Journal of Engineering Science and Technology
Vol. 9, No. 4 (2014) 477 - 489

A 3D FINITE ELEMENT ANALYSIS OF INCOMPRESSIBLE FLUID FLOW AND CONTAMINANT TRANSPORT THROUGH A POROUS LANDFILL

ADEGUN, I. K.[1,*], KOMOLAFE, O. D.[2],
AHMED KADHIM HUSSEIN[3], OYEKALE, J.O.[4]

[1,2]Department of Mechanical Engineering, Faculty of Engineering and Technology, University of Ilorin, Ilorin, Kwara State, Nigeria
[3]College of Engineering, Department of Mechanical Engineering, Babylon University, Babylon City, Hilla, Iraq
[4]Department of Mechanical Engineering, Faculty of Engineering and Technology, Ladoke Akintola University of Technology, Oyo state, Nigeria
*Corresponding Author: kadegun@unilorin.edu.ng

Abstract

The paper investigated the flow of incompressible fluid and contaminant transport through a Porous Landfill using a numerical technique. A three-dimensional finite element analysis technique was adopted for the solution. The problem was based on the Darcy's Law and the Advection-Dispersion equation. The solutions of the Darcy's and Advection-Dispersion equations were generated using Finite Element Analysis Software known as COMSOL Multi-physics. This simulation tool tracked the contaminant transport in the Landfill for 360 days at 10 days interval. It first modeled steady-state fluid flow by employing the Darcy's Law Application Mode and then followed up with a transient solute-transport simulation by employing the Solute-Transport Application Mode from the Earth Science Module of COMSOL. The solution results obtained from this model were found to be in close agreement with real-life data obtained at the 130- million ton Bukit Tagar Mega Sanitary Landfill site, Selangor near Kuala Lumpur, Malaysia. This showed that the model can effectively predict the trends in the distributions of pollutants from a Municipal Solid Waste Landfill into nearby land and water sources. The model is thus applicable to the issues of environmental protection and safety of groundwater.

Keywords: 3D-numerical, Simulation, Contaminant, Porous medium, Landfill, Ground water safety, Municipal waste management.

Nomenclatures

A	Area, m^2
C	Solute concentration, mg/m^3
C_o	Initial solute concentration, mg/m^3
D_x, D_y, D_z	Dispersion coefficients in x, y, z directions, m^2/day
k	Permeability, m^2
Q	Discharge of fluid, that is, depth of seepage per day, m/day
q	Darcy flux, m/day
q_n	Darcy flux at the defined boundary, m/day
t	Time, days
U	Velocity of the fluid, m/s
$\overline{U}_x$, $\overline{U}_y$, $\overline{U}_z$	Average fluid velocities in x, y, z directions, m/s
x, y, z	Space coordinate

Greek Symbols

∇	Gradient operator
φ	Piezometric head, flow potential, m
μ	Fluid absolute viscosity, N/m^{2}s
ρ	Fluid density, kg/m^3

1. Introduction

The search for new techniques for the exploitation of oil reservoir by petroleum engineers, as well as recent concern with groundwater pollution problems by hydrologists has provided a great stimulus to studies of hydrodynamics in porous media. Contamination of groundwater is an issue of major concern in residential areas in the vicinity of Landfills and waste disposal repositories as municipal water supplies often depend on the utilization of the groundwater resources [1-3].

Phenomena of transport in porous media are encountered in many engineering disciplines such as the flow of water in aquifers, the movement of moisture through and under engineering structures, transport of pollutants in aquifers and the propagation of stresses under foundations of structures, the movement of water and solutes in the root zone in the soil, heat and mass transport in packed-bed reactor columns and drying processes, and the flow of oil, water and gas in petroleum reservoirs [4, 5]. The theory of laminar flow through homogeneous porous media is based on a classical experiment originally performed by Henri Darcy the city engineer of Dijon [5].

Mehnert and Hensel [6] used an analytical and a numerical contaminant transport model to predict the transport of boron from coal combustion by-products Landfills. They found out that burial of a waste on the pit floor with a minimum of five feet soil cover will prevent plant uptake of elements. Badv [7] performed contaminant transport analysis for four selected solid waste Landfill designs using the computer code POLLUTE. Upon evaluation for the contamination of the underlying aquifer, using the drinking water standard for chlorine ion, he found out that a waste landfill design which included elements of a blanket type leachate collection layer and a compacted clay liner underneath the landfill has more certainty in controlling the contaminant transport. Mirbagheri [8] developed a mathematical and computer model for the transport and transformation of solute contaminants through a soil column from the surface to the groundwater. Wang et al. [9] proposed a numerical approach to model the flow in porous media using homogenization theory to generate a characteristic flow equation which is numerically solved using a penalty FEM schemes.

Polubarinnova [10] in his book discussed extensively theories that govern ground water while Krothe [11] studied fluid flow in porous rocks in the lemon landfill areas. A 3-D study of nuclear waste contaminants flow in porous medium using finite element was carried out by Ewing et al. [12]. In his research work, he found out that gravity has significant effect on contaminant transport. The same numerical tool of finite element was adopted by Whey et al. [13] to model analytically the fluid flow and transport in unsaturated zone. The concept to the application of finite element for the purpose of solving engineering problem was presented in the book of Burnett [14]. Olaniyan et al. [15] investigated surface water contamination by the landfill seepage from Maiduguri, Yola and Kaduna, Nigeria. They used US soil conservation service model in conjunction with the Streeter-Phelps dissolved oxygen balance equation and found out that the critical dissolved oxygen concentration could be as bad as 730 to 786 mg/litre and could occur within 1.98 to 2.17 days of first contact with the steam which will begin to show sign of recovery as early as the 25th day of first contact.

The few studies in Nigeria on the effect of seepage from landfill took care of the safety of surface water, not much attention was paid to ground water safety and appropriateness of the location for digging of well in the neighborhood of such a landfill; this gap in knowledge motivated the present authors to embark on this research work with the intention of determining the location that will be safe for sinking of drinkable well /borehole in the neighborhood of landfills and to advise appropriately the concern individuals and authorities.

2. Theory of Contaminant Transport in Porous Medium

2.1. Physical model

Figure 1 shows the physical model. The model adopted for this study is to take the porous Landfill as a cuboid from which a hemisphere had been removed. The choice of the coordinate is conveniently rectangular Cartesian with the length being x, the width y and the depth z. It is assumed that porous Landfill which was to be built of soil is a homogeneous and isotropic porous medium. The porous Landfill is non-deformable and stationary. Physical properties of the fluid, such as density and viscosity are constant, the fluid flow is assumed steady while the pollutant transport is assumed unsteady.

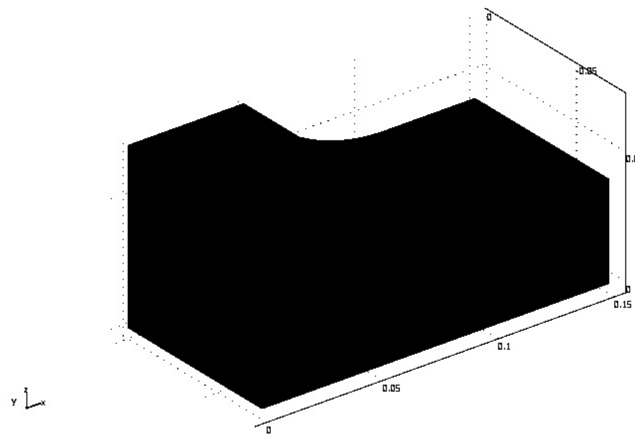

Fig. 1. Physical Model of the Landfill.

2.2. Formulation of the governing equations

In three dimensions, Darcy's law can be written as:

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2} = 0 \tag{1}$$

For contaminant transport in the Porous Landfill, the Advection-Dispersion equation (2) is as contained in the work of Frey et al. [16].

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - \overline{U}_x \frac{\partial C}{\partial x} - \overline{U}_y \frac{\partial C}{\partial y} - \overline{U}_z \frac{\partial C}{\partial z} \tag{2}$$

The Boundary conditions are:

- The initial condition:

  $C(x, y, z, t) = 0$ at time, $t = 0$, that is, there is no contamination initially in the soil.

- The boundary conditions:

  (a) $\frac{\partial C(x,y,z,t)}{\partial x} = 0$ for $x = \infty$, that is, the concentration gradient at a distant boundary does not change over time.

  $-D_x\left(\frac{\partial(x,t)}{\partial x} + \overline{U}_x C\right) = \overline{U}_x C_o$,

  (b) $-D_y\left(\frac{\partial(y,t)}{\partial x} + \overline{U}_y C\right) = \overline{U}_y C_o$, that is, the solute is added continuously

  $-D_z\left(\frac{\partial(z,t)}{\partial x} + \overline{U}_z C\right) = \overline{U}_z C_o$

  at the concentration C_o at the near boundary.

3. Solution Techniques

Numerical solutions of problems in fluid dynamics are usually formulated using one of three methods: finite-volume method, finite-difference method and finite-element method. In this work finite element was adopted because finite Element programmes such as COMSOL Multi-physics that can perform the task and that are user friendly are available commercially. The fluid flow and solute-transport equations are linked by the seepage velocity, $q = -k\,\nabla\phi$, which gives the specific flux of fluid across an infinitesimal surface representing both the solid and the pore spaces. COMSOL Multi-physics computes the Darcy velocity vector, which consists of *x, y, z* directional velocities denoted by $\overline{U}_x, \overline{U}_y$ and $\overline{U}_z$, respectively. First, it solves the Laplace equation to find the steady-state values of the hydraulic head. It then used Darcy's law to establish the seepage velocities which were included in the time-dependent solute transport equation. The numerical solutions to the Darcy's law and the Advection-dispersion equations were presented as generated by the Software COMSOL Multi-physics. The statistics of the mesh as generated by the software is given in Table 1.

Table 1. Mesh Statistics.

S/No.	Description	Value
1.	Number of degrees of freedom	6618
2.	Number of mesh points	532
3.	Number of elements	1839
4.	Tetrahedral mesh	1839
5.	Hexagonal mesh	0
6.	Prism mesh	0
7.	Number of boundary elements	814
8.	Triangular mesh	814
9.	Quadrilateral mesh	0
10.	Number of edge elements	90
11.	Number of vertex elements	12
12.	Minimum element quality	0.367
13.	Element volume ratio	0.041

4. Results and Discussion

The results are presented for seepage velocity field(m/s), Pressure gradient(Pa/m), lechate dispersive tensor(m^2/s), concentration gradient(kg/m^4), concentration (kg/$m^{3)}$, adventive flux (kg/m^2s) and dispersive flux (kg/m^2s) for arc length ranging from zero to one metre(0-1 m).

Figures 2 to 5 are the average of *x*, *y* and *z* components of the numerical values for the Darcy velocity (flux), pressure gradients, seepage velocity and Pressure head respectively. Observation of the plots shows that the porous medium are all in phase, the discharge or seepage from the landfill is lowest at an x-coordinate of 0.01m and highest at 0.1m. A porous medium has pores or void spaces which may be interconnected or non-interconnected, as flow of fluid can occur only through the interconnected pores of saturated soils [17]. This explains the saw- toothed nature of the graphs.

Figure 2 shows the average transport velocity of the leachate which was calculated by dividing the Darcy flux by the effective porosity of the porous medium. It should be noted however that the actual velocity of leachate in pores within the pore spaces remain unknown as it depends on the composition and structure of the soil and also the type of voids. For the particular application of this study the peak as well as lowest values of the pressure gradient and the pressure head within the porous system should be known.

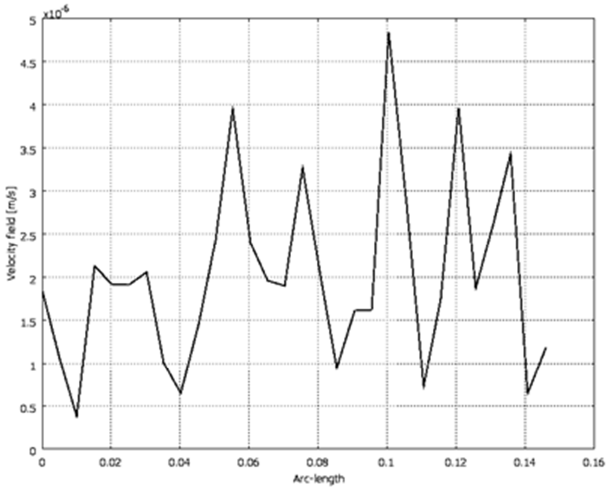

Fig. 2. Variation of Darcy Flux (Velocity) in the Landfill with Arc-Length.

Considering Figs. 3-5, it is observed that the three plots have maximum values of 3.5×10^{8} Pa/m, 4.8×10^{-6} m/m and 1.6×10^{-5} m/s of pressure gradient, pressure head gradient and velocity respectively at a distance of 0.1 m along the *x*-axis.

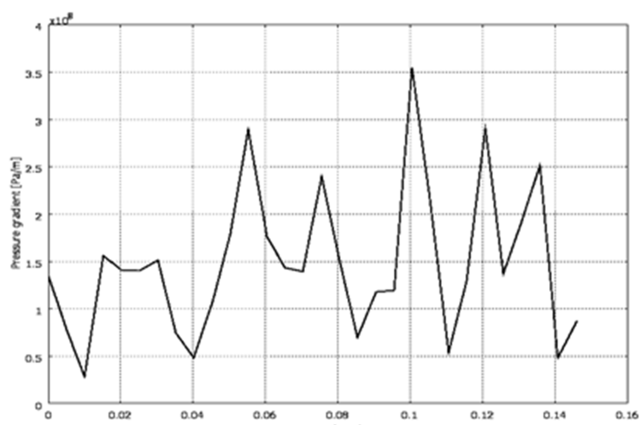

Fig. 3. Numerical Solution of Pressure Gradient across the Landfill Arc-Length.

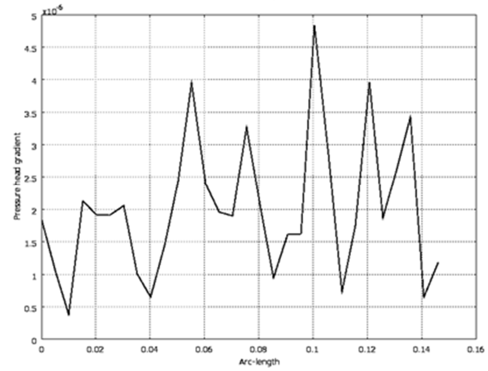

Fig. 4. Values of Pressure Head in the Landfill Arc-Length.

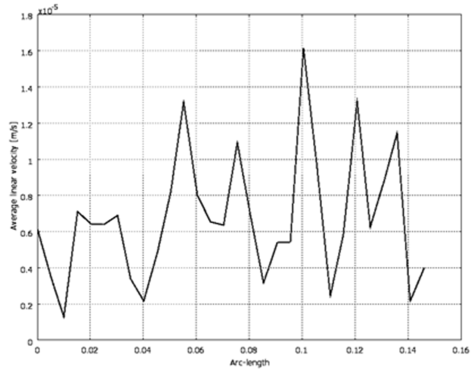

Fig. 5. Numerical Solution of Average Seepage along the Arc-Length in the Landfill.

Figure 6 shows that the velocity in the x-direction is highest at 0.1 m with a value of 4.5×10^{-6} m/s. The lowest value of -3.2×10^{-6} m/s occurs at 0.075 m. Figure 7 predicts the velocity in the y-direction and again the highest value occurs at 0.1 m. For the velocity in the z-direction, Fig. 8 shows that the lowest value is -1.8×10^{-6} m/s and occurs at 0.07 m.

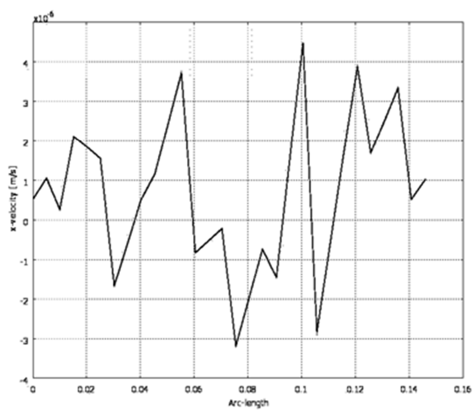

Fig. 6. Darcy Velocity in the x-direction in the Landfill.

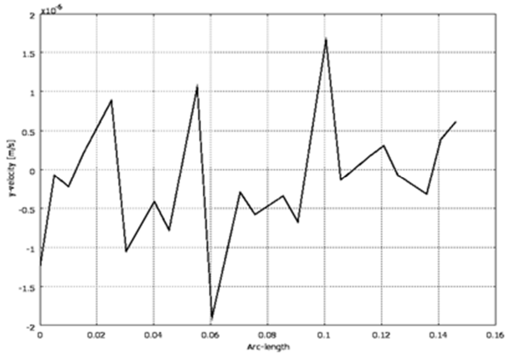

Fig. 7. Darcy Velocity in the *y*- Direction in the Landfill.

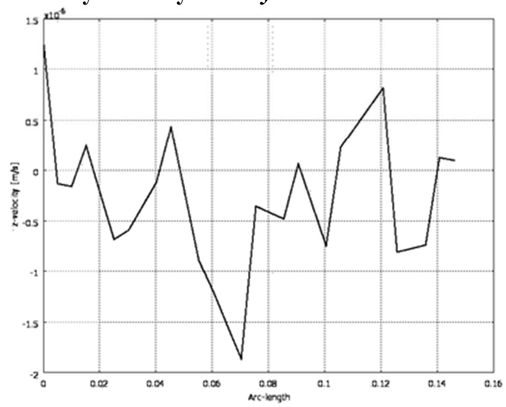

Fig. 8. Darcy Velocity in the *z*-Direction in the Landfill.

Figure 9 depicts how the pollutant concentration gradients (*x*-component) vary in the landfill at 10 days interval as the pollutant is continuously added at the near boundary at a constant concentration. Maximum value is 1.35×10^7 kg/m^4 at around 0.07 m at 360 days. Concentration gradients in the *y* and *z*-directions fall lowest at 0.03 m and 0.07 m respectively at 360 days (Figs. 10 and 11).

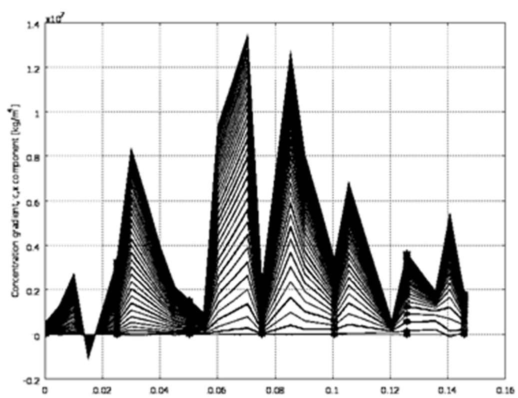

Fig. 9. Concentration Gradients in the *x*-Direction at 10 days' Interval in the Landfill.

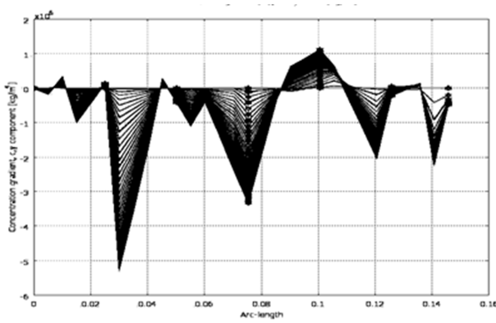

Fig. 10. Concentration Gradients in the *y*-Direction at 10 days' Interval in the Landfill.

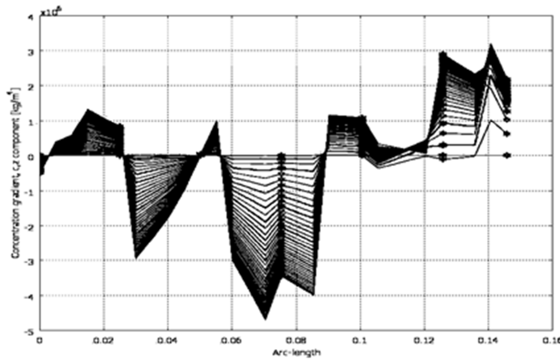

Fig. 11. Concentration Gradients in the *z*-Direction at 10 days' Interval in the Landfill.

Figure 12 is an isosurface of contaminant concentration within the porous medium. An isosurface plot displays a quantity as a colored set of surfaces on which the quantity has a constant value. This study shows that in a situation whereby there is constant injection of a pollutant into the landfill at a particular concentration C_o at source, then the pollutant concentration detectable within the porous system will be greatest at a distance of 0.14 m along the *x*-plane. In this study, the peak concentration was 8.67×10^5 kg/m^3 (Fig. 12).

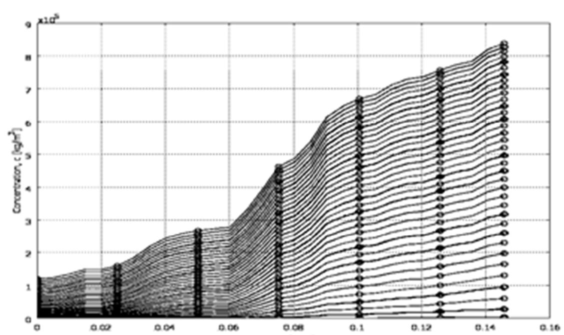

Fig. 12. Numerical Solute Concentration Values at 10 days' Interval with a Point Source on Tetrahedral Mesh.

Figure 13 shows that the concentration gradient (total or vector sum of the three-dimensional values of concentration gradients) is lowest at the point where the concentration is highest, (at around 0.14 m along the *x*-axis) and that the concentration gradient is highest at between 0.06 and 0.08 m.This can also be deduced from the results depicted in Fig. 14 which is the numerical values of solute concentration at 10 days' interval.

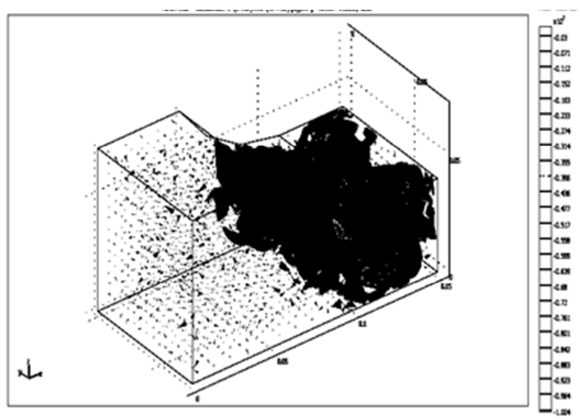

Fig. 13. Contaminant Spread in the Landfill at the End of 360 Days.

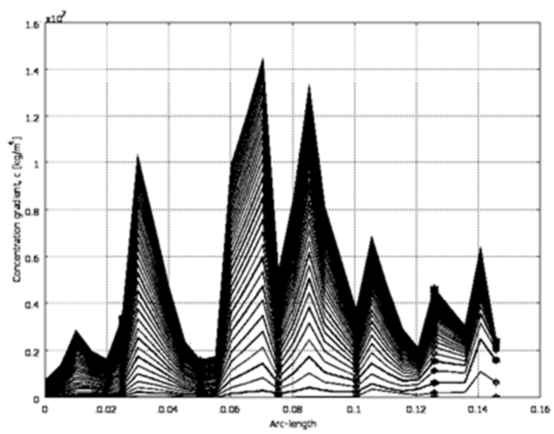

Fig. 14. Total Concentrations Gradients in the Landfill at 10 days' Interval.

Figure 15 established the fact that the pressure head is highest at the base and near the center of the landfill model.

Figure 16 shows that the greatest advective flux, that is, the spread of leachate which may contain toxic chemicals, occurs at around 0.1m along the *x*-coordinate. Also, as shown in Fig. 17, the peak value of dispersive flux is not far from where the peak advective flux is, it occurs at around 0.105 m along the *x*-coordinate.

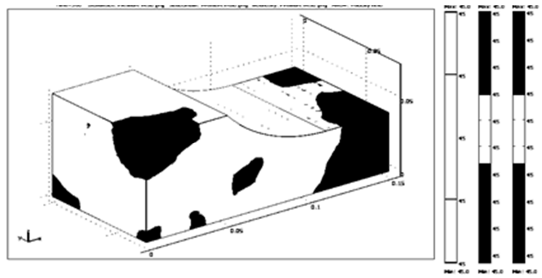

Fig. 15. Pressure Head Distribution (Input) in the Landfill.

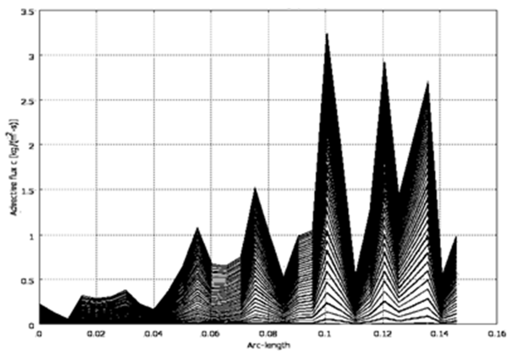

Fig. 16. Advective Flux at Points along the Landfill Arc-Length at 10 days' Interval.

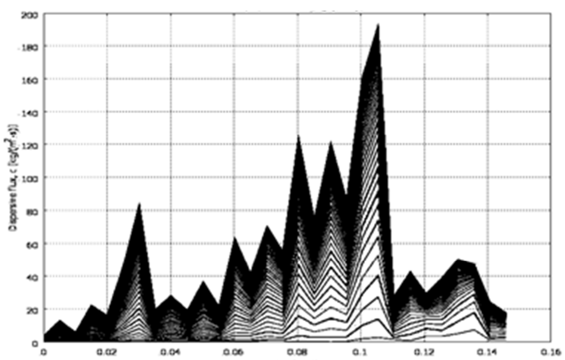

Fig. 17. Dispersive Flux at Points along the Landfill Arc-Length at10days' Interval.

- **Validation of results**

Figure 18 is a graphical representation of the actual data collected at the Bukit Tagar landfill in Malaysia [18], the trend of the behaviour of the daily leachate flow resembles the sinusoidal or saw-toothed nature of the graphs in Figs. 2-5 in the present work. This validated the numerical scheme used in this research as not being far from reality and that without the expense of building an actual landfill or constructing a real porous medium, the model of this work can be applied appropriately to get the needed results.

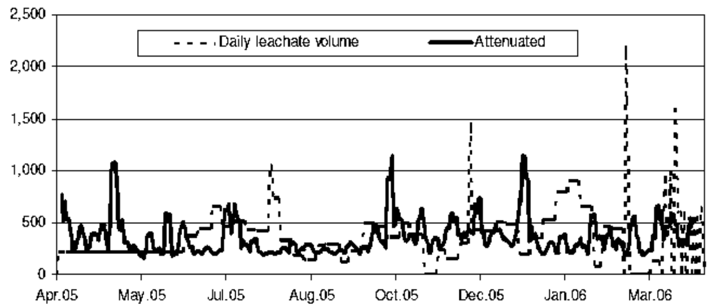

Fig. 18. Daily Leachate Flow Model Results for the Bukit Tagar Mega-Sanitary Landfill, Selangor, Malaysia. (Reproduced for Validation from [18])

5. Conclusions

A numerical solution has been presented for the flow of an incompressible fluid and contaminant transport in an isotropic porous medium using the finite element method. The domain of solution was a scale model of a Landfill which was integrated into Finite Element analysis software COMSOL Multi-physics. By comparing real-life results obtained at the Bukit Tagar sanitary landfill shown in Figure 4.5 with the results generated using this model, as shown in Figs. 2-5, this work produced results that are very close to what obtains in reality.

From the results obtained, one could conclude that the model can effectively predict the trends in the distribution of pollutants from a solid waste landfill into nearby land and groundwater sources. It is also concluded that wells in the neighborhood of such solid waste landfills are better protected from contamination if sunk behind the side of the landfill where the embankment is thickest rather than in front of the landfill. It is therefore recommended to both the government and individuals that wells or boreholes in the neighborhood of solid waste landfills should be sung behind the side of the landfill where the embankment is thickest.

References

1. Verruijt, A. (1988). *Theory of groundwater flow* (2^{nd} Ed.). University of Delft Press, Deft.
2. Bear, J. (1987). *Modelling groundwater flow and pollution.* Reidel Publishing Co., Dordrecht.
3. The report entitled "*Groundwater models, scientific and regulatory applications*'' (1990). Submitted by the Water Science and Technology

Board Committee on Groundwater Modelling Assessment to the National Academy Press, Washington D.C.

4. The United States Environmental Protection Agency, Landfills Fact Sheet, Municipal Solid Waste, available at http//www.epa.gov/epa.gov/epawaste/nonhaz/municipal/landfill.htm. Accessed on 24th February, 2010.
5. Bear, J.: and Bachmat, Y. (1990). *Introduction to modelling of transport phenomena in porous media*. Reidel Publishing Co., Dordrecht.
6. Mehnert, E.; and Hensel, B.R. (1996). Coal combustion by-products associated with coal mining. *Interactive Forum*, Southern Illinois University, Carbondale.
7. Badv, K. (2001). Evaluation of optimum landfill design by contaminant transport analysis. *Iranian Journal of Science and Technology*, 31(5), 535-545.
8. Mirbagheri, S.A. (2004). Modelling contaminant transport in soil column and groundwater pollution control. *International Journal of Environmental Science and Technology*, 1(2), 141-150.
9. Wang, J.G.; Leung C.F.; and Chow, Y.K. (2003). Numerical solution for flow. In Porous Media. *International Journal for Numerical and Analytical Methods in Geomechanics*, 27(2), 565-583.
10. Polubarinnova-Kochina, P.Y.A. (1952). *Theory of groundwater movement.* Translated from Russian by Roger De Wiest, J.M. (1962). Princeton University Press, Princeton.
11. Krothe, N.C. (2003). A study of fluid flow through porous rocks in the lemon lane landfill area. *RMZ Journal of Materials and Geo-environment*, 50(1), 177-180.
12. Ewing, R.E.; Wang, H.; Shapley, R.C.; and Celia, M.A. (2006). A three-dimensional finite element simulation for transport of nuclear waste contamination in porous media. *SIAM Journal of Numerical Analysis*, 26(6), 1513-1524.
13. Whey, B.; Tsai, F.; Tim-Hau, L.; Ching-Jen, C.; Shin-Jye, L.; and Chia-Chen, K. (2000). Finite analytic model for low and transport in unsaturated zone. *Journal of Engineering Mechanics*, 6(2), 471-479.
14. Burnett, D.S. (2009). *Finite element analysis: From concepts to applications*. Addison-Wesley Publishing Company, Ontario.
15. Olaniyan, I.O.; Alayande. A.W.; and Bamgboye, O. (2009). Predicting surface water contamination from the Kaduna, Yola and Maiduguri landfill sites. *Researcher*, 1(6), 63-68.
16. Frey, S.; Martins-Costa, M.L.; and Saldanha Da Gama, R.M. (2008). Petrov-Garlerkin approximation for advective-diffusive heat transfer in saturated porous media. *Latin American Applied Research*, 38(6), 169-178.
17. Scheidegger, A.E. (1972). *The physics of flow through porous media* (3rd Ed.). University of Toronto Press, Toronto.
18. Kortegast, A.P.; Eldridge, S.F.; Richards, B.A.; Young, S.; Chock, E.T.; Bryce, A.; Robinson, H.; and Carville, M. (2007). Leachate generation and treatment at the Bukit Tagar landfill, Malaysia. *Proceedings of the 11th International Waste Management and Landfill Symposium*, Sardinia, Italy.