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Effects Of Electrical Parameters Variation On The Dynamic Behaviour Of Two-Phase Induction Motor

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ABSTRACT : *This research work seeks to analyse the performance of a two-phase induction motor using dynamic model based on MATLAB. The 4th order Runge-Kutta was used for the solution of this dynamic model. The modelling and simulation will be strictly for Two-phase induction motors used in low power applications. Measured and simulated results would be discussed and correct mitigation techniques would also be recommended .Future work within the scope of this research work would also be discussed. With the growing concerns about low-cost operation and efficient use of energy, Two-Phase induction motors have gained interest in low-power applications, especially for domestic or commercial applications where a three-phase power supply is not available. This result from this research work would help improve the design of two-phase induction motor used in the applications above.*

KEYWORDS : *Induction motor, Two phase ,Dynamic modeling ,Flux , linkage ,Stator ,Rotor , commutator.*

I. INTRODUCTION

The dynamic models of systems are representations such as functions, sets of differential equations and so that allow estimations on the outputs based on input measurements. There are two basic ways to determine the dynamic models of a given system either using explanatory theories or with input and output measurements and system identification algorithms. In the basic approach, the dynamic models of the electric drives are obtained with the direct and quadrature-axis theory tailored to the specific class of the electric machine and power converter. The implementation of the direct and quadrature-axis theory provide models that allow estimating system's response in the time domain, Henneberger (2002). The estimate's consistency is affected by the accuracy of the measurements and the consistency of the parameters' estimates. With the space vector definition, the time domain model of the machine may be transformed into the complex representation. The complex representation of the electric machine model provides the easiest way to transform the dynamic model from one reference coordinate system to another. In addition, in the complex representation the command of the three-phase inverters can be handled in the most appropriate manner.In small power applications, asymmetrical two-phase induction motors fed by single-phase supply have been widely used in electric machines in home appliances and industrial applications requiring less than 5 kW Ojo and Omozusi (2001). Single-phase induction motors with main and auxiliary winding can be viewed as two-phase machines, since these winding mechanisms are displaced 90 degrees apart from each other. Therefore, two-phase induction motors have a configuration identical to single-phase induction motors, but the input voltage applied to the stator winding terminals is independently controlled so that a two-phase voltage is supplied. In recent years, several methods that use models for simulation of two-phase induction motors have been proposed. The dynamics of single/two phase induction motors have been studied and performance evaluated.

II. STATEMENT OF PROBLEM

In an electric drive system, the motor is part of the control system elements. Due to the complex nature of the dynamic models of induction motors, there is a need for the dynamic behavior of the motor to be considered, in order to assess the induction motor performance with electrical parameter variations.

III. OBJECTIVES

The main objectives of this research work are:

[1] To carry out the dynamic modelling of a Two- phase induction motor.
[2] To use the 4th order Runge-Kutta method for solving the dynamic model.
[3] To simulate the dynamic behaviour of a two-phase induction motor.
[4] To clarify the effects of electrical parameter variations on the dynamic behaviour of Two-phase induction motor.

IV. TWO-PHASE AC INDUCTION MOTOR

In the two-phase motor, two sets of coils are set perpendicular to each other surrounding the core. When alternating current is sent to the coils, they become electromagnets where polarity rapidly changes with each reversal of current flow. As the first coils are supplied with current, they create a magnetic field which starts the core turning. When the first coils' current supply reverses, the second coil set is at its maximum supply point and creates its own magnetic field; the core spins on. In effect the "magnetization" amount never varies and a rotating magnetic field is created. The result is a smooth-running, commutator free motor with the rotor as its only moving part.

V. TWO PHASE INDUCTION MOTOR MODEL

The two-phase induction motor is composed of two asymmetrical windings. Therefore, the auxiliary winding usually has fewer turns than the main winding and is displaced at ninety electrical degrees between these winding Jang and Won (1994). Fig. 1 shows the schematic view of a two phase induction motor, illustrating that the auxiliary (α) windings and main (β) windings are not identical sinusoidal distributed windings, but are arranged in space quadrature.

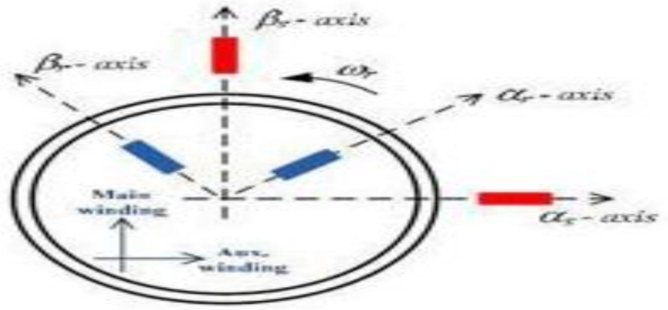

Fig 2.0 Asymmetrical Two Phase Induction Motor [Kumsuwan 2010]

The equivalent circuits representing the asymmetrical two-phase induction motor in stationary (αβ) reference frame are shown in Fig. 2. The dynamic model equation of asymmetrical two phase induction motor can be written as αβ reference frame variables. Components stator and rotor voltage of the two-phase induction motor can be expressed as follows:

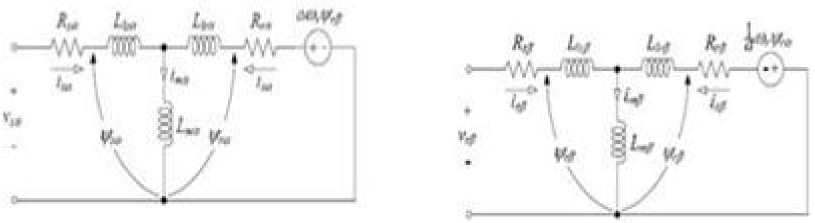

Fig 2.1 Equivalent circuit of an asymmetrical two-phase induction motor

VI. DEVELOPMENT OF MATHEMATICAL MODELS FOR TWO-PHASE INDUCTION MOTOR

In this chapter, the development of mathematical model for a Two-phase induction motor is described. Consider the figure 3.0 below; assuming that the two ends of each rotor phase are shorted together, that is, there is no voltage source for the rotor phases. With the rotor phases shorted, the expressions for the phase flux linkages and rotor torque are used to drive the nonlinear differential equation model of a two-phase sinusoidal wound induction motor.

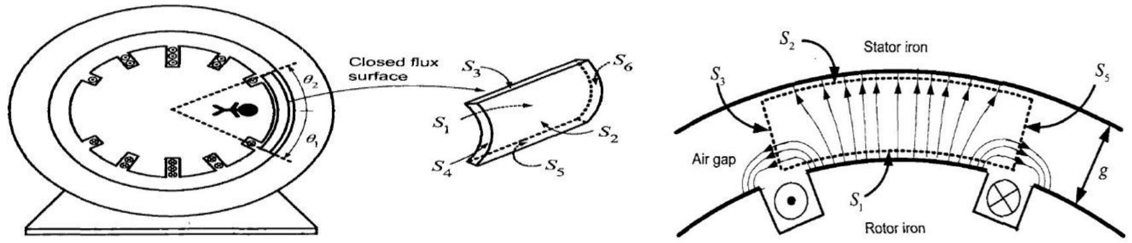

Figure 3.0 Closed flux surface to illustrate flux leakage

VII. STATOR FLUX LINKAGE PRODUCED BY THE STATOR CURRENTS

Consider a single turn (loop) of a stator phase at the angular position θ with $0 \le \theta \le \pi$ that is, one side of the loop is at θ and the other side is at $\theta - \pi$ as show in figure 3.1.

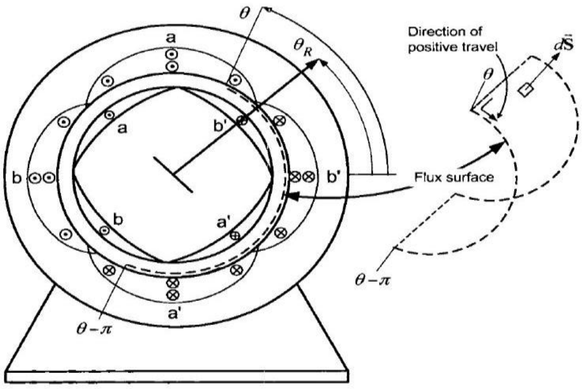

Figure 3.1 Surface to compute the flux in a single turn of stator phase ''a''

The surface element vector $d\vec{S} = r_S d\theta' dz\hat{r}$ is chosen. The flux $\phi_{Sa}(i_{Sa}, i_{Sb}, \theta)$ in each turn of stator phase a at the angular position θ of the stator is given by

$$\phi(i_{Sa}, i_{Sb}, \theta) \triangleq \int_{\theta'=\theta-\pi}^{\theta'=\theta} \int_{z=0}^{z=\ell_1} \frac{\mu_o r_R N_S}{2g} \frac{1}{r_S} \left[(i_S \cos(\theta') + i_{Sb} \sin(\theta')\right] \hat{r} \cdot \left[r_S d\theta' dz \hat{r} \right] \quad (1)$$

$$= \frac{\mu_o r_R \ell_i N_S}{2g} \int_{\theta'=\theta-\pi}^{\theta'=\theta} \left[i_{Sa} \cos(\theta') + i_{Sb} \sin(\theta') \right] d\theta'$$

VIII. DYNAMIC EQUATION OF THE INDUCTION MOTOR

To start with, the system of nonlinear differential equations which characterize its behavior as previously derived are presented. The dynamics of a n_p pole-pair two-phase induction motor are given by the system of differential equations below, Leonhard, (1985)

$$u_{sa} = R_S i_{Sa} + L_S \frac{d}{dt} i_{Sa} + M \frac{d}{dt}\left(i_{Ra} \cos(n_p\theta) - i_{Rb} \sin(n_p\theta)\right)$$

$$u_{sb} = R_S i_{Sb} + L_S \frac{d}{dt} i_{Sb} + M \frac{d}{dt}\left(i_{Ra} \sin(n_p\theta) + i_{Rb} \cos(n_p\theta)\right)$$

$$0 = R_R i_{Ra} + L_R \frac{d}{dt} i_{Ra} + M \frac{d}{dt}\left(i_{Sa} \cos(n_p\theta) + i_{Sb} \sin(n_p\theta)\right)$$

$$0 = R_R i_{Rb} + L_R \frac{d}{dt} i_{Rb} + M \frac{d}{dt}\left(i_{Sa} \sin(n_p\theta) + i_{Sb} \cos(n_p\theta)\right)$$

$$J\frac{d\omega}{dt} = n_p M\left(i_{Sb}(i_{Ra} \cos(n_p\theta) - i_{Rb} \sin(n_p\theta)\right) - i_{Sa}\left(i_{Ra} \sin(n_p\theta) + i_{Rb} \cos(n_p\theta)\right) - f\omega - \tau_L$$

$$\frac{d\theta}{dt} = \omega \quad (2)$$

With the flux linkages of the motor phases given by

$$
\begin{aligned}
\lambda_{Sa} &= L_S i_{Sa} + M\ \left((i_{Ra}\cos(n_p\theta) - i_{Rb}\sin(n_p\theta)\right) \\
\lambda_{Sb} &= L_S i_{Sb} + M\ \left((i_{Ra}\sin(n_p\theta) - i_{Rb}\cos(n_p\theta)\right) \\
\lambda_{Ra} &= L_R i_{Ra} + M\ \left(+ i_{Sa}\cos(n_p\theta) - i_{Sb}\sin(n_p\theta)\right) \\
\lambda_{Ra} &= L_R i_{Rb} + M\ \left(+ i_{Sa}\sin(n_p\theta) - i_{Sb}\cos(n_p\theta\right)
\end{aligned}
\tag{3}
$$

Where

$$
L_S \triangleq \frac{\mu_0 \pi \ell_1 \ell_2 N_S^2}{8g},\ L_R \triangleq \frac{\mu_0 \pi \ell_1 \ell_2 N_R^2}{8g}, M \triangleq \frac{k\mu_0 \pi \ell_1 \ell_2 N_S N_R}{8g} \qquad (3.25)(4)
$$

Hence N_S and N_R are the number of windings per pole-pair of the stator and rotor phases, respectively. The retarding torque produced by the friction in the ball bearings of the machine is modelled here by $-f\omega$, where f is called the coefficient of viscous friction. Of course, one could just consider this part of the load torque τ_L. However, as one can typically estimate f at the same time as estimating the motor parameters. $-f\omega$ then represents the "known" load torque and τ_L is then the "unknown" load torque. Also, to simplify the notation, the subscript "R" on θ_R and ω_R has now been dropped as there should be no confusion. According to Stephan, (1994), the mathematical model derived above is that of a sinusoidally wound, two-phase, n_p pole-pair induction motor. Nevertheless, it is standard practice to also use this model for induction motors with *squirrel* cage rotors.

IX. THE CONTROL PROBLEM

The particular set of nonlinear differential equations describing the induction motor is complicated and a control strategy is by no means self-evident. The solution lies in finding an equivalent set of equations which are simpler in form for which the control design becomes apparent. As a first step toward simplifying the above equations, the dynamic equations are rewritten in terms of some equivalent rotor flux linkages. This result in an equivalent model in which the $\cos(n_p\theta)$ and $\sin(n_p\theta)$ expression are eliminated. To proceed, define an equivalent set of rotor flux linkages as

$$
\begin{bmatrix} \psi_{Ra} \\ \psi_{Rb} \end{bmatrix} = \begin{bmatrix} \cos(n_p\theta) & -\sin(n_p\theta) \\ \sin(n_p\theta) & \cos(n_p\theta) \end{bmatrix} \begin{bmatrix} \lambda_{Ra} \\ \lambda_{Rb} \end{bmatrix} \tag{5}
$$

On substituting the expression for the rotor fluxes in (20) one obtains

$$
\begin{aligned}
\psi_{Ra} &= L_R(i_{Ra}\cos(n_p\theta) - i_{Rb}\sin(n_p\theta) + M\, i_{Sa} \\
\psi_{Rb} &= L_R(i_{Ra}\sin(n_p\theta) - i_{Rb}\cos(n_p\theta) + M\, i_{Sb}
\end{aligned}
$$

The model of the induction motor in terms of the state variables $\psi_{Ra}, \psi_{Rb}, i_{Sa}, i_{Sb}$, and ω is now derived Leonhard, (2001).Using (2) to eliminate the rotor currents the first two equations of (2) may be written as

$$
\begin{aligned}
u_{Sa} &= R_S i_{Sa} + L_S \frac{d}{dt} i_{Sa} + M \frac{d}{dt}(\psi_{Ra} - M\, i_{Sa}) / L_R \\
u_{Sb} &= R_S i_{Sb} + L_S \frac{d}{dt} i_{Sb} + M \frac{d}{dt}(\psi_{Rb} - M\, i_{Sb}) / L_R
\end{aligned}
$$

or

$$
\begin{aligned}
u_{Sa} &= R_S i_{Sa} + L_S\left(1 - \frac{M^2}{L_R L_S}\right)\frac{d}{dt} i_{Sa} + \frac{M}{L_R}\frac{d}{dt}\psi_{Ra} \\
u_{Sb} &= R_S i_{Sb} + L_S\left(1 - \frac{M^2}{L_R L_S}\right)\frac{d}{dt} i_{Sb} + \frac{M}{L_R}\frac{d}{dt}\psi_{Rb}
\end{aligned}
\tag{6}
$$

The third and fourth equations of (2) can be rewritten as

$$\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}R_R i_{Ra}\\R_R i_{Rb}\end{bmatrix}+\frac{d}{dt}\left(\begin{bmatrix}\cos(n_p\theta) & \sin(n_p\theta)\\-\sin(n_p\theta) & \cos(n_p\theta)\end{bmatrix}\right)\times\begin{bmatrix}L_R(\cos(n_p\theta)i_{Ra}-\sin(n_p\theta)i_{Rb})+M\,i_{Sa}\\L_R(\sin(n_p\theta)i_{Ra}+\cos(n_p\theta)i_{Rb})+M\,i_{Sb}\end{bmatrix}$$

Or simply

$$\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}R_R i_{Ra}\\R_R i_{Rb}\end{bmatrix}+\frac{d}{dt}\begin{bmatrix}\cos(n_p\theta) & \sin(n_p\theta)\\-\sin(n_p\theta) & \cos(n_p\theta)\end{bmatrix}\begin{bmatrix}\psi_{Ra}\\\psi_{Rb}\end{bmatrix}$$

Expanding

$$\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}R_R i_{Ra}\\R_R i_{Rb}\end{bmatrix}+\frac{d}{dt}\left(\begin{bmatrix}\cos(n_p\theta) & \sin(n_p\theta)\\-\sin(n_p\theta) & \cos(n_p\theta)\end{bmatrix}\right)\begin{bmatrix}\psi_{Ra}\\\psi_{Rb}\end{bmatrix}+\begin{bmatrix}\cos(n_p\theta) & \sin(n_p\theta)\\-\sin(n_p\theta) & \cos(n_p\theta)\end{bmatrix}\frac{d}{dt}\begin{bmatrix}\psi_{Ra}\\\psi_{Rb}\end{bmatrix}$$

Multiplying both sides on the left by $\begin{bmatrix}\cos(n_p\theta) & -\sin(n_p\theta)\\\sin(n_p\theta) & \cos(n_p\theta)\end{bmatrix}$ gives

$$\begin{aligned}\begin{bmatrix}0\\0\end{bmatrix}&=\begin{bmatrix}\cos(n_p\theta) & -\sin(n_p\theta)\\\sin(n_p\theta) & \cos(n_p\theta)\end{bmatrix}\begin{bmatrix}R_R i_{Ra}\\R_R i_{Rb}\end{bmatrix}+\begin{bmatrix}\cos(n_p\theta) & -\sin(n_p\theta)\\\sin(n_p\theta) & \cos(n_p\theta)\end{bmatrix}\\&\quad\times n_p\omega\begin{bmatrix}-\sin(n_p\theta) & \cos(n_p\theta)\\-\cos(n_p\theta) & -\sin(n_p\theta)\end{bmatrix}\begin{bmatrix}\psi_{Ra}\\\psi_{Rb}\end{bmatrix}+\frac{d}{dt}\begin{bmatrix}\psi_{Ra}\\\psi_{Rb}\end{bmatrix}\end{aligned}\tag{7}$$

This simplifies to

$$\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}(R_R/L_R)\,(v_{Ra}-M\,i_{Sa})\\(R_R/L_R)\,(v_{Rb}-M\,i_{Sb})\end{bmatrix}-n_p\omega\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}\psi_{Ra}\\\psi_{Rb}\end{bmatrix}+\frac{d}{dt}\begin{bmatrix}\psi_{Ra}\\\psi_{Rb}\end{bmatrix}\tag{8}$$

Finally, the torque equation becomes

$$\begin{aligned}J\frac{d\omega}{dt}&=n_pM\left(i_{Sb}\frac{\psi_{Ra}-M\,i_{Sa}}{L_R}-i_{Sa}\frac{\psi_{Rb}-M\,i_{Sb}}{L_R}\right)-f\omega-\tau_L\\&=n_P\frac{M}{L_R}(i_{Sb}\psi_{Ra}-i_{Sa}\psi_{Rb})-f\omega-\tau_{L1}\end{aligned}\tag{9}$$

Collecting these equations together, the dynamic model of the induction motor in terms of the state variable $\theta,\omega,\psi_{Ra},\psi_{Rb},i_{Sa}$ is then

$$\frac{d\theta}{dt}=\omega$$

$$\frac{d\omega}{dt}=\frac{n_pM}{JL_R}\left(i_{Sb}\psi_{Ra}-i_{Sb}\psi_{Rb}\right)-\frac{f}{j}\omega-\frac{\tau_L}{J}$$

$$\frac{d\psi_{Ra}}{dt}=-\frac{R_R}{L_R}\psi_{Ra}-n_p\omega\psi_{Rb}+\frac{MR_R}{L_R}i_{Sb}$$

$$\frac{d\psi_R}{dt}=-\frac{R_R}{L_R}\psi_{Ra}-n_p\omega\psi_{Ra}+\frac{MR_R}{L_R}i_{Sb}$$

$$\begin{aligned}u_{Sa}&=R_Si_{Sa}+\sigma L_S\frac{di_{Sa}}{dt}+\frac{M}{L_R}\frac{d\psi_{Ra}}{dt}\\u_{Sb}&=R_Si_{Sb}+\sigma L_S\frac{di_{Sb}}{dt}+\frac{M}{L_R}\frac{d\psi_{Rb}}{dt}\end{aligned}\tag{10}$$

Where

$$\sigma \triangleq 1-\frac{M^2}{L_R L_S}$$

is the leakage parameter. Substituting the expressions for $d\psi_{Ra}/dt$ and $d\psi_{Rb}/dt$ form the third and fourth equations of (2) into the fifth and sixth equations of (2) and rearranging one obtains the state-space from the system given by Marino et.al., (1993) ; Bodson et.al., (1994).

$$\frac{d\theta}{dt}=\omega$$

$$\frac{d\omega}{dt}=\mu\left(i_{Sb}\psi_{Ra}-i_{Sa}\psi_{Rb}\right)-\left(f/J\right)\omega-\tau_T/J$$

$$\frac{d\psi_{Ra}}{dt}=-\eta\psi_{Ra}-n_p\omega\psi_{Rb}+\eta M i_{Sa}$$

$$\frac{d\psi_{Rb}}{dt}=-\eta\psi_{Rb}-n_p\omega\psi_{Ra}+\eta M i_{Sb}$$

$$\frac{di_{Sa}}{dt}=\eta\beta\psi_{Ra}+\beta n_p\omega\psi_{Rb}-\gamma i_{Sa}+u_{Sa}/\sigma L_S \tag{11}$$

$$\frac{di_{Sb}}{dt}=\eta\beta\psi_{Rb}-\beta n_p\omega\psi_{Ra}-\gamma i_{Sb}+u_{Sb}/\sigma L_S$$

With $\eta \triangleq \frac{R_R}{L_R},\beta\triangleq\frac{M}{\sigma L_R L_S},\mu\triangleq\frac{n_p M}{JL_R},\gamma\triangleq\frac{M^2 R_R}{\sigma L_R^2 L_S}+\frac{R_S}{\sigma L_S}$ (12)

X. MATHEMATICAL MODEL SOLUTION USING LAPLACE TRANSFORM

Rewriting equation 2, we obtained

$$u_{Sa}=R_S i_{Sa}+L_S\frac{di_{Sa}}{dt}+M\cos(n_p\theta)\frac{di_{Ra}}{dt}-M\sin(n_p\theta)\frac{di_{Rb}}{dt}$$

$$u_{Sb}=R_S i_{Sb}+L_S\frac{di_{Sb}}{dt}+M\sin(n_p\theta)\frac{di_{Ra}}{dt}-M\cos(n_p\theta)\frac{di_{Rb}}{dt}$$

$$0=R_R i_{Ra}+L_R\frac{di_{Ra}}{dt}+M\cos(n_p\theta)\frac{di_{Sa}}{dt}-M\sin(n_p\theta)\frac{di_{Sb}}{dt}$$

$$0=R_R i_{Rb}+L_R\frac{di_{Rb}}{dt}+M\sin(n_p\theta)\frac{di_{Sa}}{dt}-M\cos(n_p\theta)\frac{di_{Sb}}{dt} \tag{13}$$

Taking equation 13 into Laplace transform, we obtained;

$$\frac{\mathrm{u_{Sa}}}{\mathrm{s}}=\mathrm{R_S}\overline{\imath_{Sa}}+\mathrm{L_S}[\mathrm{S}\,\overline{\imath_{Sa}}-i_{Sa}(0)]+M^{I}[\mathrm{S}\overline{\imath_{Ra}}-i_{Ra}(0)]-M^{II}[\mathrm{S}\,\overline{\imath_{Rb}}-i_{Rb}(0)]\frac{\mathrm{u_{Sb}}}{\mathrm{s}}$$

$$=\mathrm{R_S}\overline{\imath_{Sb}}+\mathrm{L_S}[\mathrm{S}\,\overline{\imath_{Sb}}-i_{Sb}(0)]+M^{II}[\mathrm{S}\overline{\imath_{Ra}}-i_{Ra}(0)]-M^{I}[\mathrm{S}\,\overline{\imath_{Rb}}-i_{Rb}(0)]$$

$$0=\mathrm{R_R}\overline{\imath_{Ra}}+\mathrm{L_R}[\mathrm{S}\overline{\imath_{Ra}}-i_{Ra}(0)]+M^{I}[\mathrm{S}\,\overline{\imath_{Sa}}-i_{Sa}(0)]-M^{II}[\mathrm{S}\,\overline{\imath_{Sb}}-i_{Sb}(0)]$$

$$0=\mathrm{R_R}\overline{\imath_{Rb}}+\mathrm{L_R}[\mathrm{S}\,\overline{\imath_{Rb}}-i_{Rb}(0)]+M^{II}[\mathrm{S}\,\overline{\imath_{Sa}}-i_{Sa}(0)]-M^{I}[\mathrm{S}\,\overline{\imath_{Sb}}-i_{Sb}(0)] \tag{14}$$

Where $M^{I}=M\cos(n_p\theta)$, $M^{II}=M\sin(n_p\theta)$, $i(s)=\bar{\imath}$ and $i(0)$=0 for initial condition

Rearranging equation 14, we have;

$$\frac{u_{Sa}}{S}=(R_S+L_S S)\overline{\imath_{Sa}}+M^{I}S\,\overline{\imath_{Ra}}-M^{II}S\,\overline{\imath_{Rb}}$$

$$\frac{u_{Sb}}{S}=(R_S+L_S S)\overline{\imath_{Sb}}+M^{II}S\,\overline{\imath_{Ra}}-M^{I}S\,\overline{\imath_{Rb}}$$

$$0=R_R\overline{\imath_{Ra}}+L_R S\,\overline{\imath_{Ra}}+M^{I}S\,\overline{\imath_{Sa}}-M^{II}S\,\overline{\imath_{Sb}}$$

$$0=R_R\overline{\imath_{Rb}}+L_R S\,\overline{\imath_{Rb}}+M^{II}S\,\overline{\imath_{Sa}}-M^{I}S\,\overline{\imath_{Sb}}$$

Casting this in matrix form, we have;

$$\begin{bmatrix} u_{Sa} \\ u_{Sa} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_S + L_S S & 0 & M^I S & -M^{II} S \\ 0 & R_S + L_S S & M^{II} S & M^I S \\ M^I S & -M^{II} S & R_R + L_R S & 0 \\ M^{II} S & -M^I S & 0 & R_R + L_R S \end{bmatrix} \begin{bmatrix} \overline{i_{Sa}} \\ \overline{i_{Sb}} \\ \overline{i_{Ra}} \\ \overline{i_{Rb}} \end{bmatrix}$$

Using Crammer's rule,

$$\overline{i_{Sa}} = \frac{\begin{vmatrix} \frac{u_{Sa}}{S} & 0 & M^I S & -M^{II} S \\ \frac{u_{Sb}}{S} & R_S + L_S S & M^{II} S & M^I S \\ 0 & -M^{II} S & R_R + L_R S & 0 \\ 0 & -M^I S & 0 & R_R + L_R S \end{vmatrix}}{\begin{vmatrix} R_S + L_S S & 0 & M^I S & -M^{II} S \\ 0 & R_S + L_S S & M^{II} S & M^I S \\ M^I S & -M^{II} S & R_R + L_R S & 0 \\ M^{II} S & -M^I S & 0 & R_R + L_R S \end{vmatrix}}$$

$$\overline{i_{Sb}} = \frac{\begin{vmatrix} R_S + L_S S & \frac{u_{Sa}}{S} & M^I S & -M^{II} S \\ 0 & \frac{u_{Sb}}{S} & M^{II} S & M^I S \\ M^I S & 0 & R_R + L_R S & 0 \\ M^{II} S & 0 & 0 & R_R + L_R S \end{vmatrix}}{\begin{vmatrix} R_S + L_S S & 0 & M^I S & -M^{II} S \\ 0 & R_S + L_S S & M^{II} S & M^I S \\ M^I S & -M^{II} S & R_R + L_R S & 0 \\ M^{II} S & -M^I S & 0 & R_R + L_R S \end{vmatrix}}$$

$$\overline{i_{Ra}} = \frac{\begin{vmatrix} R_S + L_S S & 0 & \frac{u_{Sa}}{S} & -M^{II} S \\ 0 & R_S + L_S S & \frac{u_{Sb}}{S} & M^I S \\ M^I S & -M^{II} S & 0 & 0 \\ M^{II} S & -M^I S & 0 & R_R + L_R S \end{vmatrix}}{\begin{vmatrix} R_S + L_S S & 0 & M^I S & -M^{II} S \\ 0 & R_S + L_S S & M^{II} S & M^I S \\ M^I S & -M^{II} S & R_R + L_R S & 0 \\ M^{II} S & -M^I S & 0 & R_R + L_R S \end{vmatrix}}$$

$$\overline{i_{\mathrm{Rb}}} = \frac{\begin{vmatrix} R_S + L_S S & 0 & M^I S & \frac{u_{Sa}}{S} \\ 0 & R_S + L_S S & M^{II} S & \frac{u_{Sb}}{S} \\ M^I S & -M^{II} S & R_R + L_R S & 0 \\ M^{II} S & -M^I S & 0 & 0 \end{vmatrix}}{\begin{vmatrix} R_S + L_S S & 0 & M^I S & -M^{II} S \\ 0 & R_S + L_S S & M^{II} S & M^I S \\ M^I S & -M^{II} S & R_R + L_R S & 0 \\ M^{II} S & -M^I S & 0 & R_R + L_R S \end{vmatrix}}$$

From the analysis above, we can see that finding the inverse of the Laplace transform is a rigorous work which will take long analytical time, therefore recourse is made to numerical methods, precisely the 4th order Runge-Kutta method. The Runge-Kutta method for solving first order differential equations is widely used and affords a high degree of accuracy. It is a further step-by-step process where a table of function values for a range of values of x is accumulated Stroud (1996). In general terms, the method is as follows.

To solve $y' = f(x, y)$ with initial condition $y = y_o$ at $x = x_o$, for a range of values of $x = x_o(h)x_n$.

Starting as usual with $x= x_o$, $y= y_o$, $y' =(y')_o$ and h, we have

$x_1 = x_o+ h$

Finding y_1 requires four intermediate calculations

$$K_1 = hf(x_o, y_o) = h(y')_o \tag{15}$$

$$K_2 = hf(x_o + \frac{1}{2}h, y_o + \frac{1}{2}k_1) \tag{16}$$

$$K_3 = hf(x_o + \frac{1}{2}h, y_o + \frac{1}{2}k_2) \tag{17}$$

$$K_4 = hf(x_o + h, y_o + k_3) \tag{18}$$

The increment Δy_o in the y-values from $x = x_o$ *to* $x= x_1$ is then

$$\Delta y_o = \frac{1}{6}\{k_1+2k_2+2k_3+ k4\} \tag{19}$$

and finally $y_1 = y_o+ \Delta y_o$. (20)

From Equation 19, setting the rotor position θ at 0˚and 90˚respectively and number of pole pair n_p =3, we have:

For θ = 0˚

$$u_{sa} = R_S i_{Sa} + L_S \frac{d}{dt} i_{Sa} + M \frac{d}{dt} i_{Ra} \tag{21}$$

(22)
(23)
(24)

$$u_{sb} = R_S i_{Sb} + L_S \frac{d}{dt} i_{Sb} + M \frac{d}{dt} i_{Rb}$$

$$0 = R_R i_{Ra} + L_R \frac{d}{dt} i_{Ra} + M \frac{d}{dt} i_{Sa} \tag{25}$$

$$0 = R_R i_{Rb} + L_R \frac{d}{dt} i_{Rb} + M \frac{d}{dt} i_{Sb}$$

$$J \frac{d\omega}{dt} = n_p M(i_{Sb} i_{Ra} - i_{Sa} i_{Rb}) - f\omega - \tau_L$$

$$\frac{d\theta}{dt} = \omega$$

Multiplying equation 28 by L_R and equation 30 by M, we have

$$L_R u_{sa} = R_S i_{Sa} L_R + L_S L_R \frac{d}{dt} i_{Sa} + M L_R \frac{d}{dt} i_{Ra} \tag{27}$$

$$0 = R_R i_{Ra} M + L_R M \frac{d}{dt} i_{Ra} + M^2 \frac{d}{dt} i_{Sa} \tag{28}$$

Subtracting equation 35 from 36, we obtain

$$L_R U_{Sa} = R_S i_{Sa} L_R - R_R i_{Ra} M + \left(L_S L_R - M^2\right) \frac{d}{dt} i_{Sa} \tag{29}$$

Carrying out the same procedure for equations 29 and 31, we obtain the following set of equations

$$L_R U_{Sa} = R_S i_{Sa} L_R - R_R i_{Ra} M + \left(L_S L_R - M^2\right) \frac{d}{dt} i_{Sa} \tag{30}$$

$$M U_{Sa} = R_S i_{Sa} M - R_R i_{Ra} L_S - \left(L_S L_R - M^2\right) \frac{d}{dt} i_{Ra} \tag{31}$$

$$L_R U_{Sb} = R_S i_{Sb} L_R - R_R i_{Rb} M + \left(L_S L_R - M^2\right) \frac{d}{dt} i_{Sb} \tag{32}$$

$$MU_{Sb} = R_S i_{Sb} M - R_R i_{Ra} L_S - \left(L_S L_R - M^2\right) \frac{d}{dt} i_{Rb} \tag{33}$$

$$J \frac{d\omega}{dt} = n_p M\left(i_{Sb}(i_{Ra}\cos(n_p\theta) - i_{Rb}\sin(n_p\theta)\right) - i_{Sa}\left(i_{Ra}\sin(n_p\theta) + i_{Rb}\cos(n_p\theta)\right) - f\omega - \tau_L \tag{34}$$

$$\frac{d\theta}{dt} = \omega \tag{35}$$

In order to simulate the Two phase induction motor dynamic model, the differential Equations (28 – 33) will be solved using the 4th order Runge-Kutta described above, with the aid of MATLAB.

XI. RESULTS AND DICUSSION

RESULTS

The simulation results were realized in MATLAB. See appendix for the simulation parameters used.By assuming that the leakage current and leakage inductances are constant with the saturation effects not taken into consideration, Figure 4.0 shows the behaviour of the induction motor speed as a function of time.

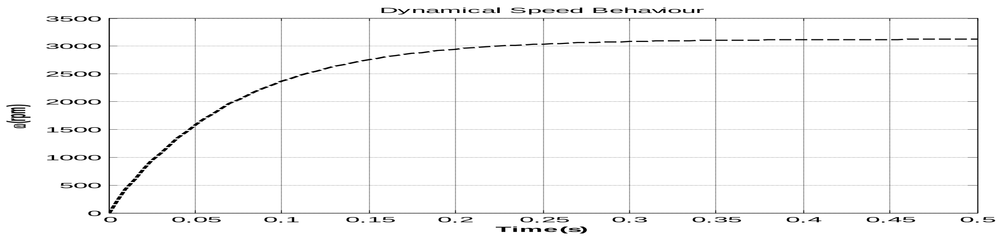

Fig 4.0 Dynamic Rotor Speed behavior

With the same conditions used above, Figure 4.1 and 4.2 show the behaviour of current in the stator phase a and b.

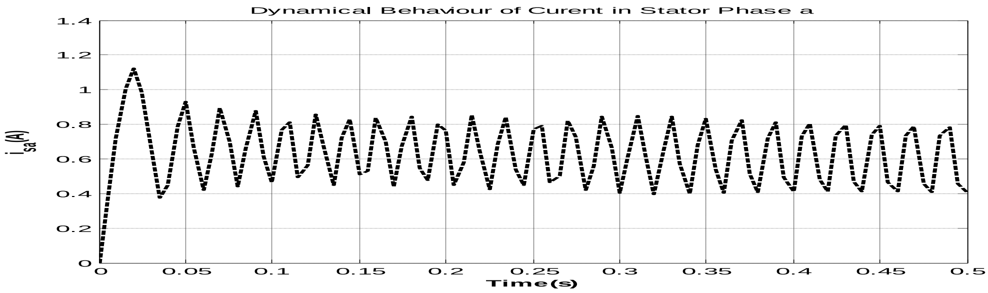

Fig 4.1 Dynamic Behaviour of Current in Stator phase a

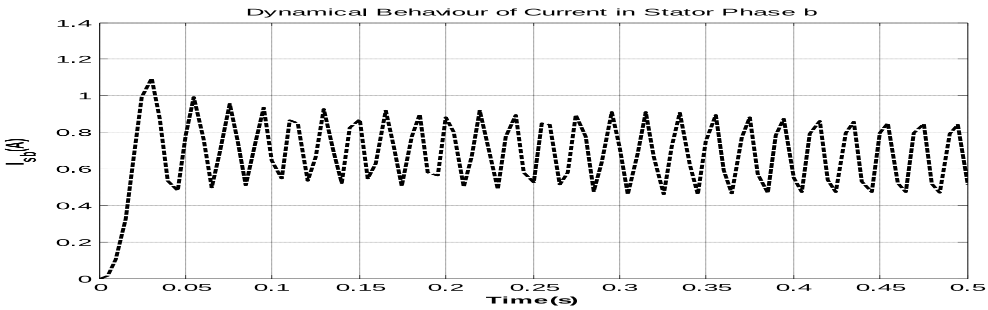

Figure 4.2 Dynamic Behaviour of Current in Stator phase b

With the same conditions mentioned above, Fig 4.3 and 4.4 show the behaviour of the induction motor current in rotor phase a and b

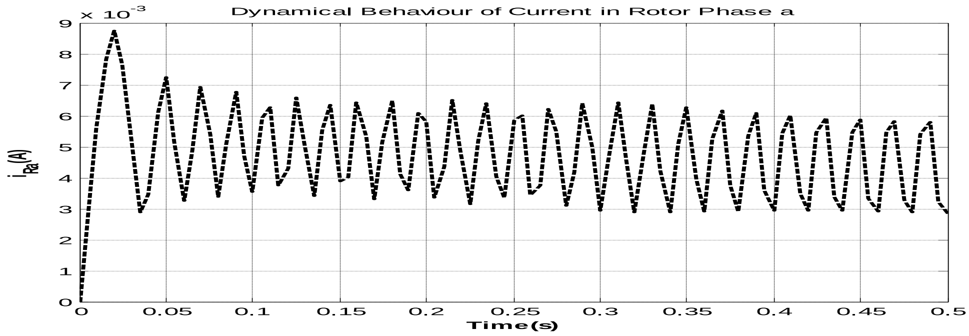

Fig 4.3 Dynamic Behaviour of Current in Rotor Phase a

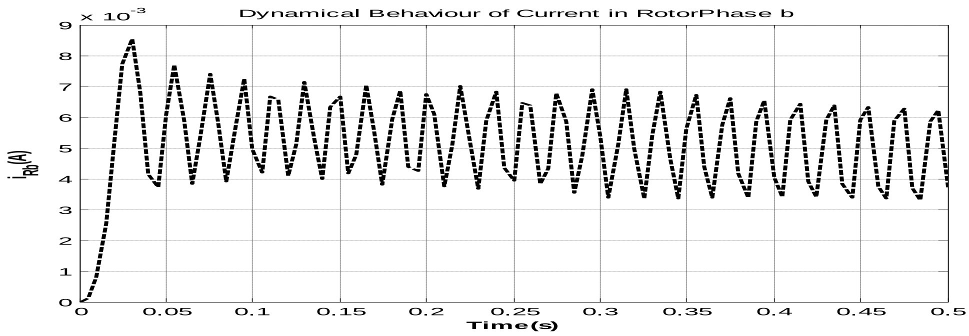

Fig 4.4 Dynamic Behaviour of Current in Rotor Phase b

With the conditions above adhere to, Fig 4.5 shows the behaviour of induction motor rotor position or angle as a function of time.

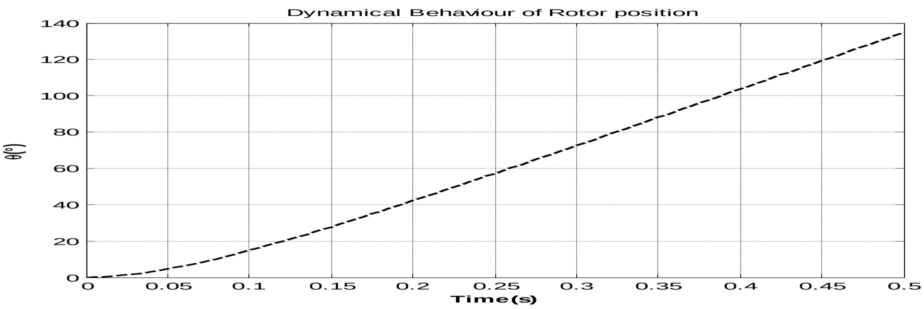

Fig 4.5 Dynamic Behaviour of Rotor Position

XII. DISCUSSION

From the results obtained from the simulation, considering figure 4.0, it was evident that keeping the electrical parameter (leakage current) constant and the non-consideration of the saturation effect have little or no effect on the rotor speed of the induction motor, when it is compared to when the leakage current is varied as shown earlier in the literature review.From figures 4.1 and 4.2, it was evident with the appearance of spiked shaped waveforms that the parameter variations affects the dynamic behaviour of the current in stator phases a and b.Considering figure 4.3 and 4.4, since the saturation effect was not taken in account, and leakage current kept constant the waveform is similar to that of stator current. Hence, it could be observed that the behaviour of the rotor currents in both phases is regular and periodic.

XIV. CONCLUSIONS

The dynamic modelling of two-phase induction motor has been done. The 4^{th} order Runge-Kutta method was used to solve this model was employed to solve the model. Some dynamic behaviour of two-phase induction motors was simulated. The simulation results have shown that electrical parameter variation and the neglect of saturation effect have considerable effects on the current in the rotor and stator phases. On the other hand, the rotor speed was not considerably affected when subjected to the same conditions. In conclusion, the two-phase induction motor does not give good dynamic and steady state response performance when subject to electrical parameter change as depicted in the results.

XV. CONTRIBUTIONS TO KNOWLEDGE

This research would help in small power applications, where two-phase induction motors are used in the manufacture of home appliances and automobile industries where fractional horse power(FHP) electric motor are widely used. The result would be of use electric machine designers and manufacturers.

XVI.RECOMMENDATIONS

The applications of two-phase induction motor are limited by its design and construction. For most industrial applications, the three-phase induction motors should be used instead of two-phase induction motor in as much power consideration allows it. The designers and modellers of electric machines should take cognizance of the relevance of parameter changes and saturation effect on the machines being produced.

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