P_{MN}\left(1-Cos\frac{m\pi x}{a}\right)\left(1-Cos\frac{n\pi y}{b}\right)$$

$$\Rightarrow P_z = \sum_{m-1}^{M}\sum_{n=1}^{N}P_{MN}\left(1-Cos\frac{m\pi x}{a}\right)\left(1-Cos\frac{n\pi y}{b}\right)$$

$$(m,n = 1,3,5,...)$$

Given

$$P_{mm} = 8P_0\pi^2 mn \quad \text{and} \quad a = a, b = \frac{a}{2}$$

substituting for P_{mm} in P_z, we have

$$\Rightarrow P_z = \sum_{m-1}^{N}\sum_{n=1}^{N} 8P_0\pi^2 mn\left(1 - Cos\frac{m\pi x}{a}\right)\left(1 - Cos\frac{2n\pi y}{a}\right)$$

$$P_z = 8P_0\pi^2\sum_{m-1}^{M}\sum_{n=1}^{N} mn\left(1 - Cos\frac{m\pi x}{a}\right)\left(1 - Cos\frac{2n\pi y}{a}\right)$$

For deflection

$$W_{mn} = \frac{1}{\pi^4 D}\left[\frac{P_{mn}(1 - Cos\frac{m\pi x}{a})(1 - Cos\frac{2n\pi y}{b})}{\left(\frac{m}{a}\right)Cos\frac{m\pi x}{a}(1 - Cos\frac{n\pi x}{b}) - 2\left(\frac{m}{a}\right)^2 Cos\frac{m\pi x}{a}Cos\frac{n\pi x}{b} + \left(\frac{n}{b}\right)^4 Cos\frac{n\pi x}{b}(1 - Cos\frac{m\pi x}{a})}\right]$$

$$W_{mn} = \frac{1}{\pi^4 D}\left[\frac{8P_0\pi^2 mn\left(1 - Cos\frac{n\pi x}{a}\right)\left(1 - Cos\frac{2n\pi y}{a}\right)}{\left(\frac{m}{a}\right)^4 Cos\frac{m\pi x}{a}(1 - Cos\frac{2n\pi x}{a}) - 8\left(\frac{mn}{a^2}\right)^2 Cos\frac{m\pi x}{a}Cos\frac{2n\pi x}{a} + \left(\frac{2n}{a}\right)^4 Cos\frac{2n\pi x}{a}(1 - Cos\frac{m\pi x}{a})}\right]$$

$$W_{mn} = \frac{8P_0}{\pi^2 D}\left[\frac{mn(1 - Cos\frac{n\pi x}{a})(1 - Cos\frac{2n\pi y}{a})}{\left(\frac{m}{a}\right)^4 Cos\frac{m\pi x}{a}(1 - Cos\frac{2n\pi x}{a}) - 8\left(\frac{mn}{a}\right)^2 Cos\frac{n\pi y}{a}Cos\frac{2n\pi y}{a} + \left(\frac{2n}{a}\right)^4 Cos\frac{2n\pi y}{a}1 - Cos\frac{m\pi x}{a}}\right]$$

since

$$W(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} W_{mn}\left(1 - Cos\frac{m\pi x}{a}\right)\left(1 - Cos\frac{2n\pi y}{a}\right)$$

$$W(x, y) = \frac{8P_0}{\pi^2}\sum_{m=1}^{M}\sum_{n=1}^{N}\left[\frac{mn\left(1 - Cos\frac{m\pi x}{a}\right)^2\left(1 - Cos\frac{2n\pi y}{a}\right)^2 a^4}{(m)^4 Cos\frac{m\pi x}{a}(1 - Cos\frac{2n\pi x}{a}) - 8(mn^2\ )Cos\frac{m\pi y}{a}Cos\frac{2n\pi y}{a} + (2(n)^4 Cos\frac{2n\pi y}{a}\left(1 - Cos\frac{m\pi x}{a}\right)}\right]$$

substituting the given values $1 \le m \le 5$ and $1 \le n \le 5$ and the maximum deflection occurs at $x = a$ and $y = \frac{a}{2}$.

When and $n = 1$, we have

$$W_{11} = \frac{8P_0}{\pi^2 D}\left[\frac{(1 - (-1)^2(1 - (-1)^2)^2}{-1(1 - (-1) - 8 - 16}\right] \Rightarrow \frac{8P_0 a^4}{\pi^2 D}\left[\frac{164}{-26}\right]$$

When $m = 3$ and $n = 3$

$$W_{33} = \frac{8P_0 a^4}{\pi^2 D}\left[\frac{(1 - Cos3\pi)^2(1 - Cos3\pi)^2 3\times 3}{(3)^4 Cos3\pi) - 8(9)^2 Cos3\pi Cos3\pi + (6)^4 Cos3\pi(1 - Cos3\pi)}\right] \Rightarrow \frac{8P_0 a^4}{\pi^2 D}\left[\frac{(1+1)^2 9}{81(-2) - 648 + [(-2)(1296)}\right]$$

$$W_{33} = \frac{8P_0 a^4}{\pi^2 D}\left[\frac{144}{-3402}\right] \Rightarrow \frac{8P_0 a^4}{\pi^2 D}\left[\frac{24}{-567}\right]$$

When $m = 5$ and $n = 5$

$$W_{55}\frac{8P_0a^4}{\pi^2 D}\left[\frac{\left(1-Cos5\pi\right)^2\left(1-Cos5\pi\right)^2(5\times 5)}{(5)^4 Cos5\pi\left(1-Cos5\pi\right)-8(5\times 5)^2 Cos5\pi Cos\pi+(10)^4 Cos5\pi\left(1-Cos5\pi\right)}\right]$$

$$=\frac{8P_0a^4}{\pi^2 D}\left[\frac{(1-(-1)^2(1-(-1)(25)}{625-1(1-(-1))-8(25)^2(-1)(-1)+10{,}000(-1)(1-(-1)}\right]$$

$$=\frac{8P_0a^4}{\pi^2 D}\left[\frac{400}{-625(2)-5{,}000-20{,}000}\right]$$

$$=\frac{8P_0a^4}{\pi^2 D}\left[\frac{400}{26{,}250}\right]\Rightarrow\frac{8P_0a^4}{\pi^2 D}\left[\frac{8}{-525}\right]$$

Now,

$$W(x,y)=\frac{8P_0a^4}{\pi^2 D}\left[\frac{16}{-26}\right]+\frac{8P_0a^4}{\pi^2 D}\left[\frac{24}{567}\right]+\frac{8P_0a^4}{\pi^2 D}\left[\frac{8}{-535}\right]$$

$$W(x,y)=\frac{8P_0a^4}{\pi^2 D}\left[\frac{-16}{26}\right]-\left[\frac{24}{567}\right]-\left[\frac{8}{-535}\right]$$

$$=\frac{8P_0a^4}{\pi^2 D}\left[-0.61538-0.04233-0.01524\right]$$

$$=-\frac{8P_0a^4}{\pi^2 D}\left[-0.67295\right]$$

$$W\max=\frac{-0.67295P_0a^4}{\pi^2 D}$$

We obtained deflections for various values of flexural rigidity. The result agrees with claims made by [12].

Table 1: The relationship between the flexural rigidity and deflection

D	W
27473	-1.737×10^{-4}
28617	-1.667×10^{-4}
29762	-1.603×10^{-4}
30907	-1.544×10^{-4}
32051	-1.489×10^{-4}

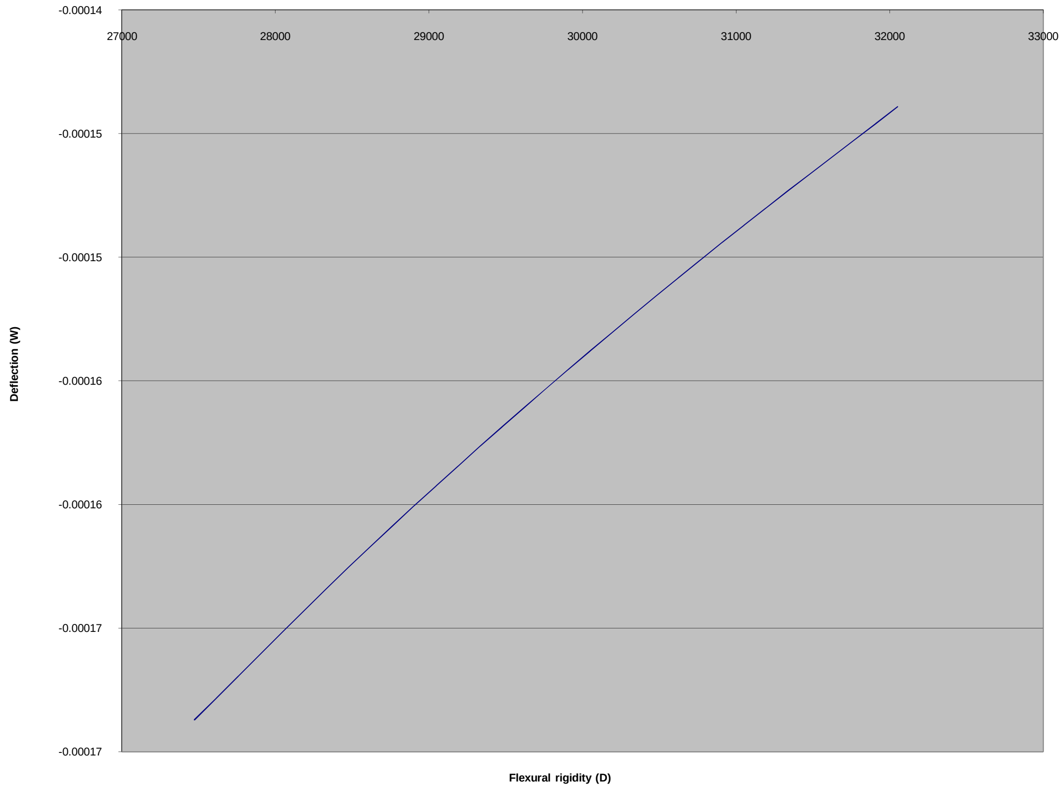

Fig 2: Effect of increasing in flexural rigidity on the deflection

6.0 Conclusion

The solution of a clamped rectangular plate using classical double Fourier series expansion technique has been considered in this paper. We obtained the analytical solution of the problem applying the above method, which satisfies the boundary conditions at the four edges. The solution contains three different terms which includes the case of a strip and the influences of the edges. The results obtained from the numerical example are in agreement with the ones presented in [11-14]. In addition, the effects of variations in flexural rigidity on the deflection are also presented

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