

# Cost-Effective Design of Structural Elements through the Process of Structural Optimization

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**Abstract** In the design of structural elements, there is usually more than one feasible and equally safe solution, even though not all of them may be cost-effective. Obtaining the cost-effective design out of the numerous designs is tedious, cumbersome and time-consuming. This work present the result of optimization process of a structural element carried out by using the numerical features of the Excel software. In the optimizing process, cost effectiveness in terms of area reinforcement and cross section is required, but subject to the constraints of deflection and cracking imposed by the Standard in operation. The result shows that optimal design of any structural element can be achieved by following the proposed methodology. The method also has the added advantage of reuse once it is put in operation.

**Keywords** Constraints, Design, Objective, Optimization, Structure

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## 1. Introduction

Optimization techniques play an important role in structural design, the very purpose of which is to find the best ways so that a designer or a decision maker can derive a maximum benefit from the available resources [1]. In the design of structural elements, it is possible to obtain more than one feasible and safe solutions. But not all these designs may be cost-effective. But out of the possible design solutions of a structural element under a given loading conditions, only one of them will be cost effective. How to obtain this one design, called optimal design, that optimally satisfies cost effectiveness and performance is the goal of structural optimization. Structural optimization can be defined or explained in a number of ways. However, its principal objective is to find the best design out of many designs that satisfies a prescribed criterion. It can be explained as a process of maximizing or minimizing a desired objective function while satisfying the prevailing constraints. Another approach looks at it as a process of determining the optimal values of design variables that maximizes or minimizes an objective function, within the limits of imposed constraints. Structural optimization involves making decision that will result in maximum benefits from available resources. Many types of optimization methods are described by [2, 3, 4]. However, according to [5], structural optimization process and methodology adopted for a given problem is influenced by

many factors like: (i) type of structure, (ii) model of structure, (iii), dimensionality of structure, and (iv) choice of design variables. The type of structure being optimized affects the way the process is carried out. According to [6], four types of structure are identifiable. These are: Truss, Shell, Solids, and Composite materials. The Truss structures occur in all areas of engineering that involve structural mechanics. Shell structures: its properties of in-plane and out-of-plane deformation are frequently used in aerospace structural analysis and optimization. Solids structures play an important role in mechanical and geo-technical engineering [7]. And lastly, the composite structures are mostly encountered in aerospace and civil engineering. All these affect the way optimization is carried out. Also the models of a structure defined in terms of how the structure is idealized for the process of optimization affects optimization process. Engineers conceive a structure in the form in which it will be built, however, the analysis must be based on mathematical and skeletal model which approximates to the behavior of the structure in under service loads. In structural optimization process, the structural engineer will model the structure accurately in this mathematical sense. The optimization models should be such that it is close to Real-World problems. Dimensionality deals with whether it is 1-D, 2-D, 3-D structural configurations [2]. In practice, 3-D models are usually used, since it is considered to be true to the real-world material realities and problems. While the choice of design variables plays an important role in structural optimization. In this regard, variables can be discreet (that is, material choices) or continuous (that is, physical dimensions). Variables should not be too little or too many [1]. Using too few variables can limit the diversity of possible solutions, and thus sacrifice the optimality of the

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results. Also too many variable can lead to an overly complex models. Structural optimization problems can be classified into three: (i) sizing optimization, (ii) shape optimization, and (iii) topology optimization [3, 8, 9]. Sizing optimization problems uses trusses or grillage member cross-sectional areas, plate or shell component thickness as design variables. In sizing optimization problems, the shape and topology of the analysis domain is fixed. This will help the designer to see the effect of the design variables on the member sizing. Shape optimization involves finding the right shape which will optimally perform a given function, subject to certain constraints. The very important aspect of optimization problems is that the topology of the analysis domain is fixed. Topology optimization problem is also called generalized shape or layout optimization [2]. In this type of problems, the optimal boundary and connectivity, as well as the optimal size, shape, location, and number of other factors in an analysis domain are sought. Three fundamental modules are involved in most structural optimization process. These are: (i) design parameters, (ii) constraints (equality or /and inequality), and (iii) objective functions (criteria). The design parameters specify the geometry and topology of the structure, as well as the physical properties of the members. These may include: cross-sectional variables (like areas, sectional modulus, etc), parameters that defines the structural configurations, or the material properties. The restraints that must be satisfied for the design to be acceptable are termed as constraints. Examples of constrains on the performance of the structural systems, are usually on stress, or deflections, or buckling, or natural frequencies, or thickness. The objective function (also called merit function) is formed by the proper choice of design parameters. This function is either minimized or maximized or balance combination of these. For example: (i) If this function is cost or weight of structure, it is minimized, (ii) if this function is performance, or reliability, or other performance-based parameters (like energy requirement, thermal capacity, sound, insulation, etc.), it is maximized, and (iii) or their combinations. According [1], the structural optimization problems are usually mathematically posed in this format:

Minimize or Maximize

$$F = F(x_1, x_2, x_3, \dots, x_n) \quad (1)$$

Subject to:

$$\left. \begin{aligned} C_1 &= C_1(x_1, x_2, x_3, \dots, x_n) = 0 \\ C_2 &= C_2(x_1, x_2, x_3, \dots, x_n) = 0 \\ &\dots \dots \dots \\ C_n &= C_n(x_1, x_2, x_3, \dots, x_n) = 0 \end{aligned} \right\} \quad (2)$$

And

$$\left. \begin{aligned} \alpha_1 &= \alpha_1(x_1, x_2, x_3, \dots, x_n) \geq 0 \\ &\dots \dots \dots \\ \alpha_n &= \alpha_n(x_1, x_2, x_3, \dots, x_n) \geq 0 \end{aligned} \right\} \quad (3)$$

where:

F = objective function

$x_1, x_2, x_3, \dots$  = design variables

$C_1, C_2, \dots$  = equality constraints

$\alpha_1, \alpha_2, \dots$  = inequality constraints

The nature of the mathematical programming problem depends on the functional forms of objective functions and equality constraints. If these are linear functions of design variables, it is a linear programming problem. Otherwise, it is nonlinear programming problem. The manner in which the best decision on design is arrived at is the subject of structural optimization. Until the advent of microcomputer, finding an optimal structural design may be cumbersome and tedious. Numerical optimization methods have been applied to the optimal structural design of satellites, spacecrafts, aircraft fuselages and similar for the purpose of reducing the structural weight and satisfying the design requirements on structural properties such as improving the structural stiffness and strength, reducing the vibration levels, adjusting the natural frequencies and increasing the buckling loads [10, 11, 12, 13]. Earlier works by [14 – 18] involves the use of artificial intelligence search algorithms such as genetic algorithms, simulated annealing, threshold accepting, tabu search, ant colonies, etc. and [19] attempted a design of reinforced concrete using MATLAB. All these either require great computation efforts or may not be available or accessible especially in developing counties. But in all personal computers, MS Excel is available. Engineers can now avail themselves of the availability, accessibility and simplicity of this software to optimize their design in day to day design practice. This work demonstrate the use MS excel spreadsheet to optimize the design of structural elements.

## 2. Methodology

In order to demonstrate this procedure, a simply supported beam of length  $l$  (m), which is part of slab system of a building, loaded with a uniformly distributed load  $\omega$  KN/m that produced a moment of 150KN.m (Figure 1) is to be designed in accordance to Limit State design principles as specified in [20], so that it is cost effective in terms of reinforcement and cross section area of concrete without exceeding the deflection specified by [20].

This is a type of structural element that carries load normal to its axis. The loads are transmitted to the supports by bending action (under flexural stress) of the beam [21]. In the Limit State method of design, the usual approach is to design a reinforced concrete beam in such a way that the steel yields or fails first before the concrete [22, 23, 24]. Beam design in this manner is said to be “singly-reinforced”. The reinforcement is placed at the tension zone, usually at the bottom at midspan and at the top at the supports. A situation of reinforcement being placed at both compressive and tensile zones is called “doubly-reinforced”.

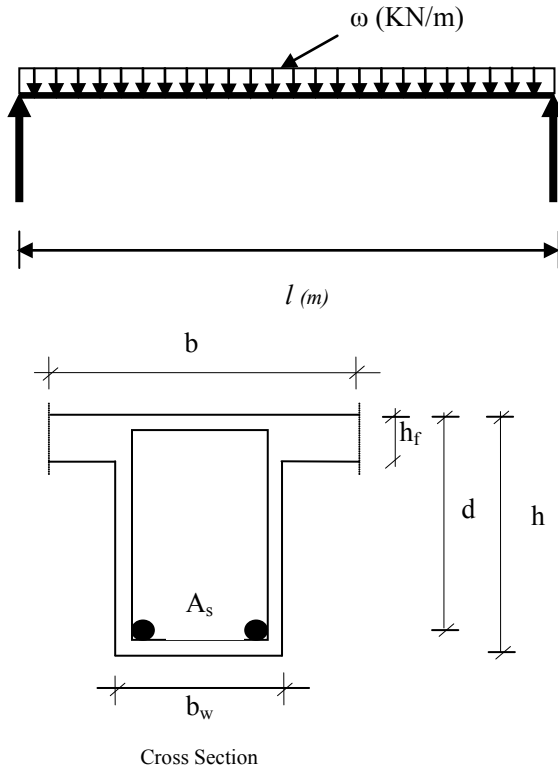


Figure 1. The beam configuration

Although there can be many feasible and acceptable designs of this simple beam, its design is to be optimized. Many methods of searching for the optimal design of structural members exist. However, depending on the environment where an engineer is expected to put into practice one of the fundamental principles guiding engineering practice: which is the use of what is available and accessible to solve problems at reasonable cost. Excel software is one of the commercial software that is present in all microcomputer. Its strength lies in its programmability to solve mathematical and iterative problems. For practicing engineers, it can serve as a powerful tool to obtain a design that is cost effective within a relatively short time with ease. Excel is a spread-sheet software program that is accessible and simple to use, is bound to enhance the performance of engineers, especially those involved in the design of projects. The data for the beam are: Moment at Ultimate = 150KN.m, Span = 6m,  $f_{cu} = 30\text{N/mm}^2$ ,  $f_y = 460\text{N/mm}^2$ ,  $b_f = 1400\text{mm}$ ,  $b_w = 225\text{mm}$ ,  $l/d = 20$ , cover to main reinforcement = 30mm. From the above, the following equations of [20] can be appropriately modified as follows:

$$k = M/(f_{cu}bd^2) = 150 \cdot 10^6 / (30bd^2) = 5000000/bd^2 \quad (4)$$

Also

$$z = 0.95d \quad (5)$$

And

$$A_s = M / (0.87 \cdot f_y \cdot z) = 150 \cdot 10^6 / (0.87 \cdot 250 \cdot z) = 689655/z \quad (6)$$

The objective functions are the minimum cost in terms of minimum area of reinforcement and minimum cross sectional area, and maximum performance in terms of cracks and deflection being kept within the limits set by [20].

The cross section of the beam is to be chosen subject to the following constraints: (i) that the deflection must meet the code requirements after making necessary modification. For this exercise, a modification factor of 1.3 is to be used by assuming a lightly reinforced section [22]. Which in this case of simply supported beam

$$d > l / (20 \cdot 1.3)$$

$$d > l / 26 \quad (7)$$

and (ii) no compression reinforcement is to be used, that is, it is to be designed as singly-reinforced. This procedure assumed the elementary knowledge of Excel Software. In the use of Excel software, the equations 4, 5, and 6 were built into Table 1. From this Table, the cross section area of the concrete that will result in minimum reinforcement and at the same time satisfying the deflection requirements can be generated through the “iterative” process of design. Column A to column G contains the geometric and the material data as defined in Figure 1. These are respectively,  $l$  = length of the beam (mm),  $b_w$  = the width of the beam (mm),  $h$  = total depth of the beam (mm),  $c$  = minimum cover to the reinforcement (mm),  $d$  = the effective depth (mm),  $f_{cu}$  = the compressive strength of concrete ( $\text{N/mm}^2$ ), and  $f_y$  = the tensile strength of the reinforcement ( $\text{N/mm}^2$ ). Column H to column L contained the mathematical parameters defined in equations 4 to 7. For example,  $k$  is as per equation 4,  $z$  is as per equation 5,  $A_s$  = the area of reinforcement ( $\text{mm}^2$ ) as per equation 6 and  $A_s$  (min) is the minimum area of reinforcement as stipulated by [20]. Column L is the value for the basic span/effective depth ratio for simply supported beam. Column M and N are where decisions are made. The first decision is to evaluate the parameter  $k$  (equation 4), and then determine if  $k >$  or  $< 0.153$ , and to display the conditions that exist. These conditions are: (i) If  $k < 0.153$ , to display “it is singly reinforced (SR)”, and (ii) If  $k > 0.153$ , to display “compression reinforcement required (CRR)”. The second decision is to determine (Column N) if the deflection conditions are satisfied. This is by evaluating and comparing the values of deflection “ $d$ ” in column E and the modified deflection in column M, and to display the results of the evaluation and comparison which is either: (i) Deflection OK, (ii) Deflection not OK. All these mathematical formulas were programmed into their respective columns. The procedure is to insert the values for “ $b_w$ ” and “ $h$ ” in columns B and C respectively, by using the FILL features in the MS Excel, and then observed the result of this action in other columns. The idea is to continue this operation until the designer is satisfied that he/she has obtained the desired solution.

Table 1. Results of Structural Optimization of A Beam Using MS Excel

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Iter.*	l (m)	b <sub>w</sub>	h	c	d	f <sub>cu</sub>	f <sub>y</sub>	k = M/f <sub>cu</sub> bd <sup>2</sup>	z = 0.95d	A <sub>s</sub> = M/0.87f <sub>y</sub> z	A <sub>s</sub> (min) = 0.0013bh	l/26	Test 1 = IF (I > 0.156, "CRR, ", "SR")	Test 2 = IF (M < F, "Deflection OK", "Deflection Not OK")
1	6000	300	550	30	520	20	250	0.092	494.000	758.730	214.500	230.77	SR	Deflection OK
2	6000	400	300	30	270	20	250	0.257	256.500	1461.258	156.000	230.77	CRR,	Deflection OK
3	6000	250	350	30	320	20	250	0.293	304.000	1232.936	113.750	230.77	CRR,	Deflection OK
4	6000	275	400	30	370	20	250	0.199	351.500	1066.323	143.000	230.77	CRR,	Deflection OK
5	6000	300	450	30	420	20	250	0.142	399.000	939.380	175.500	230.77	SR	Deflection OK
6	6000	300	650	30	620	20	250	0.065	589.000	636.354	253.500	230.77	SR	Deflection OK
7	6000	275	620	30	590	20	250	0.078	560.500	668.711	221.650	230.77	SR	Deflection OK
8	6000	250	600	30	570	20	250	0.092	541.500	692.175	195.000	230.77	SR	Deflection OK
9	6000	225	575	30	500	20	250	0.133	475.000	789.079	168.188	230.77	SR	Deflection OK
10	6000	200	560	30	530	20	250	0.133	503.500	744.414	145.600	230.77	SR	Deflection OK
11	6000	200	570	30	540	20	250	0.129	513.000	730.629	148.200	230.77	SR	Deflection OK
12	6000	225	580	30	550	20	250	0.110	522.500	717.345	169.650	230.77	SR	Deflection OK
13	6000	250	600	30	570	20	250	0.092	541.500	692.175	195.000	230.77	SR	Deflection OK
14	6000	275	620	30	590	20	250	0.078	560.500	668.711	221.650	230.77	SR	Deflection OK
15	6000	225	650	30	620	20	250	0.087	589.000	636.354	190.125	230.77	SR	Deflection OK
16	6000	250	675	30	645	20	250	0.072	612.750	611.689	219.375	230.77	SR	Deflection OK
17	6000	275	700	30	670	20	250	0.061	636.500	588.865	250.250	230.77	SR	Deflection OK
18	6000	300	690	30	660	20	250	0.057	627.000	597.787	269.100	230.77	SR	Deflection OK
19	6000	350	675	30	645	20	250	0.052	612.750	611.689	307.125	230.77	SR	Deflection OK
20	6000	375	650	30	620	20	250	0.052	589.000	636.354	316.875	230.77	SR	Deflection OK
21	6000	400	625	30	595	20	250	0.053	565.250	663.092	325.000	230.77	SR	Deflection OK
22	6000	360	600	30	570	20	250	0.064	541.500	692.175	280.800	230.77	SR	Deflection OK
23	6000	350	600	30	570	20	250	0.066	541.500	692.175	273.000	230.77	SR	Deflection OK
24	6000	320	640	30	610	20	250	0.063	579.500	646.786	266.240	230.77	SR	Deflection OK
25	6000	300	700	30	670	20	250	0.056	636.500	588.865	273.000	230.77	SR	Deflection OK
26	6000	200	725	30	695	20	250	0.078	660.250	567.683	188.500	230.77	SR	Deflection OK
27	6000	230	700	30	670	20	250	0.073	636.500	588.865	209.300	230.77	SR	Deflection OK
28	6000	250	750	30	720	20	250	0.058	684.000	547.972	243.750	230.77	SR	Deflection OK

\* Iter. = iterations

### 3. Results and Discussion

The results of the numerical process to find the most economic design subject to acceptable deflection and non-provision of compression reinforcement, using MS Excel are presented in Table 1. Twenty eight iterations were made. From the Table, it can be observed that all the combinations of “h” and “b<sub>w</sub>” if used will result in design that satisfies the deflection requirement of [15]. The other constraint is that the design must be such that the steel yields before the crushing of reinforcement. That is, the beam must be singly-reinforced, without compression reinforcement. Iterations 2 to 4 did not meet this requirement. But all others met it. Having identified the designs that satisfy the constraints of deflection and no compression reinforcement, the design with the least area of tensile reinforcement among them may be considered as the most cost-effective of the design. From the perspective of cost of reinforcement, consideration may be given to iterations where the areas of reinforcement are between 500 and 600 mm<sup>2</sup>. Iterations 17, 18, 25 – 28 fall within this category. The final choice may be influenced by other factors such as constructional convenience, but at least, the designer can form a reliable picture of how to eventually configure his/her design for cost effectiveness.

### 4. Conclusions

From the result presented, it can be seen that an optimal design of structural element with MS Excel can be carried out easily following the approach adopted. Although a simple beam is used here to demonstrate how a designer can optimize his/her design using what is available, without so much stress. The same principle can be used for a more complex structural member or structure, since a structure is formed by putting together small elements. The procedure also has the added advantage of reuse for subsequent designs just by making minor or no modifications.

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